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# AIR TECHNICAL INTELLIGENCE TRANSLATION

MAGNETIC STORMS AND SYSTEMS OF ELECTRIC CURRENTS  
(MAGNITNYE BURI I SISTEMY ELEKTRICHESKIKH TOKOV)

BY

N. P. BEN'KOVA

FROM

TRUDY NAUCHNO-ISSLEDOVATEL'SKOGO INSTITUTA ZEMNOGO MAGNETIZMA

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## NOTE

This work by N.P.Ben'kov is devoted to a study of magnetic storms and the electromagnetic processes responsible for them. It contains a survey of the literature on this topic, a classification of storms, a description of the morphology of the phenomenon, and a calculation of the extra-ionosphere, responsible for the regular parts of the magnetic disturbances. It also contains a description of individual storms and of related electric currents. One of the Chapters is devoted to the electric currents induced by the field of magnetic storms in the conducting layers of the earth. This work is of interest for specialists in geophysics, scientific workers, postgraduate students, students taking advanced courses, and specialists in the field of ionospheric physics.

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## INTRODUCTION

Magnetic storms, i.e., rapid random oscillations of the intensity vector of the geomagnetic field which from time to time disturb the normal march of the magnetic elements, constitute one of the most interesting geophysical phenomena. They were first discovered at the very dawn of the development of geomagnetic research, when the only magnetic instrument available was the magnetic needle, and have long attracted the attention of both navigators and scientists. The Arkhangel'sk seafarers, sailing on voyages in the basins of the White Sea and the North Arctic Ocean, noted unexpected and random fluctuations of the needle, frequently coinciding with auroral displays in the sky. "Our little mother deceives us when the North glows" \* is a well-known maritime proverb which runs back to the middle of the Eighteenth Century. At present, when not only magnetic, gyro and astro-compasses but also complex radio-navigation and radio control systems are used for marine and aerial navigation, when shortwave radio is the principal means of communication in times of peace and war, the study of magnetic storms has become of still greater practical interest, being a necessary element of the theory and application of ionospheric propagation of radio waves.

The theoretical significance of the study of magnetic storms is likewise very great and not primarily, for geomagnetism itself, in which the problem of the irregu-

\* The discovery of magnetic storms is usually attributed to Hiorter who, in 1741, discovered the irregular fluctuations of the magnetic needle. There is, however, reason (Bibl.30) to assume that they were known to Russian sailors in Northern waters.

lar variations occupies a particularly important place, but also for other divisions of geophysics, such as the physics of the upper layers of the atmosphere, the study of the aurora polaris, the cosmic rays and the earth currents. Since the electromagnetic processes of the earth's atmosphere are primarily due to solar radiations of all forms, there is an intimate relation between the departments of geophysics and of heliophysics. Magnetic storms, in particular, were the first geophysical phenomena for which a correlation with solar activity was discovered and which yielded abundant material from the solution of a number of problems related to solar radiation and behavior of the active regions of the solar envelopes.

The study of magnetic storms, the regularities in their course, and the electromagnetic processes causing them, constitute the subject matter of the present work.

#### Section 1. General Discussion of the Theories of Magnetic Storms

Despite the great efforts made by geophysicists of several generations in studying the morphology and nature of magnetic storms, many essential questions still remain controversial. This is explained, both by the complexity of the phenomenon which requires the attentive study of a large amount of empirical material for the clarification of any regularities at all, and its intimate connection with ionospheric physics and heliophysics. A quantitative theory of magnetic storms is given its necessary empirical base only when we know reliably the composition of the upper layers of the atmosphere, the velocity and laws of motion of air masses, the laws of radiation by the undisturbed solar surface, and by the active formations of the sun. The exceptionally rapid development of ionospheric and solar physics, which owes much to the work of Soviet scientists, allows us to expect that the combined efforts of geophysicists and astronomers will lead in the near future to a solution of these problems.

But even today, the basic stages of the theory of magnetic storms have already been marked out. As far back as 200 years ago, the hypothesis was postulated that magnetic storms are caused by minute particles of matter flying from the sun. The

data subsequently accumulated on the geographical distribution of magnetic activity, on its fluctuations with time, and on its correlation with solar activity, confirm this view, and the works of a number of geophysicists (Arrhenius, Angenheister, and mainly Stoermer, Birkeland, Chapman, and Alfven) laid the scientific foundation for the corpuscular theories of magnetic storms. The existence of a corpuscular radiation from the sun, proposed to explain magnetic storms and the aurora polaris and successfully used to solve a number of other problems, still remained a hypothesis until recent years. It was only in 1950-51 that measurements of the Doppler shift of the hydrogen lines in the spectra of the aurora confirmed the penetration of a stream of particles into the upper layers of the earth's atmosphere.

The modern corpuscular theories of storms are based on a chain of independent problems, beginning with the emission of the sun's geoeffective corpuscular radiation, the dynamics and electrodynamics of the corpuscular stream en route between the sun and the earth, and ending with the electromagnetic processes taking place on the earth's surface as a result of the interaction of the corpuscular stream with the permanent magnetic field of the earth and the earth's atmosphere. The construction of the system of electric currents, which constitutes the immediate cause of the fluctuation of the magnetic field during the time of a storm, occupies a position of considerable importance among the links of this chain. The mechanism of excitation of these currents is in many respects still obscure, and the very existence of the currents has not yet been confirmed by direct observations, as has been done for the currents responsible for the regular diurnal variations of the magnetic field \*. In its present phase, however, geophysics offers no other hypothesis of equal value to

\* Measurements of the magnetic field at great altitudes, by means of remote-reading magnetometers installed in rockets, have shown the existence of a discontinuity in the variation of the field at the height of the E layer of the ionosphere. This discontinuity confirmed the existence of electric currents at the level of 90-105 km, which might, judging by their intensity and diurnal variation, explain the quiet diurnal variations ( $S_q$ -variations) of the magnetic field. The hypothesis of currents flowing in the upper layers of the atmosphere was postulated by B. Stewart long before the experimental detection of the conducting properties of the ionosphere by radio methods.

explain the field of geomagnetic variations, without assuming electric currents external to the earth's surface. According to the Chapman-Ferraro and Alfven corpuscular theories of magnetic storms, which are widely recognized today, the excitation of these currents in the upper layers of the earth's atmosphere, and beyond it, as a result of the action of a stream of solar corpuscles is a physical reality. Other authors consider the field of magnetic disturbance to be the direct field of flying, charged corpuscles of solar origin. This view evokes two remarks. First, motion of electric charges at high velocity is identical with an electric conduction current, and thus, this view cannot be opposed to the current theory of magnetic storms; second, considerably more theoretical and empirical arguments can be opposed to it than can be cited in its favor. The ultraviolet theory of magnetic storms\*, according to which the prime causes of magnetic disturbances are outbursts of wave radiation, likewise reduces the effect of the disturbance of the upper layers of the atmosphere to the formation of certain additional current systems.

For an explanation of the  $S_q$ -variations, a diamagnetic theory had been advanced previously. According to this theory, the upper conducting layers of the atmosphere, due to the rotation of charged particles about the lines of force of the permanent magnetic field are, as it were, magnetized. The magnetic field of these layers, superimposed on the permanent field, forms the diurnal fluctuations of the magnetic elements. As a result of the work by Tamm (Bibl.31) and others, this hypothesis has been recognized as unfounded. However, even if the possible existence of a diamagnetic effect were not open to fundamental objections, it would be quite impossible to use the hypothesis for explaining such complex fluctuations as are observed during magnetic storms. For any view of the mechanism of action of a geoeffective solar stream on the magnetic field of the earth, it seems that the immediate causes of the fluctuations of the magnetic field during a disturbance are electric

\* This theory developed by Meyers and Hulbert, is at present time the object of violent criticism.

currents\* excited in some manner outside the earth's surface itself, and by induction in its depths. Thus an explanation of the morphology and nature of these currents is of fundamental significance for the development of the theories of magnetic storms. The calculation and discussion of the electric currents responsible for magnetic storms is the primary purpose of the present work.

## Section 2. The Electric Current Systems of Magnetic Storms

The problem of finding the density and configuration of the currents from the magnetic field observed on the earth's surface is, in the general case, a many-valued one. However, by calling on supplementary information from other fields of geophysics, the number of possible solutions is narrowed, leaving only one or two parameters indeterminate. For example, the very plausible hypothesis was formulated

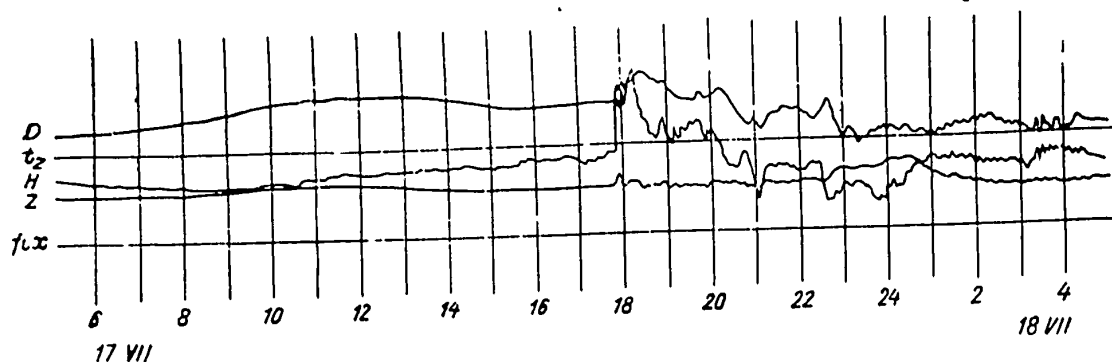


Fig.1 - Storm of 17 July 1947

Magnetograms of Krasnaya Pakhra Observatory

that the currents, responsible for the quiet diurnal variations, flow in a spherical layer concentric with the earth's surface. This allowed calculation of the system of currents of the  $S_q$  variations by means of spherical analysis and served as a basis for the formulation of the physical theories of the  $S_q$ . Only one parameter, the height of the current layer, still remained indeterminate in the calculations of the  $S_q$ -layer. Its value was found by consideration of experimental data on the ioniz-

\* Not necessarily conduction currents. It is possible that the disturbances in the polar region are connected with a peculiar type of discharge currents.

0. ation of the D and E layers of the ionosphere.

2 The situation with respect to questions of the construction of the current.  
4 systems responsible for the field of perturbation is considerably less favorable.  
6 In spite of the large number of papers devoted to this subject, it has not yet been  
8 definitively solved. The main reason for this is the above-mentioned complexity of  
10 the fluctuations of the magnetic elements during a storm. If the calm (quiet) diurnal  
12 variations are so regular (cf. left part of Fig.1) that a simple averaging of the  
14 data for a few days in a month is sufficient to determine the law of variation of the  
16 magnetic elements, magnetic storms (as will be seen from the right side of the same  
18 figure) belong to those very capricious and at first glance completely random phen-  
20 omena which are so abundant in geophysics. Magnetic storms are characterized not  
22 only by complexity in the fluctuations of the vector of the magnetic field with time  
24 (rapid fluctuations of various amplitudes and frequently of utterly irregular form,  
26 follow each other without apparent regularity). The distribution of the vectors of  
28 the disturbing force in space is also extremely complex. The form and amplitude of  
30 the oscillations at different stations, particularly those located in different lat-  
32 itudes, often bear little similarity to each other (Fig.2). The rough qualitative  
34 characteristics of the field of magnetic storms (the disturbance is greater in high  
36 than in low latitudes, greater in the evening than in the morning, etc.) have long  
38 been known. However, in order to study with more rigor the morphology of the field,  
40 by its spatial and time variations, the accumulation of a large amount of empirical  
42 material was necessary, with long series of observatory data at a large number of  
44 geographical points. While Schuster disposed of the annual data of seven observa-  
46 tories in his calculation of the potential of the quiet diurnal variations, which  
allowed him to get an idea of the system of currents that well represents the mean  
features of the field, the workup of materials from 22 observatories to a few years  
permitted Chapman to find only the general outlines of the morphology of the storm  
field.

The second difficulty produced by the complex structure of the storm field in studying the causative electric currents, is the need for a special mathematical apparatus suitable for an analytical representation of the field and for the calculation of the current function. Spherical analysis, which is successfully used to represent the permanent field and the quiet diurnal variations, has permitted solutions of a number of fundamental problems of the structure of these fields, but it is practically useless for the investigation of fields with a complex geographical distribution.

All attempts made until now to construct a system of electric currents with fields equivalent to the fields of magnetic storms, were based on modest empirical material and were calculated by an approximate method (Chapman), or else were based on data relating to only a limited part of the earth's surface (for instance, a few polar stations) and were calculated under very narrow a priori assumptions (for instance, the postulate advanced by Birkeland, Gnevyshev, and others as to linearity of the current). As a result, these systems of electric currents do not represent (with the accuracy that is desirable for theoretical and practical problems) the geographic distribution and time regularities of the field of magnetic storms. Moreover, they have been constructed without proper division of the field observed on the earth's surface into the parts of external and internal origin, without investigating the question of potential, and without considering a number of other questions whose solution could be obtained only by means of the analytic representation of the field.

Most of the known current systems, and in particular the system of Chapman, which is cited in all manuals and textbooks on terrestrial magnetism, are average systems, equivalent to an average magnetic storm. The literature contains only few works devoted to the study of electric currents of individual magnetic storms and to the relations between the average and individual pictures. It follows from this that there is very great need for a new construction of the current systems of magnetic storms, based on the most complete possible empirical material and performed

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by analytic methods.

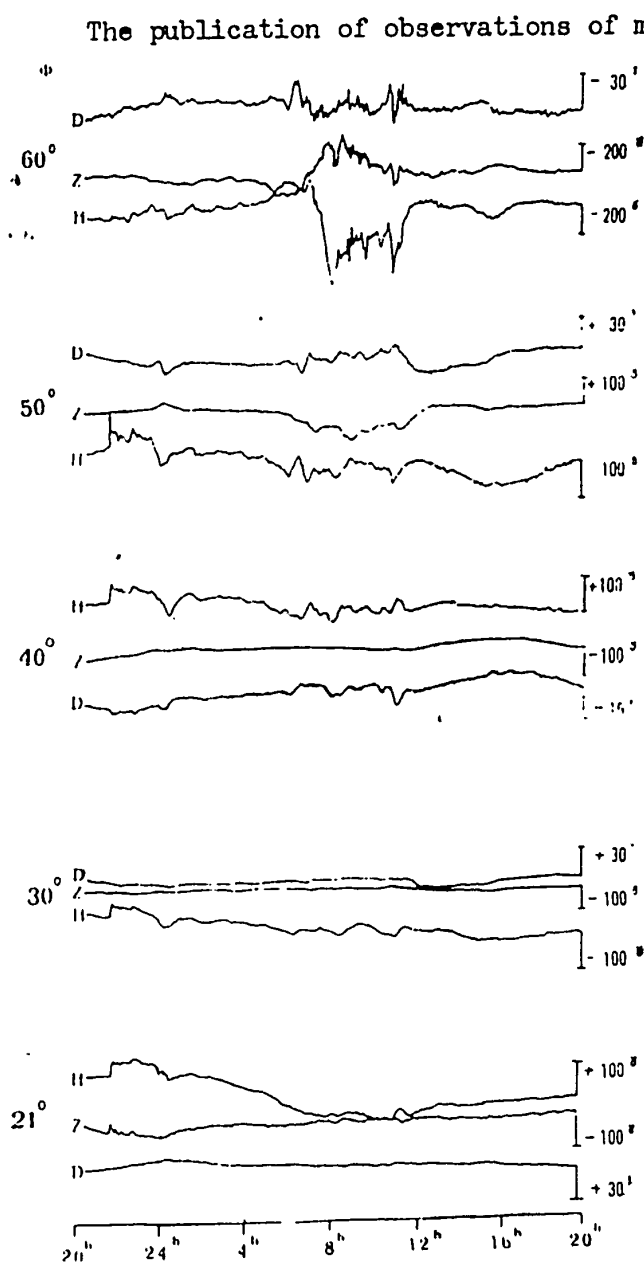


Fig.2 - Storm of 8 April 1947

Magnetograms of Observatories:

Sitka (60°), Tucson (50°),  
Cheltenham (40°), San Juan (30°),  
and Honolulu (21°)

The publication of observations of magnetic observatories during the Second International Polar Year (1932/33), was completed in the 1940's. During this year, over 60 observatories were in action. The publication of the observations of a number of Arctic observatories for later years, places a rather broad empirical material at our disposition today. The use of this material, particularly abundant for the high latitudes of the Northern Hemisphere, and the application of new analytic methods, has enabled me to construct systems of electric currents that are more reliable than those heretofore known. The discussion of the electric currents so obtained, from the viewpoint of modern ideas on the morphology of the ionosphere on a disturbed day, helped to explain the parameters of these currents and to formulate certain conclusions on the mechanism of their excitation. A consideration of the magnetic field on individual days made it possible to follow the development of the electric current systems of individual storms, and it was found that the current systems of individual storms may be regarded the re-

sult of fluctuations of an average system. The use of analytic methods made it pos-



sible to divide the field of disturbance into parts of external and internal origin, and, on the basis of these parts, to judge the electromagnetic parameters of the interior of the earth.

### Section 3. Content of this Report

It follows from the objects of this report, given in Section 2, that it includes two parts, a geophysical part comprising a study of the morphology of magnetic storms and of the disturbed ionosphere, and a mathematical part giving the development of practical methods of calculating the electric currents from the observed distribution of the magnetic field, to satisfy the specific requirements of our problem. The geophysical part covers the following points:

1. Classification of magnetic disturbances and separation of the perturbation field into individual parts. I consider that two groups of storms must be distinguished: world (M) and polar (P). The field of a worldwide storm, as stated by Chapman, is made up of three parts: an aperiodic part or, as it has been customarily called in all the earlier literature on geomagnetism, "the stormtime variations" ( $D_{st}$ ), the disturbed diurnal variations ( $S_D$ ), and the irregular part ( $D_i$ ). However, the worldwide storms are always accompanied by a series of superimposed polar disturbances. The subdivision of the field of a worldwide storm must therefore be made by means of a four-term equation

$$D_{st} + S_D + D_i + P.$$

The methods of calculating the various parts of the field are described while Chapter II is devoted to the exposition of these questions.

2. Chapter III is devoted to a description of the geographical distribution of the field of  $D_{st}$ . The same Chapter gives the calculation results for the potential and currents of the field of  $D_{st}$ . The comparative simplicity of the field allowed us to use the method of spherical analysis. Two alternate systems of currents were calculated: the ionospheric layer of current, and an equatorial extra-ionospheric current ring. The ratio between the external and internal parts of the field was

obtained in good agreement with analogous data of other investigators.

3. The regularities of the  $S_D$ -variations has been considered: the dependence on geomagnetic coordinates, the role of Universal Time, the features of the distribution of  $S_D$  on the polar cap, and the longitudinal members. The external system of currents of the  $S_D$  variation was calculated by the method of surface integrals and compared with the Chapman system. The ratio of the external ( $V_e$ ) to the internal ( $V_i$ ) parts of the potential is discussed. We find that the ratio  $\frac{V_e}{V_i}$  depends on the latitude and that its mean value is 0.89.

The above-enumerated questions are discussed in Chapter V.

4. A current system of an idealized polar storm (Chapter VI) is discussed, constructed from data of Silsbee and Vestine by expansion of the storm field into a series of Bessel functions. A resemblance of this system to the system of currents of the  $S_D$  variations was found.

5. The seasonal and 11-year fluctuations of the currents of the  $D_{st}$ - and  $S_D$ -variations are described (Chapter VII).

The current systems for individual seasons and years were calculated by approximate methods. It was found that the intensity of the  $D_{st}$ -current has cyclic fluctuations resembling the fluctuations of solar activity. The seasonal march of the  $D_{st}$  current has two waves (one annual and the other semiannual) and is completely explained from the viewpoint of the corpuscular theory of storms. The seasonal and 11-year fluctuations of the current systems of  $S$  are much more complex. The material presented by a number of observatories has shown that, during the course of the 11-year cycle and during the course of the year, both the intensities of the current eddies and the position of the auroral zone vary. The intensities of the currents in the middle and high latitudes obeys different regularities.

6. The  $D_{st}$ - and  $S_D$ -variation of the density of ionization of the  $F_2$  layer of the ionosphere are discussed. It was found that the  $D_{st}$ -variations in the ionization of the  $F_2$  layer cannot, either from their geographical distribution or from their

absolute value, be responsible for the ionospheric system of electric currents required for explaining the  $D_{st}$ -variations of the magnetic field. On this basis, the conclusion is drawn that the most probable cause of  $D_{st}$ -variations is an equatorial ring with a radius of 3-4 earth radii. A comparison of the  $S_D$ -currents may be explained, both in intensity and in form, under the assumption of a drift of the charged particles of the  $F_2$  layer under the action of the earth's permanent magnetic field and of its gravitational field. Chapter VIII is devoted to an exposition of these questions.

7. The electric systems of currents of individual magnetic currents are calculated in Chapter IX. It is found that in all cases the currents, at a given instant of time, may be represented as the result of the superimposition of typical systems of  $D_{st}$ -,  $S_D$ -, and P-currents. However, the intensities and configurations of the  $D_{st}$ -,  $S_D$ -, and P-currents vary within wide limits from case to case.

8. The external (principal) part of the field of magnetic disturbances induces, secondary currents in the inner conducting parts of the earth, which in turn influence the magnetic field observed on the earth's surface. The separation of the potential of the field of  $D_{st}$  and the potential of the P-storms into an external and an internal part made it possible to calculate the conductivity of the deep parts of the earth and the thickness of the upper nonconducting layer. The calculations were made under three assumptions: 1) the conductivity of the deep parts of the earth is constant; 2) the conductivity increases with depth; and 3) the currents induced in the oceans and wet soil are allowed for. For the estimate of conductivity we use not only the data on the P-storms and the first harmonic of the  $D_{st}$ -field, but also the data on the  $S_q$ -variations. The results so obtained on the variation of conductivity with depth differ somewhat from those of previous authors and are in good agreement with modern ideas on the internal structure of the earth, based on seismic data. The division of the field for the harmonic  $P_3$  of the  $D_{st}$ -variations cannot be

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explained within the scope of the Chapman-Price induction theory. Chapter X is devoted to these questions.

The mathematical part of the work includes the following factors:

1. The method of surface integrals proposed by Vestine in 1941 was used for calculating the external and internal potentials of the  $S_D$ -field. This method, which is used for the first time in geomagnetism, required the development of practical methods of processing the material and of a technique of computation.

2. The author of the present work has proposed a method of calculating the current function on a sphere with a radius of  $a$ , if the potential observed on the surface of a sphere  $R (R < a)$  is assigned in numerical or graphical form. The method is based on finding the current function for regions internal with respect to the sphere  $R$ , and on its extrapolation to outer space. The finding of the current function from the known potential on the sphere leads to the solution of the inner Dirichlet problem by the aid of a Fredholm equation of the second order. Practical calculation methods were worked out. The method is applied to a calculation of the currents of  $S_D$ . The questions connected with the integral method of analysis are discussed in Chapter IV.

The principal conclusions from the work are collected in the Conclusion.

Chapter I is devoted to a survey of the literature. Since this work is primarily devoted to questions of the morphology of the perturbation field and of the construction of the electric currents equivalent to it, out of the wide and varied literature on magnetic disturbances only studies devoted to the solution of these very questions are mentioned in the survey. Works devoted to other divisions of the theory of magnetic storms, to descriptions of individual phenomena, or to statistics of magnetic activity are not considered in the survey.

The equations are separately numbered in each Chapter. In referring to an equation given in the same Chapter, only its number is stated. In referring to an equation from a different Chapter, its number and the Chapter number are given.

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## CHAPTER I

## SURVEY OF THE LITERATURE

Section 1. Basic Properties of Magnetic Storms. The Works of Birkeland

The magnetic field of the earth is rarely completely quiet. Very often, the smooth march of the magnetic elements, due to the quiet periodic variations (solar-diurnal,  $S_q$ ; lunar-diurnal,  $L$ ; annual,  $A$ ) is disturbed by irregular fluctuations of varied form and amplitude. Any deviations of the magnetic field from the normal march are called disturbances. Some of them are so small (tenths and hundredths of a gamma) as to be detected only by special high-precision instruments (Bibl.16). The strongest disturbances, expressed in large and sharp fluctuations of the magnetic elements and lasting from several hours to several days, are called magnetic storms. Storms are observed simultaneously either over the entire earth or, at least, in the high latitudes. The amplitudes of fluctuation of the elements during extremely strong storms exceeds 1,000  $\gamma$  in the middle latitudes and 2,000-3,000  $\gamma$  in the high latitudes. During the time of a medium (moderate) storm, the fluctuations are of the order of 200-400 to 500-1,000  $\gamma$  depending on the latitude. The rate of variation of the elements likewise fluctuates over a wide range, sometimes exceeding a few tens of gammas a second. Occasionally, very slow and smooth variations of the elements are observed (especially in the low latitudes, in the Z-component). The fluctuations of the magnetic elements during a storm are so diverse that, during the entire period over which the observatories have been recording the magnetic elements, i.e., for over 100 years, no two identical storms can be found.

Despite such randomness of fluctuations, statistical regularities obeyed by magnetic storms have long been known. These regularities are as follows: The intensity of storms (characterized by the frequency and amplitude of the fluctuations, the mobility of the curves, and the magnitude of the deviation from the normal values) depends on the latitude. It reaches its maximum values in the high latitudes, in the zone of maximum visibility of the aurora; as the pole is approached, the degree of disturbance again decreases. The number and intensity of the storms has a seasonal march with maxima at the epoch of the equinoxes, and also has an 11-year cycle. The maxima of the magnetic cycle lag 1-2 years behind the maxima of the solar cycles. There is a correlation between individual magnetic storms and the manifestations of solar activity: sunspots, flares, eruptions. This correlation is of a statistical nature for the weak and moderate storms. The strong storms, as a rule, are uniquely related to solar phenomena. Tendencies to a repetition of storms after a synodical revolution of the sun and to a lag of storms behind the passage of an active region across the central meridian, have been noted. Finally, the distribution of the intensity of a storm during the course of the day, the "diurnal march of magnetic activity", has been found.

An extensive section of the literature has been devoted to these regularities, and served, as already stated, as the basis for the development of the corpuscular theories of magnetic storms. Considerably fewer papers have been devoted to the study of the structure of the field of the storm field itself. A.Schmidt and van Bemmelen (Bibl.40) were among the first investigators who attempted to find the regularities obeyed by the storm field. According to them, the vector of the disturbance systematically varies its direction during the course of the storm, and the "eddies" into which the storm is divided are displaced along the earth's surface. Without taking up this idea of the storm in detail, based as it was on the erroneous assumption that storms are local and of terrestrial origin, let us turn to an exposition of the memoirs of Birkeland (Bibl.38), which have not lost their significance even

today. Having set himself the problem of studying the distribution of the vector of disturbance over the earth's surface and of explaining the origin of the storms, Birkeland commenced his investigations by accumulating observational data. He understood the particular importance of high-latitude observations and organized two special expeditions, in 1899/1900 and in 1902/03, during which a special system of temporary stations, provided with apparatus of the same type and operating under a common program, was used. In working up this material subsequently, Birkeland compiled, for several storms, maps of the geographical distribution of the vector of disturbance for successive most characteristic instants of time. Birkeland defines the vector of disturbance as follows:  $F_d = F - F_n$ , where  $F$  denotes the observed value of the magnetic field and  $F_n$  the normal undisturbed value. The construction of these "synoptic" maps showed Birkeland that, despite the apparent randomness of the fluctuations of the magnetic elements, a certain systematic character is manifest in the distribution of  $F_d$ . The vectors at closely adjacent stations are almost parallel; a definite relation exists between the vectors and the longitude of the station and, in particular, the latitude. Birkeland divided the listed magnetic storms, about 30 cases in all, into five types. Type 1, the most frequent, is characterized by the almost everywhere negative horizontal component of the vector  $F_d$ . The maximum magnitude of the vector is reached in the polar zone, declines sharply in the middle latitudes, and again increases somewhat in the equatorial belt. Storms of this type were called negative equatorial storms by Birkeland. Type 2, positive equatorial storms, are storms with a positive horizontal component of  $F_d$ ; the least disturbance embraces all latitudes, but its value is usually much weaker than the disturbance of negative storms. This type was rarely observed. Type 3 and 4 are positive and negative polar storms and are characterized by the fact that the vector of disturbance reaches high values only in the high latitudes, while the magnetic field of middle and low latitudes remains in fact almost undisturbed. Type 5, the cyclo-median storms, of small value, reach their greatest development on the daylight side of the

earth in low latitudes. As Chapman later pointed out, the cases of disturbances classified by Birkeland as Type 5 would be more correctly included among the bay disturbances accompanying sudden ionospheric disturbances and due to outbursts of ultraviolet radiation. These bay disturbances are anomalous intensifications of these  $S_q$ -variations, but not ordinary disturbances of corpuscular origin.

With respect to the equatorial storms Birkeland confined himself to the hypothesis that they were presumably due to certain systems of electric currents flowing not far from the equatorial region, and devoted all of his attention to a study of the polar storms. The most characteristic feature of the polar storms is a sharp increase of the H-component of the vector of disturbance and the passage of the Z-component through zero in the auroral zone. From this, Birkeland concluded that the polar storms were caused by a powerful linear current flowing at a certain height along the zone. Elementary counts, based on the use of the Biot-Savara law, allowed Birkeland to make an approximate estimate of the height (100-300 km) and the intensity ( $4 \times 10^5$  to  $9 \times 10^5$  amp) of the current. The short duration of these storms (lasting from one to several hours) forced the assumption that the extension of the current along the zone is short:  $10^\circ$ , or a few tens of degrees. Birkeland postulated that his horizontal current was a part of a U-shaped current system, whose vertical branches extend beyond the limits of the atmosphere. The diagram of a typical field of a polar storm (Fig.3) shows the distribution of horizontal projections of the lines of force of the magnetic field (the solid curves), a graph of the variations of the vertical component (the lower part of the figure), and the system of isopotential lines (the broken lines). The hypothetical linear current flows in the direction of the principal axis of the disturbance, marked by the arrow. The maximum value of the vector  $H_d$ , as will be seen from the diagram, should be observed at the point C, the center of the disturbance.

The question of the closure of the Birkeland current system remained open, assuming the possibility of the existence, at a great distance from the earth, of a



very diffuse branch closing the current system, and also assuming the possibility of an unclosed system.

This current system is in agreement with the views of Birkeland and Stoermer on the origin of magnetic storms. According to the well-known Stoermer-Birkeland theory,

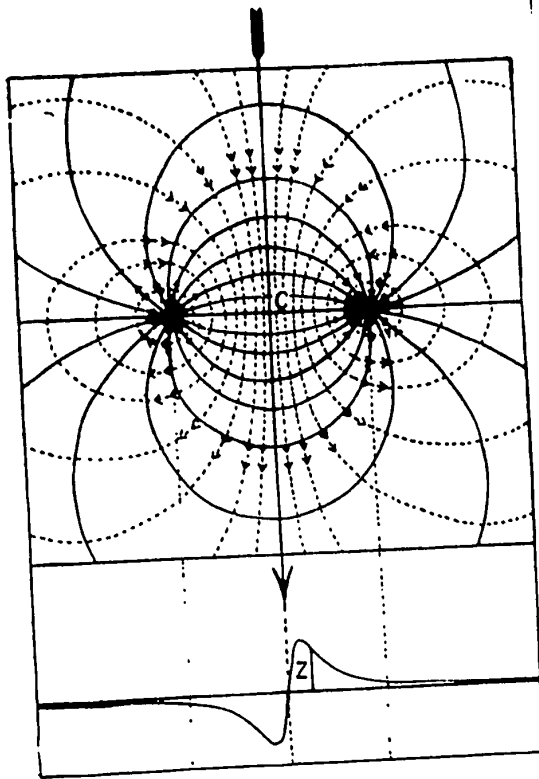


Fig.3 - Diagram of Magnetic Field of an Elementary Polar Storm (according to Birkeland).

The arrow shows the direction of the principal axis of the disturbance (C = Center of Disturbance; — Lines of Force; - - - Equipotential lines; Z = Vertical component of the storm field)

which is frequently set forth in the literature, the storm field is the field of a solar stream of charged particles of a single sign, deflected by the earth's magnetic field toward the polar zones. Solving the equation of motion of a charged particle, Stoermer calculated the possible forms of the paths and, in particular, obtained paths explaining the above described U-shaped current: the particles, moving along these paths, approach the earth from space, penetrate the atmosphere in the high-latitude region down to a height of 100-300 km, take a horizontal segment of their path in the atmosphere, as a rule along the auroral zone, and then once more leave the neighborhood of the earth. Experimental studies by Brueche (irradiation of a magnetized sphere by a narrow beam of cathode rays), which allowed him to follow the paths of individual particles, confirmed the possibility of such paths and thereby gave still greater significance to the Stoermer-

Birkeland theory. This theory is thus an attempt to systematize the data on magnetic disturbances, to establish an idea of the typical picture of a disturbance, to calculate the electric current equivalent to it, and to explain its origin. The

criticism of the physical bases of this theory is commonly known. Serious objections to the theory (a stream of particles with only a single sign could not reach the earth, due to the electrostatic repulsion of the particles; the invasion of the earth's atmosphere by particles of a single sign must lead to great fluctuations of electric potential during a storm, etc.) forced the various investigators subsequently to abandon the hypothesis of a singly-charged stream. Let us discuss the remarks provoked by the morphological part of the study. Since Birkeland's system of station was located in a narrow longitudinal sector of the Arctic (Iceland, Spitzbergen, Norway, Novaya Zemlya), he did not discover the fact that positive and negative polar disturbances are always observed simultaneously, but in different hemispheres. In reality, however, a polar disturbance usually covers all the longitudes of the polar region, the direction and magnitude of the vector of disturbance being different at different longitudes. It would thus seem more expedient to construct the system of electric currents determining the distribution of the magnetic field at all longitudes. Further, Birkeland had too small an observational material on the course of disturbances in moderate latitudes. The morphology of the equatorial storms therefore remained actually unstudied by him, and he did not get a clear idea on the currents responsible for them. The classification of storms introduced by Birkeland, as shown below, likewise does not seem usable.

## Section 2. Chapman's Investigations and their Revisions

A completely different approach to the study of the morphology of magnetic storms is contained in the works of Chapman (Bibl.40). As far back as the beginning of the Twentieth Century, the works of Moos, Director of the Bombay Magnetic Observatory, contained indications that, during the storms, the horizontal component first increases (first phase of the storm), then decreases below the normal (second or chief phase, during which the fluctuations of the magnetic elements are greatest) and then slowly return to the normal state. The return to the normal state [in the literature, various terms are used - restoration phase, aftereffect, Nachstoerung,

postperturbation, and noncyclic variation noncyclic change] takes several days, even when the field is no longer disturbed by irregular fluctuations. The work of Moos gave Chapman ground for postulating that the storm field contains regular parts, for which certain stable systems of electric currents influencing the distribution of the vector of disturbance of the entire earth are responsible. The varied fluctuations of these currents result in the individual features of each storm, the random fluctuations superimposed on the average picture of the magnetic variations. Chapman worked up the variations of the magnetic elements H, D, and Z for 22 observatories located between  $22^{\circ}$  and  $60^{\circ}$  North Latitude. His calculations consisted in averaging of the values of the magnetic elements by hours, counting from the beginning of the storm. As a result of averaging a rather large number of cases (Chapman used the data of 40 moderate storms), the influence of the irregular fluctuations and of the regular part of the disturbance connected with the local time was to a large extent eliminated. He succeeded in finding the regular part of the storm field taking place at the same World Time at all longitudes of the same latitude. He termed this part of the field of a magnetic storm, the stormtime variations, i.e., the variations taking place according to a time reckoned from the beginning of the storm. During the 1930's and 1940's, the English term "stormtime variations" was still used in the Russian literature on terrestrial magnetism to designate this part of the storm field. It seems to us preferable to use the term "aperiodic disturbed variations" as we will do in future, while retaining nevertheless the symbol  $D_{st}$ -variations or  $D_{st}$ -field which is generally used today in the world literature. The  $D_{st}$ -variations of the H- (or X) component at all latitudes (or at least at the middle latitudes) were described similarly by Moos for Bombay. The  $D_{st}$ -variations of the Z-component, on the other hand, reduced to the decrease of the element in the first phase of the storm and to its increase in the second stage. The amplitude of the  $D_{st}$ -variations of the Z-component is smaller than the amplitude of the H-component. No regular aperiodic part could be found in the element D. During the entire storm, the fluctuations of

$D$ , no matter how large they were, usually take place about the normal value of the element. This is evidence that the horizontal component of the vector of disturbance on the average is most often directed along the magnetic meridian. The  $D_{st}$ -variations during the course of moderate and great magnetic storms are the same in form and differ only in intensity. This made it possible for Chapman to conclude that the average picture of the  $D_{st}$ -variations was constant (or, more accurately, stable).

In analyzing the classification of storms proposed by Birkeland, Chapman concluded that the positive equatorial storms of Birkeland correspond to the first phase of the ordinary storm, and the negative ones to the second phase. The insufficiency of the material, in Chapman's opinion, prevented Birkeland from noting that the two types of storms are in reality only two successive phases of a single phenomenon. The averaging of the value of the magnetic elements (after eliminating the  $D_{st}$ -part for each storm) in accordance with the hours of the local days allowed discovery of the relation of the field of the magnetic storm on the time of day. This second regular part of the field of a magnetic storm is customarily termed the disturbed diurnal variation (abbreviated  $S_D$ ). The existence of regular diurnal variations on days of magnetic storms, differing from the diurnal variations on quiet days, was noted, independently of Chapman, by a number of investigators. Chapman's calculation showed the existence of the  $S_D$ -variations in all the elements, and their regular change with latitudes. A characteristic feature of the  $S_D$ -variations in the  $H$  and  $Z$  components in the temperate latitudes is the minimum value of the elements in the morning hours and the maximum values in the evening.

The third part of the storm field, in Chapman's opinion, is the irregular fluctuations ( $D_i$ ) superimposed on the regular parts and giving a random appearance to the variation of the magnetic field on disturbed days. Considering the regular parts of  $D_{st}$ - and  $S_D$ - to be the principal and most interesting parts, Chapman directed his efforts toward their further investigation, leaving the irregular part aside. Considering that the  $D_{st}$ - and  $S_D$ -variations we have described to be due to

electric currents flowing near the earth's surface (in the atmosphere itself or beyond it), Chapman formed an idea, from the observed magnetic variations, of the configurations and intensities of these currents. His systems of electric currents of magnetic storms entered the geophysical literature as the most probable representation of the electric currents, and served as a starting point for the development of the modern theoretical views on the nature of the phenomenon. His systems were constructed by an approximate method, without calculating the potential of the observed fields of variations. He started out from the following postulates:

1. By analogy with the  $S_q$ -variations it may be assumed that the field of  $D_{st}$  or  $S_D$  observed on the earth's surface is the result of the composition of an external main field and an internal field due to induction in the conducting part of the earth. The ratio of the external field  $E$  to the internal field  $I$ , i.e.,  $E/I = 3/2$ .
2. The external system of electric currents is a spherical nonuniform current layer concentric with the earth's surface. The height of the current above the earth's surface  $h = 200$  km.
3. The direction and density of the current may be calculated from the observed magnetic field by the Biot-Savara law, by replacing at each point the action of the nonuniform spherical layer by the action of a uniform, plane current sheet of infinite extension. The current systems of the  $D_{st}$ - and  $S_D$ - variation so obtained are presented in Figs. 4a and 4b. It will be seen that the  $D_{st}$  currents flow everywhere westward in the direction of the parallels of latitude. The intensity of the current increases somewhat in the equatorial region, and increases strongly in the polar cap. The current along the auroral zone is represented in the form of a linear current of high density. The total intensity of the current flowing in each hemisphere is 200,000 amp; the current lines on the figures are drawn in such a way that a current of 10,000 amp flows between adjacent lines. During the first phase of magnetic storms (increasing  $H$ ) the current should flow in the eastern direction. The current system of the  $S_D$ -variations is much more complex. An analysis of

the material as shown that on the whole the field of  $S_D$ -depends on the latitude and local times. For this reason, without taking into account the possible slight longitudinal asymmetry in the distribution of the field, Chapman constructed the current system in the same way as the system of the  $S_q$ -currents, i.e., fixed, if viewed from the sun. In order to explain the variation of the magnetic elements during the course of the day, the earth must be imagined to rotate inside this fixed system. The system  $S_D$  consists of four current loops in the moderate latitudes and a layer of almost parallel currents flowing about the polar cap. As in the case of  $D_{st}$ , an increased intensity of the current is observed in the auroral region. The currents presented in Figs. 4a and 4b correspond to a moderate magnetic storm with decrease of  $H$  in the principal phase equal to about  $40\gamma$ . During very strong storms, the currents can be expected to increase by a factor of 10-15.

The systems of currents of  $D_{st}$  and  $S_D$ , according to Chapman, call forth the following remarks:

1. The empirical material that served for their construction is, absolutely without question, insufficient. If the workup of the data of 22 observatories gave a sufficient idea of the distribution of the  $D_{st}$ - and  $S_D$ -variations in the moderate latitudes, then the regular part of the storms in the high latitudes would still remain in fact, unknown. Chapman judged the intensification of the  $D_{st}$ -variations in the auroral zone by the geographic distribution of the value of  $D_m$ . The symbol  $D_m$  denotes the difference between the mean diurnal values of the horizontal component on disturbed and quiet days, that is,  $D = \bar{H}_q - \bar{H}_d$ . Since the principal effect of the aperiodic storm variations reduces down to the decrease in the horizontal component, it follows that the difference of the mean diurnal values of  $H$  on quiet and disturbed days may serve as a certain characteristic of the value of the  $D_{st}$ -variations. Chapman judged the  $S_D$ -variations inside the zone by the diurnal march for all days, at the Antarctic Station of Cape Evans. The data from observatories lying in the auroral zone itself were not fully available to Chapman. As shown by the

materials collected by us for a number of high-latitude stations (cf. Chapter V), the distribution of variations is in reality somewhat different.

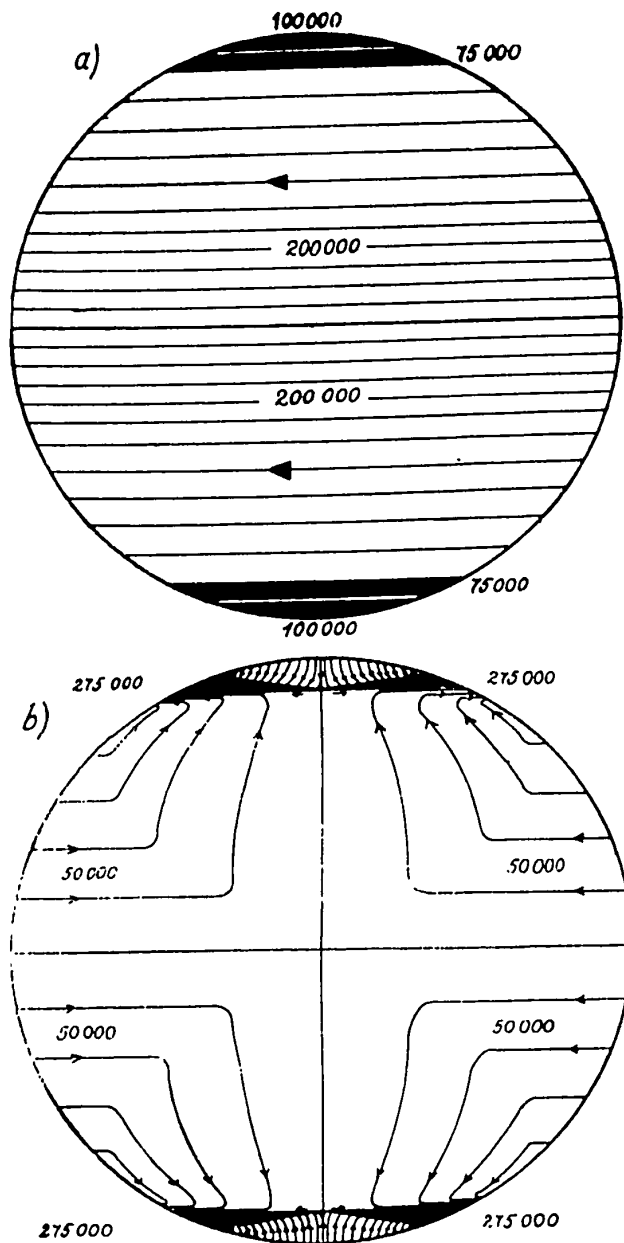


Fig.4 - Currents of the Regular Parts of the Storm Field (after Chapman). Current strength in amperes (a -  $D_{st}$ -Variations; b -  $S_D$ -Variations)

2. The systems constructed by Chapman actually corresponds to worldwide storms (those observed over the entire earth). The absence of a distinct boundary line between world wide and polar storms (Chapman did not pay proper attention to the question of the classification of storms) lead to a certain distortion in the current systems in high latitudes (cf. Chapter II and III).

3. My own calculations of the potential of the external and internal parts of the  $S_D$ -variations have shown that the ratio  $I/E$  is not the same at all latitudes, and that in any case, the value  $I/E = 0.6$  adopted by Chapman is exaggerated.

4. The height of the currents  $h = 200$  km seems too low, which in turn would affect the numerical values of the current intensities found.

On comparing the Chapman and Birkeland systems, Vestine, in his paper written in collaboration with Chapman (Bibl.60) states that the Chapman system better reflects the actual course of a storm, and that the

Birkeland system does not withstand the test of comparison with empirical data. It seems to me that in comparing the Chapman and Birkeland systems it must above all be

borne in mind that these systems are responsible for different types of disturbances and have been constructed from material that is not entirely of full value: Chapman had almost no data on the high latitudes available to him, while Birkeland made little use of information on the course of disturbance in the middle and low latitudes.

The question of the necessity of verifying and elaborating the Chapman system from more complete magnetic data and applying more accurate methods to the calculation of the currents has repeatedly been raised in the literature. The most exhaustive revision made in the above mentioned work by Chapman and Vestine. Up to 1937, (the work was published in 1938) certain worked-up materials of magnetic observations made during the Second International Polar Year (II MPG 1932/1933) were available to the authors. In particular, there were observations within the auroral zone (the Thule and Godhavn Observatories) and immediately in the region of that zone (Bear Island, Matochkin Shar, etc). The values of the  $D_m$ - and  $S_D$ -variations had been calculated for all observatories, the  $S_D$ -variations being taken as the difference between the diurnal marches for international disturbed and quiet days\*. This method of calculating  $S_D$  involves very little work and was subsequently used by a number of investigators.

The mathematical difficulties connected with the calculation of the electric currents corresponding to magnetic fields as complex in geographical distribution as  $S_D$  and  $D_{st}$ , forced the authors to abandon the solution of the direct problem (calculation of the currents from the observed field) and to take up instead the inverse problem (calculation of the magnetic field of the Chapman system of currents and its comparison with the observed field). For this purpose, the current systems of Figs. 4a and 4b were broken down into several principal forms:  $S_1$ , surface current in the

\* The International Association for Terrestrial Magnetism and Electricity, since 1905, has been selecting, from the magnetic characteristics of a worldwide system of observatories, the five quietest and the five most disturbed days in each month.



0 polar cap in the  $D_{st}$ -system;  $L_1$ , linear current in the auroral zone in the  $D_{st}$ -system;  
 2  $S_2$ , current layer between two zones; etc. The magnetic field of each component part  
 4 of  $S$ ,  $L$ , etc. was separately calculated by applying the Biot-Savara law to the el-  
 6 ements of the current and performing the corresponding integration over the surface  
 8 or outline. This method led to rather complicated computational work and made major  
 10 simplifications necessary. As an example we may say that the evaluation of the mag-  
 12 netic field of the  $L_1$ -current, assuming it to be of circular form, led to the calu-  
 14 lation of integrals of the type  $\int_0^{2\pi} \frac{P - a \cos \varphi}{r^3} d\varphi$ , which are reduced to tabulated el-  
 liptic integrals; the surface currents over the polar cap in the middle latitudes  
 were assumed to be plane, and the evaluation of the surface currents  $S_4$  (the middle-  
 latitude eddies in the  $S_d$  system) was not performed at all. As a result of graphic  
 integration, curves of the latitude dependence of the components of the  $S_d$  and  $D_{st}$   
 fields were obtained. Their comparison with the observational data showed that the  
 Chapman systems do not contradict them, but still did not remove the question of the  
 desirability of a new construction of the systems, using all available material.

An attempt to elaborate the Chapman system was made in the paper by Vestine  
 (Bibl.58), based on the same starting material as the above-discussed work. It was  
 found that the Chapman systems had been constructed without allowing for the lati-  
 tudinal asymmetry in the distribution of the field. In the low and medium latitudes,  
 this asymmetry is actually small, but it is impossible to ignore it in the high lati-  
 tudes. Vestine expressed the very interesting thought that the asymmetry in the high  
 latitudes is due primarily to the noncoincidence between the magnetic and geographic  
 axes of the earth due to which fact the auroral zone is of an elliptical shape in-  
 stead of circular and is elongated in the direction of a line joining the magnetic  
 and geographic poles. For this reason, if we allow for the distance of a given point  
 of observation from the auroral zone, instead of simply taking into account the geo-  
 magnetic latitude of the point, then the longitudinal asymmetry is considerably dim-  
 inished. Vestine, on the basis of the magnetic data, determined the location of the

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zone of linear polar current, which he found to be rather close to the position of the maximum isochasm obtained as early as 1867 by Fritz (for more details see Chapter V, Section 7). Thus Vestine's work not only solved certain questions as to the morphology of the  $S_D$ - and  $D_{st}$ -variations, but also disclosed the possibility of using magnetic data for pinpointing the position of the auroral zone. The magnetic data, being the result of continuous recording independent of the meteorological conditions can provide more reliable conclusions than those based on auroral statistics.

Among the work that followed the investigations by Chapman, the paper by Chynk (Bibl.41) is worth mentioning. It points out the existence of a seasonal asymmetry in the distribution of the field of  $D_{st}$ -variations. According to Chynk, the seasonal march of  $D_m$  has a maxima in spring and autumn, like various measures of magnetic activity. However, in addition, it also has another maximum in the winter.

### Section 3. Analytical Representation of the $D_{st}$ -Variations

Attempts at an analytical representation of the potential field of disturbance are also contained in the geomagnetic literature. These attempts related only to the simplest part of the storm field, the aperiodic disturbed and noncyclic variations or, more exactly, only to the middle-latitude parts of these fields. All known papers on this subject (cf. Bibl.40 and 15) followed a definite object, namely separation of the observed field into an external and internal part, explanation of the internal part on the basis of the induction hypothesis, and definition of the conductivity in the depths of the earth required for such an explanation. Chapman and Whitehead calculated the external and internal potentials of the  $D_{st}$ -variations by expanding the spherical functions of the H and Z components of the field into series from the same data that had been used by Chapman for his approximate calculation of the  $D_{st}$ -currents. Since it was assumed that the field of  $D_{st}$  depends only on universal time and geomagnetic latitude, it followed that the values of the potential for a definite instant of time were represented by series of Legendre polynomials, and, since the potential was supposed to be symmetric with respect to the equator,

only the odd harmonics were retained in the series of polynomials. Thanks to the fact that the distribution of the  $D_{st}$ -field in the high latitudes was not taken into account, it was possible to represent the middle-latitude part of the field rather well by the three first harmonics  $P_1$ ,  $P_3$ , and  $P_5$ . The division of the field into an external and internal part (see Chapter III for more details) gave the following ratio:  $I/E = 0.39$ . It turned out that this ratio requires, for the explanation of the I-part within the framework of Lamb's induction theory, somewhat different electromagnetic parameters of the earth than those that follow from an analysis of the  $S_q$ -variations. A more detailed analysis of the results obtained by Chapman and Whitehead and other authors, and a comparison of those results with our own calculations, will be given in Chapter X.

McNish and Slautsitays, who performed the spherical analysis of the values of  $D_m$ , calculated the intensity of the external currents corresponding to those values and obtained interesting conclusions as to the ratio between the internal and external part.

No attempt has been made to date at an analytical representation of the distribution of the potential of the  $S_D$ -variations or of the irregular part of the disturbance.

#### Section 4. Position of the Points of Magnetic Storms. The Equatorial Ring

The papers enumerated in the preceding Sections exhaust all the studies of the morphology of the regular parts of the perturbation field and the calculation of the surface currents responsible for them. It goes without saying, however, that the construction of these systems is not a proof for their existence. If we make no supplementary postulates, then the problem of finding the currents from the magnetic field is an indeterminate, many-valued problem, and an infinite number of such systems can be calculated, each of a different configuration or at a different distance from the earth, whose field will likewise well represent the observed field of magnetic storms. The postulate made by Chapman, however, that the layer carrying

the currents is spherical appears entirely reasonable in the light of our knowledge of the structure of the ionosphere. In fact, if we assume for example that the height of the layer varies with the latitude or with the local time, by 100 km, then this would mean a variation of only 1.5% in the radius of the spherical surface. Under the assumption that the current layer is spherical, its radius (or the height of the current above the earth's surface) can be theoretically determined from the magnetic data just like the configuration of the lines of current or the intensity of current. If we assume that the field potential is represented by a series of spherical harmonics, then the cofactors of the expression  $(\frac{a}{R})^n$  enter into each term, where  $a$  is the radius of the spherical current layer and  $R$  the radius of the earth. Then, by comparing the weight of the  $n_1^{th}$ ,  $n_2^{th}$ , etc. terms in the expansion, the value of  $\frac{a}{R}$  can be estimated. In practice, however, in view of the low accuracy in determining the coefficients with spherical functions, it is impossible to determine  $\frac{a}{R}$  with an error of less than a few percent. Thus, to define the layer of the ionosphere in which the  $D_{st}$ -currents flow, using a spherical analysis of the type proposed by McNish and Slautsitays as basis, would hardly be possible. For any conclusion as to the height of the current layer, data on the structure of the ionosphere would have to be used, together with an attentive study of the structure, ionization density, number of collisions, and other parameters of the ionosphere, which would help to answer the question as to how far a certain layer meets the requirements that a current-carrying layer must meet. While it seems almost unquestionable today that the currents of the  $S_q$ -variations must be related to the lower part of the E layer and the D layer, there are still doubts as to the perturbation currents. The hypothesis that the disturbance currents are concentrated in the  $F_2$  layer seems the most probable, since this layer shows the closest correlation with the magnetic disturbances. Nevertheless, in discussing the possible position of the  $S_D$ -currents, Chapman pointed out that their most characteristic feature was the evening maximum of intensity, while a maximum of ionization density in the evening is not observed

in a single one of the known layers\*. In view of this, the question of the height of the  $S_D$ -currents remain unexplained in the papers of Chapman and his colleagues. As for the currents causing the  $D_{st}$ -variations, the thought is developed in the papers of Chapman and a number of other authors that these currents flow far beyond the limits of the earth's atmosphere, encircling the earth with a ring located in the equatorial plane. The idea of an equatorial ring current was first expressed by Störmer to explain the great polar distance of the auroral zone ( $\theta_0 = 22 - 23^\circ$ ). The calculation of the paths of the particles under reasonable assumptions as to their velocities, and allowing only for the permanent magnetic field of the earth, leads to much lower values of  $\theta_0$ , equal, for example, to  $-2$  to  $-4^\circ$  for cathode rays, and to  $-16^\circ$  to  $19^\circ$  for alpha particles. The magnetic field of the ring current in a westerly direction, reducing the horizontal component of the geomagnetic fields, leads to an increase of  $\theta_0$  to the necessary values. There are also other geophysical arguments in favor of the existence, in storm time, of an extra-ionospheric current ring. But it is precisely the great regularity in the course of the magnetic storms in the low latitudes (where the irregular fluctuations distort the quiet march of the elements only slightly and where the return of  $H$  to the normal state, is slow) that make both these phenomena difficult to explain under the assumption of an ionospheric location of the sources of the field. Forbush (Bibl.43), in studying the correlation between the magnetic storms and the cosmic rays, discovered such variations in the intensity of the cosmic rays as confirm the generation, at a certain distance from the earth, of a magnetic field diminishing the  $H$  component of the earth's magnetic field. A detailed theoretical consideration of the possible influence of the equatorial ring is also presented in the papers by Vallarta and Hess (Bibl.35). In recent years, a number of papers devoted to the effect of magnetic storms on the

\* We will show later that the disturbed diurnal variations of ionization density of the  $F_2$  layer satisfy this requirement, and thus eliminate the objection against placing the  $S_D$  currents in the  $F_2$  layer.

cosmic rays have been published, most of them likewise confirming the Forbush hypothesis.

According to Stoermer's calculations, the ring should be of a very great radius, of the order of the distance from the earth to the moon, and should be formed as a result of the curvature of the paths of charged solar particles by the earth's magnetic field; the ring is not necessarily a closed one. The energy of such a ring must be great (current strength  $\sim 10^7$  amp). As an argument in favor of Stoermer's calculated parameters, the "universe echo", i.e., the great delay of a radio signal returning to the earth, has sometimes been advanced. It has been supposed that, in passing through the ionosphere, a radio signal is reflected from the Stoermer electronic current\*.

All later papers, however, express a different idea on the equatorial ring. Thus, according to the Chapman-Ferraro theory of magnetic storms, the solar stream, encountering the earth's magnetic field, forms a ring of much smaller radius, of the order of two to four earth radii. Since the corpuscular stream is assumed by these authors to be neutral, it follows that the formation of a ring current is explained by the difference in the velocity of motion of the positive and negative particles. The papers by Chapman and Ferraro contain no rigorous mathematical treatment of the question as to the formation of a ring out of the bodies of the corpuscular stream. They give only a system for the physical explanation of the process based on the retardation of the stream by the magnetic field of the earth\*\*. The question as to the stability of a ring, if such a ring is actually formed, is treated with considerable rigor, explaining the conditions of dynamic equilibrium of the ring (i.e., determining the allowable fluctuations of radius and current density) and demonstrating the

\* Special observations made in 1947-1949 with high-power transmitters (Bibl.65), failed to detect greatly lagging echoes.

\*\* The USSR literature contains expositions of the Chapman-Ferraro theory [cf. for example, Eygenson (Bibl.34)].

impossibility of a prolonged existence of a ring with Stoermer's parameters. In 1951, Martin (Bibl.48) considered the process of formation of the ring on the basis of the analogy between the electrodynamic processes connected with the motion of plasma in the magnetic field and hydrodynamic phenomena. The ionized stream flowing around a magnetic dipole is compared to a stream of incompressible fluid flowing around a body submerged in the fluid; in this case, the pressure due to the interaction between the electric currents induced in the body of the stream, with the magnetic field is identified with the hydrodynamic pressure. The parameters of the rings so obtained ( $a = 5.5 R$  and  $I = 10^6$  amp) proved to be of the same order as those calculated by Chapman and Ferraro. The literature also contains an attempt at determining the radius of the ring directly from empirical data, independent of any theoretical views on its formation. As is generally known, one of the most widely used characteristics of magnetic activity is the u-measure, equal to the difference between the diurnal values of the horizontal component on successive days. Considering that the descent of H during a storm, and, consequently, the value of the u-measure, is due to the magnetic field of the equatorial ring, the day-to-day variability of H may be equated to the increase in the horizontal component of the field of the current  $\Delta H$ , thus permitting an evaluation of the ring parameters  $a$  and  $I$ , Yu.D.Kalinin (Bibl.19), who made these calculations under the assumption that the increment  $\Delta H$  was due either to the variation in  $a$  from day to day (with the constant  $I$ ), or to the variation in  $I$  (with the constant  $a$ ), found that the radius of the ring must be of the order of two to four earth radii. As shown below in Chapter III, the spherical analysis of the field of  $D_{st}$  permits determining the quantities  $a$  and  $I$  independently, without assuming invariability of one or the other.

The above-mentioned investigations by Forbush also confirm the small radius of the ring (amounting a few earth radii). Indications pointing to other results have appeared in the literature. The studies of Hayakawa, Nagata, et al (Bibl.46) have shown that the observable effect of magnetic storms in the distribution of the

currents of cosmic rays cannot be explained under the assumption of a ring radius of  $1.1 R < a < 100 R$  within the scope of the Stoermer-Forbush theory. These authors assume that the recalculation of the data based on the modifications introduced into the theories by Lemaitre and Vallart might help to explain the phenomenon. One of the authors of this report, Nagata (Bibl.51), estimated the parameters of the ring on the basis of the southward displacement of the auroral zone during the storm of 30 April 1933 and on the assumption of an intensification of the current in the equatorial ring during the course of the storm. It was found that, for agreement with empirical data, it is necessary to adopt a radius of the order of 20 earth radii for the ring.

Thus most investigators today tend toward the idea that an extra-ionospheric current ring exists, whose field explains the variations of the magnetic field and other geophysical phenomena (position of the auroral zone, influence of magnetic storm, and cosmic rays). But no definitive clarification has been obtained with respect to the parameters of the ring. The possibility of the existence of an ionospheric system of currents of the variations is likewise not completely excluded. At one time, Chapman (Bibl.40) advanced the following argument in favor of an ionospheric system. In view of the fact that the separation of the storm field into two parts is somewhat formal, it is necessary to approach with caution any attempts to explain these two parts by completely different causes; on the other hand, it would be extremely desirable to explain both regular parts of the field of disturbance by one and the same physical process. It is difficult to give an explanation of the disturbed diurnal radiations within the scope of a theory of an extra-ionospheric ring (for this it would be necessary to assume one of two improbable propositions: an elliptic ring or eccentricity of the earth's position). It would seem more natural to explain the  $S_D$ -variations under the assumption of currents flowing in the ionosphere, whose parameters depend explicitly on the diurnal rotation of the earth. This, in Chapman's opinion, does not allow complete rejection of the hypothesis of



an ionospheric system of  $D_{st}$ -currents. Sugiura (Bibl.56), without denying the existence of an extra-ionospheric ring current, considers a partial location of the  $D_{st}$ -currents in the ionosphere possible, in connection with the inductive action of the external ring on the conducting layers; as one of his arguments, he postulated a seasonal march of  $D_m$ , which is easily explained from the standpoint of ionospheric current systems.

These views on the electric currents of the perturbation field are based fundamentally on a study of the middle-latitude picture of magnetic storms and have the object of explaining this picture. In the following Section, we will discuss papers devoted to the electric currents flowing in the polar zones.

#### Section 5. Electric Currents of the Auroral Zone

The intensification of the regular and irregular parts of the perturbation fields in the high latitudes has compelled many investigators to assume that a powerful electric current flows along the auroral zone. We have already learned of Birkeland's ideas as to this current. Other authors have made similar calculations for individual instants of time of individual storms. These include McNish (Bibl.49).

McNish's work is interesting since the potential of the field on a bounded area of the surface, taken as a plane, is represented by the series

$$V = A_0 x + \sum_n e^{n^2 z} (A_{ns} \cos nx + B_{ns} \sin nx) + \sum_n e^{-n^2 z} (A_{ni} \cos nx + B_{ni} \sin nx),$$

where  $x$  is the abscissa of the point (distance from the zone of polar current) and  $z$  the ordinate (height about earth's surface). The expansion of the H and Z components of the field into analogous series, from the data of certain observatories located near the polar zone, has allowed separation of the potential into parts of external and internal origin ( $I/E \approx 1/4$ ) and has made it possible to calculate the height of the current. The best agreement with the observational data was given by a height of  $h \approx 100$  km and the assumption that the current flows in a wide belt, a few tens of kilometers wide, over this zone. It must be noted that the simple in-

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0 inspection of the magnetograms of polar observatories, without any computational op-  
 2 erations, forces us to assume the existence of electric currents at rather low levels  
 4 in the polar regions. The high degree of local variation in the distribution of the  
 6 vector of disturbance would be difficult to explain under the assumption that the  
 8 sources of the field were located far from the earth's surface.

10 The calculations of the linear electric currents responsible for the high-  
 12 latitude part of the  $S_D$ -variations were made by Harang (Bibl.44) and Sucksdorff  
 14 (Bibl.55). The materials used in both cases were the  $S_D$  variations for the Second  
 16 International Polar Year for several stations located close to the auroral zone. By  
 18 combining the observations of pairs of stations (cf. Chapter V for the formulas), the  
 20 authors calculated the position of the zone, the strength of the current flowing in  
 22 the zone, and the height of the current for various hours of the day. According to  
 24 Harang, the most probable value of the height is 100-200 km. According to Sucksdorff,  
 26 the height varies over a very wide range, from 100 to 1,000 km, and shows an obvious  
 28 dependence on the time of day. The great discrepancies between the values of the  
 30 parameters found by Harang and Sucksdorff indicate that a more careful selection of  
 32 the empirical material is necessary. Harang found the linear horizontal current to  
 34 be doubled in the polar zone, which is also confirmed by Yu.D.Kalinin (Bibl.20) on  
 36 the basis of the geographical distribution of the u-measure. Sucksdorff found that  
 38 for a better representation of the observed material, it is necessary to assume a  
 40 system of vertical currents descending from outer space to the earth's atmosphere in  
 42 regions close to the magnetic field. Sucksdorff's vertical current does not repre-  
 44 sent a closed contour, and it must be assumed that it is scattered due to a recomb-  
 46 ination of particles in the lower layers of the ionosphere. In performing his calcu-  
 48 lations, Sucksdorff started from Chapman's hypothesis that the internal part of the  
 50 perturbation field is equal to  $2/3$  of the external part. Harang in comparing the  
 52 calculated variations of the horizontal and vertical components, found that such an  
 54 idea leads to a poor correspondence of the H and Z variations, and that a lower

0 value of the ratio  $I/E$ , namely one of the order of 0.1, is more probable. As we will  
2 see later in Chapter V, the calculation of the external and internal potentials of  
4  $S_D$ , for high latitudes, leads precisely to such small values of  $I/E$ .

6 The explanation of the field of a magnetic storm by a vertical current descending  
8 in the auroral zone is likewise contained in the very interesting but somewhat con-  
10 troversial papers by M.N.Gnevyshev (Bibl.12, 13). Let us first discuss the classi-  
12 fication of storms proposed by him. In comparing the magnetic and ionospheric data  
14 and considering the geographic distribution of the perturbation field, he came to the  
16 conclusion that there are two types of storms: polar and worldwide, but his classi-  
18 fication is not identical with that by Birkeland. He includes most of the moderate,  
20 great, and violent storms usually observed over the entire earth, into the category  
22 of worldwide storms. He considers that the maximum of the vector of disturbances  
24 during these storms is observed in the auroral zone. He places a relatively small  
26 number of storms in the second, or polar category; these are storms for which the  
28 auroral zone plays no particular role and whose disturbance reaches a maximum at the  
30 magnetic pole or near it. He supports this classification of storms with graphs of  
32 the disturbance vector plotted against the geomagnetic latitude, and with comparisons  
34 of the magnetic and ionospheric disturbance. The worldwide storms, according to  
36 Gnevyshev, are caused by a vertical electric current descending from outer space and  
38 reaching heights of 80-100 km (the lower boundary of the region at which the aurora  
40 can be seen). He takes this stand on the basis of the Stoermer-Birkeland theory, as-  
42 suming the direct superimposition on the geomagnetic field of particles of a singly-  
44 charged stream. He circumvents the objections raised against that theory by assuming  
46 very low flux densities (of the order of  $5 \text{ cm}^3$ ), fairly high particle velocities, as  
48 well as a pulsating radiation (noncontinuous) of the particles by the active foci of  
50 the sun. Postulating a linear vertical current, he successfully calculates, from the  
52 observed geomagnetic variations, the current strength, the polar distance of the zone,  
54 the penetration of particles into the atmosphere, the cross section of the flux, the

velocity of the particles, and the nature of the particles (a mixture of doubly- and singly-ionized helium atoms). The decreased amount of disturbance in the middle and low latitudes is explained by him by their remoteness from the sources of the field, not admitting the existence of any special currents responsible for the disturbance in the middle and low latitudes. Gnevyshev's papers are of great value, since they show how the magnetic data can be used for judging the nature of the particles and the geometry of the corpuscular stream, but they do provoke certain objections. First, the classification adopted by the author is doubtful. A consideration of a series of magnetograms (cf. Chapter II for more details) shows that there are no grounds for putting storms with intensities increasing toward the pole in a separate category. Second, it is very difficult to explain the regular part of the perturbation field without assuming currents flowing above the middle latitudes (cf. Chapter V, Fig. 27). The flux densities obtained by Gnevyshev seem slightly too low\*; in this connection, the question remains open as to how essential the theoretical objections, raised at one time against the Stoermer-Birkeland theory, are for Gnevyshev's works.

#### Section 6. Penetration of Corpuscles into the Earth's Atmosphere. The Alfvén Theory

The penetration of solar charged particles of both signs into the earth's atmosphere in the high latitudes is considered by the majority of modern authors to be beyond doubt. This conviction is based on the factual data on the magnetic variations and the aurora. For example, a comparison of the intensity of the auroral displays with that of the magnetic disturbances enabled Harang (Bibl. 44) to calculate the velocity/spectrum of the particles penetrating the earth's atmosphere to various depths. In a number of other papers, radiosonde data are used to demonstrate the deep penetration of particles into the atmosphere. For example, the observations by Wells (Bibl. 64) showed that the intense polar storms, as a rule, are accompanied by the formation of

\* We recall that the current density according to Chapman's data is equal to  $10^2$   $\text{cm}^{-1}$ .

0 a strongly absorbing region at the level of the D layer. Less intense storms are  
 2 connected with the formation of a layer of abnormally high ionization at the level of  
 4 the E layer. In this case, the energy is obviously less, and, consequently, pene-  
 6 tration of the particles into the earth's atmosphere is not as deep. A correlation  
 8 in the high latitudes between the night layer  $E_n$  and the magnetic activity is noted  
 10 in a number of papers (cf. for example, Bibl.1 and 2).

12 The definitive experimental proof for the penetration of solar particles into  
 14 the earth's atmosphere was obtained by spectrophotometry of the aurora. It is well  
 16 known that the absence of hydrogen lines in the auroral spectra was long a puzzle to  
 18 those interested. From the 1930's on, it has been possible, owing to the improved  
 20 technique of spectroscopic work, to find (but rather rarely) the  $H_\alpha$  line in the a-  
 22 uroral spectra. In connection with this fact, Vegard postulated that hydrogen is not  
 24 a permanent component of the earth's atmosphere in the high latitudes, but is only a  
 26 stray brought in by the solar corpuscular stream. During a few auroral displays of  
 28 1950, he was able to discover (Bibl.29) a Doppler broadening in the  $H_\alpha$ ,  $H_\beta$  and  $H_\gamma$   
 30 lines, indicating the vertical displacement of hydrogen atoms at velocities of 800-  
 32 3,000 km/sec. Thus the question of the introduction of solar particles into the  
 34 earth's atmosphere must be considered definitively solved.

36 Much still remains unclarified with respect to the excitation of the currents  
 38 directly responsible for the fluctuations of the magnetic field. Since, with a  
 40 neutral stream (and there are not many grounds for doubting such a stream) the direct  
 42 influence of the field of particles of Birkeland-Gnevyshev is inapplicable. Therefore,  
 some mechanism responsible for the separation of the charges or the excitation of  
 some kind of currents must be introduced into the argument. The Chapman-Ferraro  
 theory, which is merely a theory on the formation of the equatorial ring (i.e., a  
 theory of the middle-latitude part of the  $D_{st}$ -variations), gives no answer to this  
 question. It is true that the above-mentioned note by Martyn does contain indi-  
 cations of the direction in which the theory would have to be further developed to

0 obtain an explanation of the polar part of the disturbances\*. But these indications,  
 2 unsupported by any calculations, cannot be considered a reliable explanation of the  
 4 phenomenon.

6 An explanation of the disturbances in the high latitudes is worked out in more  
 8 detail in Alfven's theory, which is an attempt to reconcile the view of Stoermer and  
 10 Birkeland with those of Chapman and Ferraro. The Alfven theory (Bibl.4) is based on  
 12 the assumption of a stream consisting of charged particles of both signs, whose  
 14 motion from the sun to the earth takes place under the action of the inhomogeneous  
 magnetic fields of the sun and the earth and of the electric field created by the  
 stream itself. As in the Chapman-Ferraro theory, as a result of the encounter of  
 the corpuscular stream with the geomagnetic field, a "hollow" is formed, which is a  
 region surrounding the earth and which does not allow penetration of the charged  
 particles. Under the action of the inhomogeneous magnetic field of the earth, the  
 paths of the positive and negative particles separate, but since their total number  
 in unit volume is the same, the volume charge remains equal to zero. The electrons  
 which, in the equatorial plane, bend around the forbidden region in an easterly di-  
 rection, and the positive particles moving toward the west, form an electric current  
 of westerly direction, which is eccentric with respect to the earth (Fig.5). The  
 current comes closest to the earth on the evening side on which, in addition, the  
 greatest density is observed. This part of the current system is responsible for  
 the  $D_{st}$ - and  $S_D$ -variations of the middle latitudes. The dimensions of the forbidden

\* It is postulated that the charged particles in the equatorial current torus, close to its surface, might become detached from the body of the flux and be displaced toward the earth along the lines of force of the magnetic field, which are lines of very high conductivity. The entrance of these particles into the earth's atmosphere at the latitude of the auroral zone causes luminescence of the atmosphere, and an elevated ionization at the 90-100 km level, and would also produce a strong electric field of meridional direction (the positive and negative particles would bombard different edges of the zone). This field, interacting with the permanent magnetic field of the earth, would produce a drift of the ionospheric ions of both signs in a westerly direction. If the velocity of the positive and negative ions is assumed to be different ( $v_+ > v_-$ ), then a Hall current would flow along the zone, eastward on the evening side of the earth and westward on the morning side, which would explain the polar part of the current system of  $S_D$ -variations.

zones depend on the parameters of the particles and therefore differ for the positive and negative particles; the positive particles come closer to the earth than the negative ones. The penetration of positive particles into the region forbidden to the electrons produces a great potential difference at the boundary of the region (accumulation of positive particles on the day side and negative particles on the night side). The conductivity of very rarefied gases in the presence of a magnetic field is considerably greater along the direction of the field than in a direction perpendicular to this. In view of this fact, positive and negative charges, tending to neutralize each other, will move along the lines of force of the magnetic field from the boundary of the forbidden zone to the upper layers of the earth's atmosphere in the high latitudes. The invasion of the ionosphere by the corpuscles takes place in the auroral zone. The conductivity of the polar ionosphere is relatively high so that the positive particles will be displaced from the day side along the auroral zone to the night side, and the negative particles in the opposite direction. Thus a peculiar kind of discharge current, counterclockwise in the first half of the day (0-12<sup>h</sup>) and clockwise in the second half (12-14<sup>h</sup>) will be established. Figure 6 gives a diagram of the formation of the current. This current, in Alfven's opinion, is able to explain the polar disturbances and the aurora. This system of motion of charges was obtained by Alfven as a result of the solution of the equations of motions of particles in inhomogeneous magnetic and electric fields, allowing (it is true, only approximately) for the interaction of the particles. The Alfven theory successfully explains certain regularities of the morphology of the aurora (position and diurnal displacement of the auroral zone, certain forms of the auroral displays, etc.), as well as the penetration of corpuscles into the polar ionosphere and the formation there of currents causing the polar disturbances. Thus, the investigations by Alfven contributed greatly to the development of the theory of magnetic storms. The concept of the aurora as the result of discharge currents appears very probable in the light of recent work by several Soviet scientists.

In spite of this, the work by Alfven evokes certain comments. First, according to Alfven, the polarization of particles in the stream and their motion from the sun

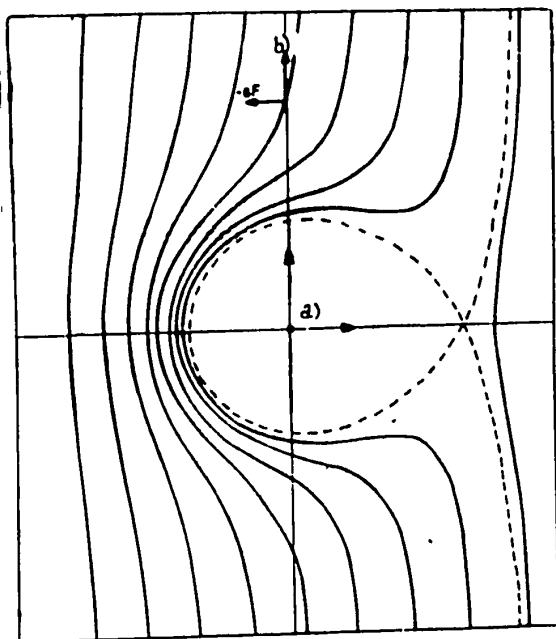


Fig.5 - Curvature of Stream in the Earth's Magnetic Field, According to Alfven (- - - Boundaries of Forbidden Zone) a) Dipole; b) To the Sun

to the earth is due to the sun's magnetic field; Alfven takes  $10^{34}$  cgs as the value of the magnetic moment of the sun, according to Khell's (Hull's) determinations. Repeated measurements by German and American authors resulted in much lower values, so that the question as to magnitude and constancy of the sun's magnetic field cannot be considered settled at present. The only fact which appears to be beyond doubt is that the magnetic field of the sunspots is many times greater than the general magnetic field of the sun. It would, therefore, appear that the magnetic field of the sunspots should have an influence on the process of formation and flight of the corpuscular polar stream, the more so

since the ejection of particles is undoubtedly from regions located near the spots. It is possible that the replacement in the Alfven equations of the value of the general magnetic field of the sun by the field of the sunspots, somewhat modifies the parameters of the stream, particularly on the first half of the path from sun to earth. Second, the explanation of the  $S_D$ -variations in the geomagnetic field by assuming an asymmetric location of the equatorial ring current seems improbable from the point of view of the morphology of the field of magnetic storms; according to the Alfven theory we would expect the field of  $S_D$  to increase toward the equator, just as is done by the  $D_{st}$  field. In reality, the  $S_D$ -variations are, on the contrary, almost entirely absent at the equator (cf. Chapter V). Third, according to Alfven, the polar

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system of currents causing disturbances in the high latitudes is formed as a result of the accumulation of space charges in the equatorial ring. In this way, the increase in the amount of disturbance in the high latitude during worldwide storms could be explained, but this leaves the polar storms completely without explanation, i.e.,

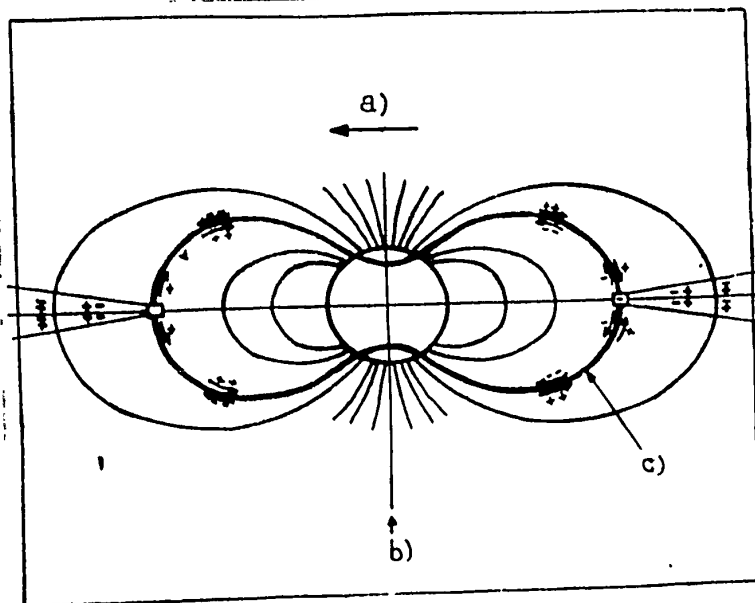


Fig.6 - Formation of Current Systems According to Alfven (— Lines of Force of Magnetic Field; - - - Boundary of Forbidden Zone)  
a) To the sun; b) Magnetic axis; c) Boundary of Forbidden zone

it does not explain the disturbances affecting only the polar regions, which are obviously not connected with the formation of the equatorial ring. Besides these, and a number of analogous concrete objections, two other shortcomings of the Alfven theory, of a more general nature, must also be pointed out. First, the Alfven theory considers the formation of magnetic storms and of the aurora without any connection with the physics of the disturbed-day ionosphere. At the present stage, it would seem impossible to separate the explanations of the magnetic and

ionospheric disturbances, just as it is impossible to explain them independently of the aurora. Second, many stages of the theory have not been vigorously developed. There are numerous assumptions requiring physical or mathematical justifications (for example, the assumption that the electronic and ionic temperatures in the stream are unequal, and the like).

Thus, in spite of the fact that the Alfven theory has contributed much of value to the development of the ideas on magnetic storms and the aurora, it cannot be considered to resolve completely all questions of their origin.

## Section 7. Dynamo Theory of Magnetic Storms

In this Section we will briefly discuss the dynamo theories of magnetic disturbances. It is well known that, during the first stage of development of the doctrine of magnetic storms, the dynamo theories occupied a prominent position (the Angenheister theory, the first version of the Chapman theory, the Lindemann theory, etc.). These authors explain the formation of the electric currents of magnetic storms by the vertical motion of the upper conducting layers (due either to thermal expansion, or to mechanical displacement under the action of the stream of particles penetrating into the atmosphere) in the permanent magnetic field of the earth. All these investigations, in time, were found to be unsound (Bibl.40), and today they are only of historical value. In 1946, Yu.D.Kalinin (Bibl.18) proposed a modification of the dynamo theory, explaining the middle-latitude part of the electric currents of the  $S_D$ -variations by the tidal motions of the upper layers of the atmosphere, under the action of the gravitational attraction of the mass of the flux. He described the conductivity of the ionosphere by the two-term expression  $\rho = \rho_v + \rho_c$  where  $\rho_v$  denotes the portion of the conductivity due to the ionization under the action of the wave radiation of the sun, and  $\rho_c$  is the conductivity due to ionization by the corpuscular stream. Naturally,  $\rho_v$  depends on the zenith angle of the sun and  $\rho_c$  on the geomagnetic latitude of the point of observation. It was assumed that the integral conductivity of the entire layer of the ionosphere  $\rho_v = 2 \times 10^7$ , and  $\rho_c = 5,100$  cgs. The calculations made by Kalinin showed that, under these assumptions, the current system would consist in each hemisphere of two eddy currents with centers located at latitudes  $\pm 60^\circ$ . On the evening side of the earth the eddy with the positive current direction is in the northern hemisphere, and on the morning side, the eddy with the negative direction. I myself consider that the mechanism of action of the stream on the earth's magnetic field, proposed by Kalinin is possible in principle, but I will demonstrate that the values of  $\rho_v$  and  $\rho_c$ , adopted by him are in need of review. The value of the  $\rho_v$  of the  $F_2$  layer, in a direction perpendicular to the magnetic field (with which the  $S_D$ -

variations can be correlated), is possibly somewhat lower (for details, see Chapter VIII), while the value of  $\rho_c$ , taken by Kalinin as equal to 5,100, requires the corresponding justification.

Great attention has been paid in recent years by Japanese geophysicists to theoretical and morphological investigations of magnetic disturbances. A proposition common to them is the assertion that a dynamo effect is the fundamental cause of geomagnetic disturbances. The revival of the dynamo theories in the works of the Japanese authors is connected with the discovery, by radio methods, of horizontal motions in the ionosphere and of an anomalous rise in the ionization (and, consequently, in the conductivity) of the lower layers of the ionosphere in high latitudes. Here some authors connect the field of disturbance with motions of the  $E_s$  clouds. The correlation between the appearance of  $E_s$  in the high latitudes and magnetic activity, as we have already remarked, is no longer doubted and indicates the corpuscular nature of the  $E_s$  ionization. But the explanation of the  $S_D$ -currents flowing around the entire earth, as being due to the motion of the  $E_s$  clouds (and referring the current of the L-variations, on the other hand, to the level of the  $F_2$  layer) would seem unreasonable. As shown below, the very frequent appearances of  $E_s$  in the equatorial regions well explain the behaviour of  $S_q$  and L at Huancayo and by no means fit in with the regularities of the storm field. The papers by Nagata (Bibl.52) are of considerably greater interest. He considers only the polar part of the disturbance (position and displacement of the auroral zone, intensity of the current in it, and width of the zone) and shows that the polar current could be formed at the 60-100 km level as a result of an increase in ionization at this height by 20-40 times above its level on quiet days. In their papers, Nagata and Fukushima conduct a polemic with Alfven, pointing out that the configuration of the currents does not correspond to the form of the discharge currents postulated by Alfven. Nagata, using the McNish formulas for the representation of the field near the polar zone, also investigated the ratio of the external to the internal parts of the field, and obtained  $E/I = 2.6$ , which

0 does not contradict the data of other authors. Hasegawa (Bibl.45), in considering  
 2 the diurnal variations on quiet days and disturbed days in 19 polar observatories,  
 4 made use of harmonic analyses and calculated the electric conductivity of the upper  
 6 layers as well as the gradient of electric potential required to explain the  $S_D$ -  
 8 variations by dynamo currents. He also studied the form and displacement of the au-  
 10 roral zones during the course of the day and with the seasons of the year.

12 It seems that the excitation of dynamo currents in the lower layers of the iono-  
 14 sphere at high latitudes is very probable, taking account of the increase in ioniza-  
 16 tion during storms and of the existence of vertical and horizontal winds. It may be  
 18 that these play an important role in the formation of local polar disturbances. But  
 20 for all that, it does appear unclear, without more detailed investigations whether  
 22 these currents are responsible for the polar part of the  $S_D$ -variations, what role is  
 24 played by the current excited in the polar regions of the  $F_2$  layer, and, a fortiori,  
 by what mechanism the low-latitude part of the  $S_D$ -currents is formed.

#### Section 8. Bay-Like Disturbances

In all the above papers, except for the investigations by Birkeland, the subject  
 of study was the fields of worldwide storms, i.e., of storms during which substantial  
 variations of the magnetic field were observed in all latitudes. After the investi-  
 gations by Birkeland, which unfortunately have not been sufficiently developed in  
 subsequent papers, it became clear that the polar magnetic storms accompanied in the  
 middle latitudes by small bay disturbances, constitute a special phenomenon charac-  
 terized by a different field structure and a different origin. A few papers devoted  
 to the statistics of bays in the middle latitudes are known (Bibl.8, 25). Thanks to  
 them, the questions of the diurnal, seasonal, or 11-year march of the frequency of  
 bays have been partially settled. Papers devoted to the considered of individual  
 bays are also known.

The most complete investigation of bay-like disturbances was made by Silsbee  
 and Vestine (Bibl.54) who subjected the bay-like disturbances observed by 13 obser-

0 vatories during the Second International Polar Year to statistical processing. They  
 2 investigated the problems of distribution of positive and negative bays during 24  
 4 hour periods and during the course of the year. This led to a concept as to the  
 6 mean (or more accurately the typical) bay, and a system of currents corresponding  
 8 to this typical bay was constructed. We will discuss the work of Silsbee and Ves-  
 10 tine in more detail in Chapter VI, which is specially devoted to bay-like distur-  
 12 bances. Silsbee and Vestine also calculated the linear polar currents for 20 indi-  
 14 vidual cases of bays. The direction of the current, in almost all cases, was paral-  
 16 lel to the auroral zone. The height of the current, even according to data from  
 18 closely adjacent observatories, varied over a wide range (for instance, 1<sup>h</sup> 25<sup>m</sup> on  
 20 3 July 1938, according to the data of one pair of stations, h was 100 km, while ac-  
 22 cording to the data of another pair, h was 560 km. At 0<sup>h</sup> 50<sup>m</sup> on 26 June 1933, h was  
 24 190 and 550 km, respectively). Such fluctuations of height must be considered an in-  
 26 dication that a linear current does not always well approximate the observed distri-  
 28 bution of the field of a polar storm.

### 32 Section 9. Current Systems of Individual Storms

34 As will be clear from the foregoing, the fundamental trend in geomagnetism dur-  
 36 ing the past two decades has been toward detection and explanation of the mean regu-  
 38 lar features of the perturbation field. A relatively small number of papers have  
 40 been devoted to a study of individual magnetic storms. In addition to the above-  
 42 mentioned calculations of the polar current of individual disturbances, there are  
 44 also a few cases where the surface-current systems of individual storms have been  
 46 considered. Thus, Yu.D. Kalinin (Bibl.21) in 1938 constructed isopotential lines by  
 48 the method of graphical integration for three successive instants of time for the  
 50 disturbance of 17 March 1933 for the northern part of Eurasia. Since the distribu-  
 52 tion of isopotential lines allows the configuration of the lines of the surface sys-  
 tem of currents to be judged to some extent, the conclusion may be drawn from these  
 maps of Kalinin that the current systems responsible for the disturbance of 17 March  
 1933 bear some resemblance to the mean systems of  $D_{st}$  and  $S_D$  represented in the works  
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of Chapman (Figs. 4a and 4b). From each map, the part of the middle latitude eddies and of the polar current of  $S_D$  may be found. Specifically for a solution of the question as to how much the electric currents, calculated for individual instants of time resemble the currents of the regular parts of the storm fields, Vestine (Bibl. 62) constructed current systems for five magnetic storms, of 14 October 1932, 30 April 1933, 5 August 1933, and 15 October 1932. As starting material, the mean-hourly values of the magnetic elements from observations of about 40 stations in the northern hemisphere were used. The methods of constructing the current lines was approximate, i.e., the same as that used by Chapman for the construction of the  $D_{st}$  and  $S_D$ -variations. The part due to the internal induced currents was eliminated from the observed values of the elements by multiplying the observed values of  $Z$  and dividing the observed values of  $H$ , by 0.8. This factor was compiled by Vestine instead of the factor 0.6 used earlier by Chapman, in connection with the papers cited in References 41 and 46, whose results were mentioned in the preceding Section. The maps of the current lines presented in the cited work show very plainly that, in all cases, it is possible to detect two intense eddies characteristic for the  $S_D$ -variation, a densification of current lines in the polar zone, and parallel current lines in the low latitudes, characteristic for the  $D_{st}$ -variations. The direction of the current corresponded in all cases to what would have been expected from a consideration of the average systems. Thus it may be considered that Vestine's experimental calculations yielded an interesting result substantiating the investigations by Chapman, devoted to the regular part of the fields of magnetic storms. It goes without saying that for a more trustworthy solution of the question posed by Vestine as to the ratio of the average to the individual current systems, it would be necessary to accumulate a large amount of material and to replace the mean values of the magnetic elements by their instantaneous values.

#### Section 10. Irregular Part of the Storm Field

The question of the irregular part  $D_i$  of the perturbation field is one of the

most obscure questions in geomagnetism. The statistical investigation of the magnetic disturbance, or activity, is usually conducted by using magnetic characteristics, i.e., by estimates of the degree of magnetic disturbance. Usually these characteristics take account of both the regular part of the field, the irregular fluctuations during storms, and the small disturbances. Thus the statistical regularities of the irregular part are studied in rather great detail. Unfortunately, only a few attempts have been made to correlate the study of the regular and irregular variations and to explain them within the framework of a single hypothesis. The well-known monograph by Chapman and Bartels, which considers in rather great detail the questions of magnetic disturbances, devotes a page and a half to the irregular part, in which it is stated that the high intensity of the field of  $D_1$  in the polar cap may possibly be connected with local and rapid fluctuations in the ionization, which follow from the variability of the form and brightness of the auroral displays. The explanation of the irregular fluctuations in the temperate latitude differs according to what systems of currents (ionospheric or extra-ionospheric) were adopted to explain the regular part of the variations. Here Chapman states that, so long as no authentic theory of the regular parts of the disturbance has been constructed, scientists will be unable to give explanations for the irregular fluctuations.

A.P. Nikol'skiy (Bibl.26) has again raised the question as to the ratio of the irregular fluctuations to the regular parts of the disturbance, in a series of papers devoted to the statistics of magnetic activity in polar observatories. Like M.N. Gnevyshev, whose papers were mentioned above, Nikol'skiy is a proponent of the theory of the direct influence of the field of charged particles on the earth's magnetic field. Each pip or pulse on the magnetogram is in his opinion, a direct result of the invasion of the earth's atmosphere by a group of particles. The regular parts of the field of  $D_{st}$ -disturbances, at least in the high latitudes, are the results of formal averaging of individual and independent pulses and do not correspond to any real physical phenomenon. Since the distribution of the frequency of positive and

negative pulses during the course of a day obey definite regularities, the averaging of the values of the magnetic elements for a series of disturbances creates the impression that a regular diurnal march of the perturbation vector of  $S_D$  does actually exist. Since, in most cases, the pulses in the horizontal component are negative in sign, this leads to the false conclusion that  $H$  decreases uniformly during the time of a storm. The mean diurnal value of  $H_q^d$  for disturbed days, taken at selected quiet intervals in these days, coincides within an accuracy of 2-3 % with the mean diurnal value  $H_q$  for quiet days. This result, obtained by Nikol'skiy for a number of high-latitude observatories, indicated the absence of a general regular depression on stormy days. Without dwelling here on a criticism of this fundamental proposition by Nikol'skiy (we will return to it in the next Chapter) we may state that his work is of great value even if only because it has attracted attention to the study of  $D_i$ , and has forced a reconsideration "de novo" of the question as to the ratio of the regular to the irregular parts of the disturbance. Without a correct solution of this problem, to which an undeservedly small amount of attention has been given, a proper approach to the solution of a number of problems in the theory of magnetic disturbances is impossible.

#### Section 11. Conclusions

On the basis of the brief survey of recent papers on magnetic disturbances, given in the preceding sections, the following conclusions may be drawn:

1. The morphology of the field of disturbance has not yet been adequately been studied. More specifically, there is no complete clarity with respect to the classification of magnetic storms; the regular variations in the high latitudes have been little investigated (not only is their form unknown, but even, as indicated by the works of A.P. Nikol'skiy, there is not even confidence in their existence), etc.
2. In spite of the large number of papers devoted to the construction of electric currents, this question is still far from a definitive solution. Chapman's current systems, which are the most trustworthy, are based on insufficient material



and have been constructed by an extremely approximate method. It would be desirable to use all the observational material available today and to use objective analytical methods for continuing the investigations in this direction.

3. Too little material has been accumulated to judge the ratio of the regular parts of the field of disturbances to the irregular fluctuations, and to estimate the correspondence between the average current systems and the currents of individual storms.

4. There are also inadequate data with respect to the separation of the observed field of disturbance into parts of external and internal origin. The separation of the  $D_{st}$  field has shown that the ratio  $I/E$  for the disturbed variations and the quiet variations ( $S_q$  and  $L$ ) is not the same. A knowledge of the ratio  $I/E$  is also important for a correct evaluation of the intensity and configuration of the external currents and for evaluating the conductivity of the deep layers of the earth.

5. There is almost a complete lack of comparisons of the current systems constructed on the basis of magnetic data with our modern ideas on the ionosphere of the disturbed day. The use of ionospheric data is necessary for any judgment as to the reality of the currents calculated and for refining their parameters.

6. The construction of a system of currents corresponding sufficiently well to the observed geomagnetic variations, is a necessary basis for working out physical explanations of the nature of magnetic storms.

The extensive materials on the geomagnetic variations, accumulated up to now in connection with the published summaries of the observations of a worldwide system for the Second International Polar Year and with the data of Soviet observatories, worked-up for a number of years, have induced me to review certain obscure questions in the morphology of magnetic storms and to carry out a new construction of the current systems. The following Chapters of the present work are devoted to a discussion of the results obtained.

## CHAPTER II

### DIVISION OF THE FIELD OF MAGNETIC STORMS

#### Section 1. Classification of Storms. Polar Storms

Before proceeding to a discussion of the regularities obeyed by the field of magnetic disturbances, the most acceptable classification of magnetic storms must be selected. After the work done by Birkeland, Gnevyshev, Vestine, and other authors, it is indisputable that magnetic storms, in all of their diversity, may be still subdivided into two main groups: polar storms and worldwide storms. The most correct definition of a polar storm, in my opinion, is the definition given by Silsbee and Vestine, according to which an elementary polar storm is a disturbance lasting from one to several hours, which in its form recalls a bay, of great amplitude in the polar latitudes and very small amplitude in the temperate latitudes. Polar storms (see Fig.7 for examples) may be observed on both quiet and disturbed days. It will be seen from the diagram that, at high latitudes, the amplitudes of a polar storm may be very great, over 1000  $\gamma$  but that, with increasing distance from the auroral zone, the value of the amplitudes drops sharply. Figure 8 gives the latitude-dependence of the H and Z components of a polar storm (according to McNish). Figure 8 shows that the correspondence between the field of a polar storm and the field of the linear electric current flowing in the auroral zone is excellent. The decrease of the field in the temperate latitudes may be considered the result of the increasing distance from the sources of the field. Thus the polar storms, as we understand them, have the same geographical distribution as the worldwide storms in Gnevyshev's class-

ification. As for Birkeland's definition of polar storms, as already remarked in Chapter I, there is no necessity of dividing polar storms into positive and negative according to the sign of the horizontal component of the perturbation field. Figure 31 (cf. Chapter VI), which gives the distribution of the vectors of the field of a

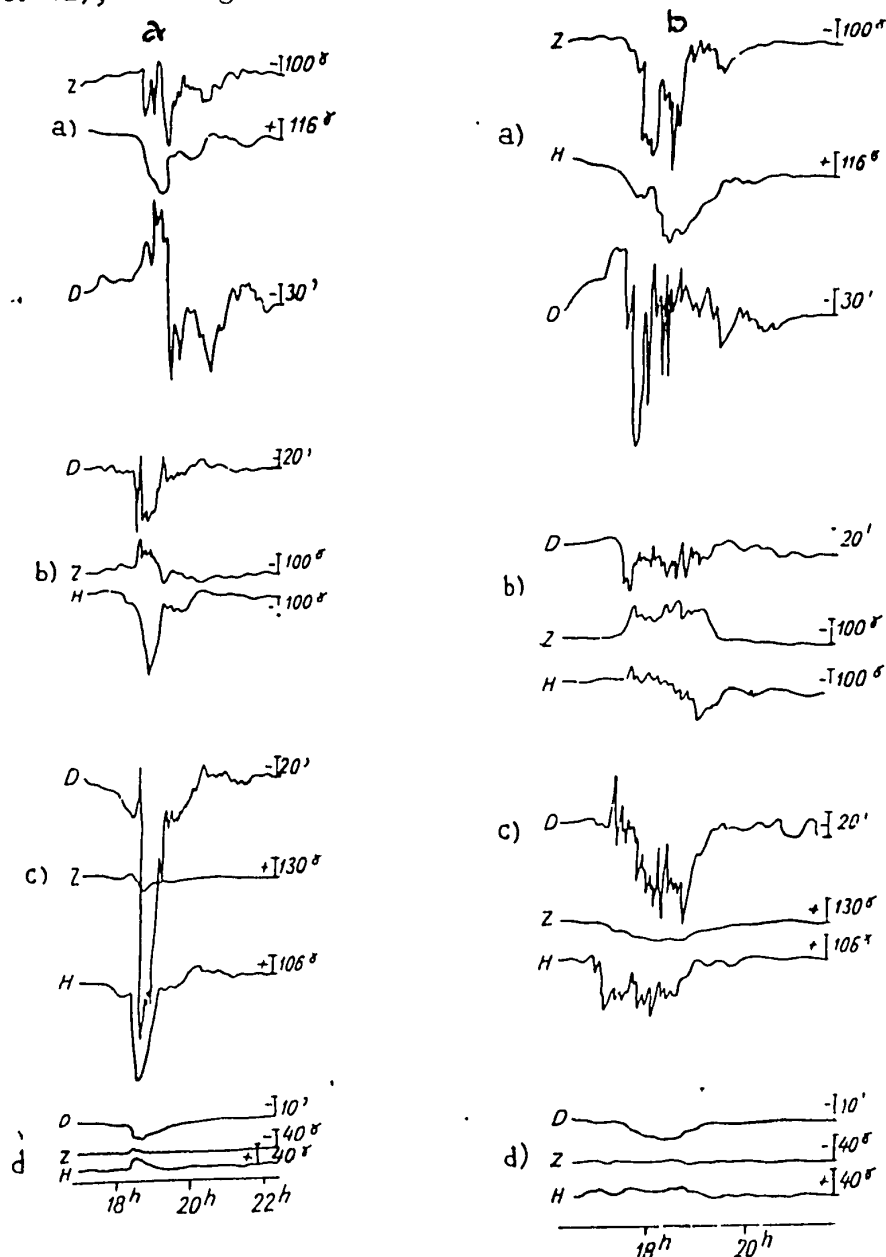


Fig.7 - Polar Storms. Universal Time (a - 14 March 1946; b - 13 March 1946).

a) Tikhaya Bay; b) Matochkin Shar; c) Dickson Island; d) Moscow

typical bay, shows very clearly that positive and negative polar storms are observed

simultaneously in the same latitude but in different longitudes. Polar storms are short-lived phenomena; the values of the magnetic elements return relatively rapidly

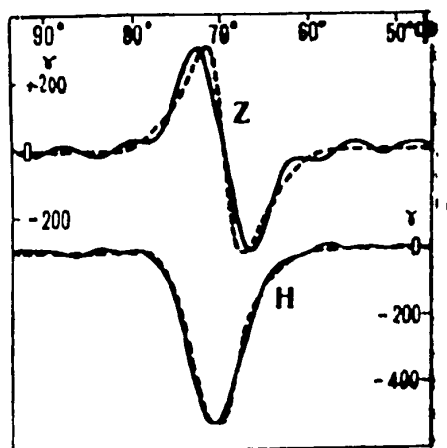


Fig.8 - H and Z Components of the Field of a Polar Storm, According to McNish ( — Observed Values; ----- Linear Current Calculated for the Field)

to normal, and no prolonged aftereffect can be detected on the magnetograms. Nor is it possible to detect such a persistence by statistical methods. At the Tikhaya Bay Observatory, the mean values of H and Z were calculated for 38 quiet intervals following immediately after Polar bay disturbances, and for entirely quiet days during the same period of time. The small difference obtained ( $2\gamma$  in H and  $-3\gamma$  in Z) indicates the absence of any aftereffect. A consideration of individual cases of polar storms (Figs.7 and 9) and of the typical picture (Fig.31) forces us to consider that the field of a polar storm also lacks the aperiodic dis-

turbed variation ( $D_{st}$ ) in the sense given to it in the papers by Chapman. Indeed, the field of the  $D_{st}$ -variations, by definition, depends only on the stormtime\* and does not depend either on the longitude nor on the local time, while the disturbance vector of a polar storm differs at different longitudes of the same parallel of latitude. On the other hand, the disturbance vector of a polar storm depends explicitly on the local time, and thus, retaining the Chapman notation, we might consider that the two parts  $S_D$  and  $D_1$  were present in the field of a polar storm. However, in view of the fact that the duration of a polar storm is much shorter than a day, it would be meaningless to speak of its diurnal periodicity; it is more correct,

\* The time reckoned from the beginning of a storm will everywhere henceforth be denoted by  $\tau$ ; for this reason, we will denote the frequently aperiodic disturbed variations by the symbol  $D_{st}(\tau)$ .

without delimiting the regular and irregular parts of the storm field, to consider that the field of a polar storm (which we will denote by the symbol  $P$ ) depends on the local time, i.e.,  $P(t)$ . The frequency of distribution of the positive and negative  $P$ -storms is unequal during the course of a day (cf. Table 13), and the mean intensity of the disturbance is likewise unequal. It follows from this that, in calculating the mean diurnal values of the elements for days with  $P$ -storms, we will have a systematic decrease in  $H$  with respect to the quiet days and a systematic elevation in  $Z$ . This must be borne in mind in considering the distribution of  $D_m = H_d - H_q$ . Thus, it seems to us that the accuracy of Nikol'skiy's conclusions that there are

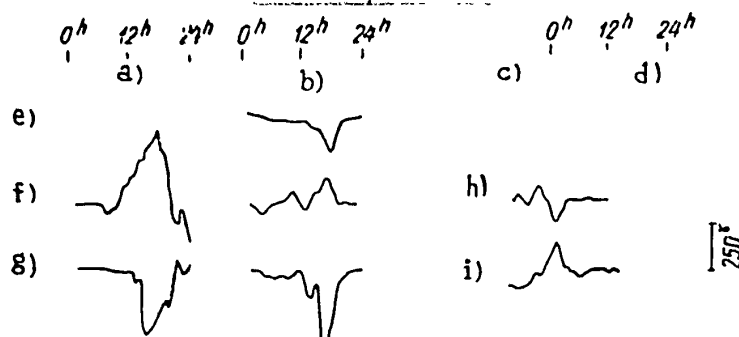


Fig.9 - Polar Storms of 1, 2 and 4 May 1933; Y-Component

Sveag. (Sveagruvan),  $\Phi = 74^\circ$ ,  $\Lambda = 131^\circ$ ; Chester. (Chesterfield),

$\Phi = 73^\circ$ ;  $\Lambda = 324^\circ$ ; M. o-va (Bear Islands),  $\Phi = 71^\circ$ ;  $\Lambda = 124^\circ$ ;

R.S. (Rude Skou),  $\Phi = 56^\circ$ ,  $\Lambda = 98^\circ$ ; Azhin. (Agincourt),  $\Phi = 55^\circ$ ,  $\Lambda = 374^\circ$

a) 1 May; b) 4 May; c) 1 May; d) 2 May; e) Sveag.; f) Chester.;

g) Bear Island; h) Rude Skou; i) Agincourt

neither regular  $D_{st}$ -variations nor stable aftereffects is greatest for the  $P$ -storms.

## Section 2. Worldwide Storms. $D_{st}$ -Variations

For worldwide storms (cf. Figs. 1 and 2), the most characteristic feature is the distribution of the disturbance over all latitudes, although the high latitudes are still the most disturbed. A second feature of worldwide storms is the existence of

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0 regular components of the field, the  $D_{st}$ - and  $S_p$ -variations. There can hardly be a  
 2 doubt as to the existence of  $D_{st}$ -variations in the temperate latitudes. Indeed, if  
 4 we consider magnetograms of low-latitude observatories, where the irregular fluctu-  
 6 ations distort only slightly the smooth parts of the elements, even during disturb-  
 8 ances, then, without any statistical treatment whatever, we will in each storm dis-  
 10 tinguish the positive and negative phases in the variations of the horizontal com-  
 12 ponent. The statistical computations described in Chapter I and the existence of a  
 very stable aftereffect, are other convincing arguments for the existence of these  
 variations. The slowness of the return of the magnetic elements to normal may be  
 judged from the noncyclic variations. The noncyclic variations, measured by the dif-  
 ference between the value of the element at  $24^h$  and at  $0^h$  on one and the same day,  
 are always positive for H and negative for Z, on international quiet days. This is  
 evidence that even on the quietest days, which are free from irregular fluctuations,  
 a slow return is observed (increase of H and decrease of Z) to the normal values of  
 the elements, i.e., the action of the  $D_{st}$ -current system is not interrupted.

However, the work by A.P.Nikol'skiy has made it necessary to answer the follow-  
 ing question: Do the  $D_{st}$ -variations exist in high latitudes? If they do, what are  
 the values of  $D_{st}$  in the auroral zone and on the polar cap? To answer this question  
 it is necessary to take account of the fact that, in the high latitudes, the irregu-  
 lar fluctuations are so great as to exceed, by many times, the possible value of the  
 regular part of the field. For this reason A.P.Nikol'skiy is justified in stating  
 that, if the irregular fluctuation obey any law (for instance, the predominant de-  
 crease of the H component), they will prevent elucidation of the true regularities  
 of the regular part of the field. In view of this fact, I decided, as A.P.Nikol'skiy  
 did, to evaluate the  $D_{st}$  field by calculating the mean diurnal value of the elements  
 at intervals for the stormy days free from irregular fluctuations. This method of  
 estimating  $D_{st}$  is somewhat arbitrary, since there are very few quiet intervals dur-  
 ing great magnetic storms, and their selection is not without subjectivity. In most

cases, the quiet intervals can be detected after the extinction of the great irregular fluctuations of the main phase, during the period of the aftereffect. By this method, I calculated the values of  $H_q = H_q^d$  and of  $Z_q - Z_q^d$  for the observatories of Honolulu, San Juan, Moscow, Cheltenham, Tucson, Sitka, Bear Islands, and Tikhaya Bay, whose magnetograms were available. As an example, Fig. 10 gives graphs of  $H_q - H_q^d$  and  $Z_q - Z_q^d$  for 1932/33 and 1947\*. Both parts of Fig. 10a show, in agreement, that the difference  $H_q - H_q^d$ , which reaches an order of 100  $\gamma$  in the equatorial region, declines monotonously with increasing latitudes, reaching practically zero at latitude  $70^\circ$ .

No increase of  $H_q - H_q^d$  in the auroral zone is detected, while the difference  $D_m = H_q - H_q^d$ , after a certain decrease at latitude  $50^\circ$ , increases sharply at latitude  $65-70^\circ$ \*\*. The values of  $Z_q - Z_q^d$  are negative, and in absolute value increase from small values in the low latitudes to large values for the observatories at Sitka and Bear Islands. In the low latitudes, we have  $H_q > D_m$ , which may be explained most easily by the fact that  $H_q - H_q^d$  was calculated for a few of the strongest storms, while  $D_m$  is the result of averaging the international disturbed days, most of which coincided with moderate and even small storms. The excess of  $H_q - H_q^d$  over  $D_m$  in the high latitudes is to be explained, in all probability, by the fact that in these latitudes, there is another factor besides  $D_{st}$ , which likewise systematically lowers  $H$  during the time of a disturbance. This factor, in my opinion consists of the irregular fluctuations superimposed on the regular part of the field of worldwide disturbance.

\* The values of  $H_q - H_q^d$  for Tikhaya Bay are taken from the paper by A.P. Nikol'skiy, and since the data for the Second International Polar Year and 1947 were unavailable, the data for 1934 and 1946 were used instead. During the entire 12-year period of 1934-1946 for which Nikol'skiy gives data, however, the value of  $H_q - H_q^d$  is at all times of the order of 2  $\gamma$ .

\*\* The graph of the latitude dependence of  $D_m$  for 1933, 1936, and 1938, constructed from data collected by me, is presented in Fig. 32 (Chapter VII).

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A consideration of magnetograms with polar storms on quiet days, magnetograms of moderately disturbed days, and of great magnetic storms will convince the investi-

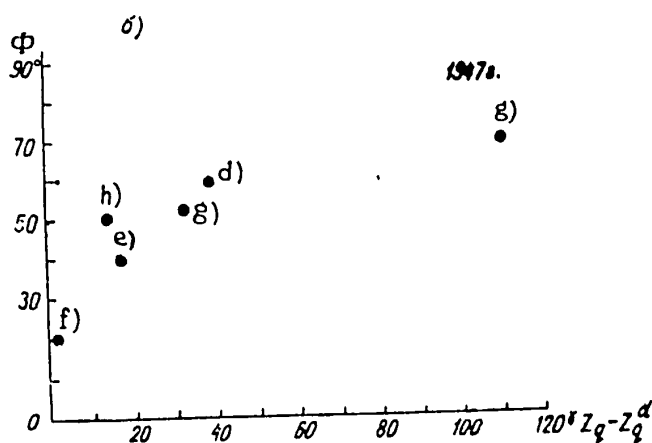
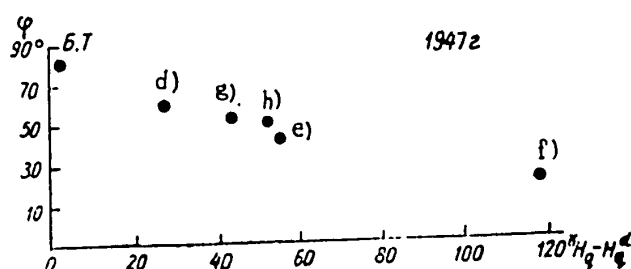
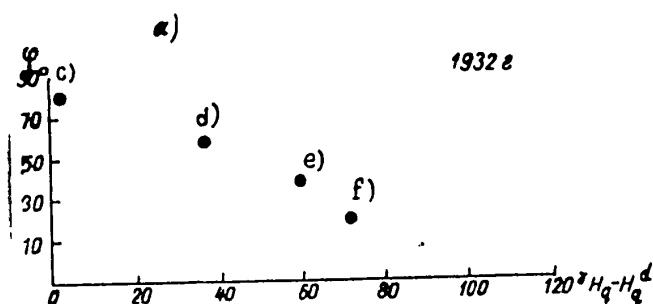


Fig.10 - Latitude Dependence: a) of  $H_q - H_q^d$ ; and b) of  $Z_q - Z_q^d$ ; c) Tikhaya Bay; d) Sitka; e) Thule; f) Honolulu; g) Bear Islands; h) Cheltenham

gator that the polar storms of a quiet day and the great irregular fluctuations during a worldwide storm are one and the same phenomenon; during a worldwide storm, a large number of polar storms of various amplitude and forms follow each other or are superimposed on each other, and, being accompanied by other forms of disturbances, give the impression of complex random fluctuations. This proposition has served as the basis for the conclusion drawn by Nikol'skiy that a magnetic storm is the sum of individual pulsations piled one on the other, a conclusion which in my opinion is erroneous. It is correct to assert that, during worldwide storms, a multitude of polar storms is always superimposed on the regular parts of the disturbance. A worldwide storm without polar storms is impossible; they are an inseparable part of it. But the worldwide storm is not a result of the simple summation of the fields of the individual

pulsations of polar storms. It has a fundamentally new property, the regular parts of the  $D_{st}$  and  $S_D$  field, which do not belong to the individual polar storms. This idea of the classification of storms into polar and worldwide, and of their inter-

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relation, may well solve the contradictions between individual investigators on the questions of the morphology and classification of magnetic storms. For instance, Nikol'skiy's conclusion that the regular lowering of  $H$  is absent during storms, due to the presence of  $D_{st}$ -currents, can apparently be explained as follows: In calculating the mean values of  $H_q^d$ , Nikol'skiy used the tabular material on the hourly amplitudes of  $r_H$  and the mean hourly values of the  $H$  component. He selected the values of  $H$  in those quiet hours (at small values of  $r_H$ ), which immediately followed strongly disturbed hours (at large values of  $r_H$ ). In most cases, such sequences of a disturbed hour followed by a quiet hour take place on days of polar storms since in the days of worldwide storms, the number of quiet intervals in general is very small. It is therefore natural enough that the statistical treatment should have disclosed a regularity inherent in the P-storms but not in the M-storms\*, i.e., an absence of any decrease in  $H$ . In the selection of quiet intervals for the calculation of  $H_q^d$  for Sitka, Bear Islands, and elsewhere, we used magnetograms of worldwide storms and, as shown by Fig.10, we obtained values of  $H_q - H_q^d$  different from zero.

Chapman, as already stated, used the values of  $D_m$  to construct the polar part of the  $D_{st}$ -currents. In the high latitude, however, the value of the horizontal component of the field of P-storms, superimposed on the regular parts of the field of a worldwide storm, is considerably greater than this same component of the  $D_{st}$  part. It is, therefore, only natural that the calculation of  $D_m$  by simply taking the average should reveal the properties of P-storms, i.e., the sharp increase of  $H_q - H_q^d$  in the polar zone.

We may also attempt to explain, from this point of view, M.N.Gnevyshev's views on the latitudinal distribution of the vector of disturbance. From Fig.1 and Table 1 of the Gnevyshev paper (Bibl.13) it would appear that, by the vector of disturbance  $\Delta F$ , he means the deviation from the normal values at the instant of some distinct maximum (for example, 2000 Y at the Matochkin Shar Observatory), due to a great

\* We will designate worldwide storms in this way to save space.

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P-storm, considerably exceeding the  $D_{st}$ -part of the worldwide storm in value. It is understandable from this that the relation between  $\Delta F$  and the distance from the auroral zone, which is depicted in Fig.4 of the paper cited, characterizes the geographic distribution of the field of a P-storm rather than that of an M-storm. This and analogous graphs served as grounds for Gnevyshev to dispute the current systems of the regular parts of the storm (the equatorial ring or the ionospheric systems of surface currents).

As for the storms in which the vector of disturbance increases toward the pole ("polar" storms, according to Gnevyshev's terminology), I am unfortunately unable to confirm or refute the existence of such storms, in view of the lack of empirical material that would be necessary for this. It goes without saying that the discovery of such storms, if indeed they exist, would be of great interest for the morphology and theory of magnetic disturbances.

The examples given above show very plainly the extent to which the ideas of an investigator about the morphology of a disturbance determine his theoretical views on the physical explanation of the phenomenon.

### Section 3. $S_D$ -Variations

Let us now discuss another regular part of the disturbed field, the  $S_D$ -variations, whose existence was doubted by Nikol'skiy. To study  $S_D$  I used the same method applied to  $D_{st}$ , namely, calculation of the variations for quiet intervals of disturbed days. Here I obtained about the same results for middle-latitude and high-latitude observatories as in calculating  $S_D$  by conventional methods, i.e.,  $S_D = S_d - S_q$ . The diurnal marches for the Sitka observatory presented in Fig.11 show that there is a great resemblance between  $S_d - S_q$  and  $S_q^d - S_q$ , except that the amplitudes of  $S_d - S_q$  are greater than the amplitudes of  $S_q^d - S_q$ . The calculation of  $S_d - S_q$  for the low-latitude station of Honolulu did not yield the expected results. In the low latitudes, the  $D_{st}$ -variations are so great that statistical treatment of a very large amount of material would be necessary in order to eliminate them, in spite of the

fact that the number of quiet intervals on stormy days there is very great. The quantity  $S_q^d - S_d$  was not calculated for the polar observatories, due to the lack of the necessary number of magnetograms; however, a simple examination of the individual disturbed days is enough to show, as could hardly have been expected, that the amplitude of the  $S_q^d - S_q$ -variations will decrease north of  $60^\circ$ . On the other hand, it would appear that these variations will behave in the high latitudes in the same way as  $S_D$ , i.e., their intensity will sharply increase in the auroral zone. Figure 11 allowed me to conclude that the second regular part of the field of worldwide storms, the disturbed diurnal variations, likewise has an existence quite as real as that of

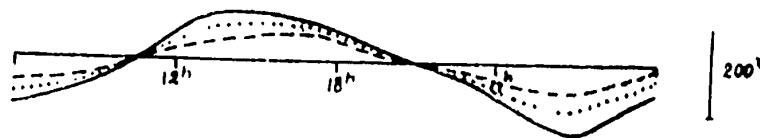


Fig.11 -  $S_D$  Variations of the Z-Component for Sitka Observatory  
(Local Time)

—  $S_q^d - S_q$ , - - -  $S_d - S_q$ , .....  $P(t)$

the  $D_{st}$ -variations, being found in all cases where the field is free from polar disturbances.

A second argument in favor of the existence of regular  $S_D$ -variations, evidently connected with the formation of a stable current system during worldwide storms, is the repetition of the active periods of a storm on successive days, which is well known to magnetologists. This repetition is manifested not only in the fact that the disturbance increases at one and the same hour of the day, but also in the fact that the main features and form of the fluctuations are sometimes repeated for several days in succession. This phenomenon is easily explained under the assumption of a current system encompassing the entire earth and fixed, if viewed from the sun. The

system exists for several days, at first developing and then weakening, repeating the fluctuations at the very same hours of each day. The  $S_D$ -current are apparently weaker in the low latitudes and considerably more intense in the high ones.

Returning to Fig.11, we may say that the  $S_D$ -variations of worldwide storms are very similar to the time-dependence of the field of polar storms. For comparison, we present in Fig.11 the diurnal variations of the Z component of the field of a typical polar storm (taken from the vector chart of Fig.31) for the latitude  $\phi = 60^\circ$ . It will be found that both curves, while differing somewhat in amplitude, have the same shape and the same times of the extremes. Thus the currents of the  $S_D$ -variations of worldwide storms and the currents of the P-storms, when superimposed, intensify each other without distorting each other.

#### Section 4. Division of the Field of Magnetic Storms

It follows from the above that the field of a worldwide magnetic storm may, in my opinion, be separated into four component parts:

$$M = D_{st}(\tau) + S_{DM}(t) + P(t) + D_i.$$

The value of the different parts in high and low latitudes is not the same. The part  $P(t)$  has a great weight (greater than the first terms) in the high latitudes, while in the moderate and low latitude it is so small that here, without great error, we may adopt the Chapman three-term equation.

$$D_{st} + S_D + D_i$$

and calculate the regular parts of  $D_{st}$  and  $S_D$  with conventional methods, by appropriately averaging the available data for the mean hourly values of the magnetic elements. The value of  $D_m$  can serve as a good estimate of the order of  $D_{st}$  in these latitudes; the  $S_D$ -variations can be calculated as the difference  $S_d - S_q$ . Approaching the auroral zone, all the weight of the terms  $P(t)$ ,  $S_D(t)$ , and  $D_i$  increases so much that it becomes difficult to separate the part of  $D_t$  by simple averaging, the more so since the value of  $D_{st}$  in the H component decreases; although it does increase in the Z component it still remains, in all probability, of the same order as in the temper-

ate latitudes\*. As will be seen later (cf. Chapter III), an attempt to calculate the  $D_{st}$  of a polar observatory by the ordinary method is unsuccessful. The value of  $D_m = H_q - H_d$  in the polar latitude ceases to characterize the value of  $D_{st}$ ; the predominance of negative polar storms has a strong influence on it. The  $S_d$ -variations in the polar latitudes, as in the low latitudes, may be taken as to  $S_d - S_q$ , bearing in mind the fact that in this way we are estimating both the regular part of the worldwide storm and the part due to the superimposition of the polar disturbances.

The properties and features of the polar storms are more easily studied by considering the isolated polar storms encountered on days free of worldwide storms, as has been done repeatedly by a number of authors.

The proposed division of the field of magnetic storms can be justified not only from the morphological point of view, but from the genetic as well. The  $D_{st}$ -variations can be considered as the field of the equatorial current ring, the  $S_d$ -variations as the field of the ionospheric currents encompassing the entire earth, and the P-storms as the result of the invasion of the ionosphere in high latitudes by corpuscles. For a worldwide storm, the presence of all three phenomena is characteristic: the formation of a ring current, the formation of ionospheric currents, and the deflection of the corpuscles toward the high latitudes. Penetration of the corpuscles in the high latitudes always accompanies the formation of great ionospheric and extra-ionospheric current systems, but such penetration can also take place without the formation of such systems. In such cases, only polar storms will be observed.

\* If we assume that the  $D_{st}$ -variations are really caused by the equatorial current ring, whose field close to the earth's surface is almost uniform, then the value of  $Z$  at the pole should be about equal to  $H$  at the equator. While the graph of  $Z_q - Z_q^d$  (cf. Fig. 10b) does not confirm this hypothesis, it still does not, in any case, contradict it.

The above point of view on the classification and division of the field of magnetic storms was used by us as a foundation for the workup and analysis of the material on magnetic disturbances. The  $D_{st}$ - and  $S_D$ -variations were isolated by statistical methods from the data on worldwide storms, and the three independent systems of electric currents, those of the  $D_{st}$ - and  $S_D$ -variations, and those of the P-storms, were calculated. The electric currents of several individual polar and worldwide storms were also studied, and their connection with the mean systems was shown.

## CHAPTER III

THE  $D_{st}$ -VARIATIONSSection 1. The Starting Materials

To assure uniformity of the starting material, the observations at all observatories were taken for one and the same interval of time, namely for 1931-1933. There were two reasons for selecting these years: first, the largest number of data have been published for 1931-1933; second, these years are years of minimum solar activity. In years of high activity, the superimposition of one storm on another makes it difficult to separate the storms and complicates any statistical investigations.

For 1931-1933, I succeeded in collecting data of the hourly values of the magnetic elements for 66 observatories, whose names and coordinates are given in Table 1. The Table shows that there are a sufficient number of stations located at various latitudes in the eastern hemisphere. The number of stations in the western hemisphere is definitely inadequate.

For 1931-1933 I selected 65 moderate and violent storms with amplitudes at Slutsk ranging from 180 to 450  $\gamma$ . It would have been desirable to determine the time of the beginning of the storm separately for each observatory. However, the lack of magnetograms from all observatories, that would be necessary for this, forced me to assume that the storms begin simultaneously over the entire earth, and to take the incipient moment according to the data of the Slutsk Observatory. A comparison of the beginnings of the storms for several observatories showed that the

Table 1

| No. | Observatory                      | $\phi$ | $\lambda$ | $\psi$ | $\varphi$ | $\lambda$ | $\eta$  | $\phi'$ |
|-----|----------------------------------|--------|-----------|--------|-----------|-----------|---------|---------|
| 1   | Thule (Tu.)                      | 88° 0  | 0° 0      | 0° 0   | 76° 5     | 291° 1    | - 19° 9 | 86° 9   |
| 2   | Godhavn (God.)                   | 79.8   | 32.5      | - 17.5 | 69.2      | 306.5     | - 11.2  | 78.2    |
| 3   | Scoresby Sound (S.Z.)            | 75.8   | 81.8      | - 36.2 | 70.5      | 338.0     | - 6.8   | 73.8    |
| 4   | Angmassalik (Ang.)               | 74.2   | 52.7      | - 22.5 | 65.6      | 322.4     | - 6.8   | 73.8    |
| 5   | Sveagruvan (Sv.)                 | 73.9   | 130.7     | - 46.2 | 77.9      | 16.8      | - 8.4   | 75.4    |
| 6   | Chesterfield (Ch.)               | 73.5   | 324.0     | + 14.9 | 63.3      | 269.3     | - 8.5   | 75.5    |
| 7   | Tikhaya Bay (B.T.) Calm Bay      | 71.5   | 153.3     | - 32.2 | 80.3      | 52.8      | - 8.8   | 75.3    |
| 8   | Bear Islands (M.O.)              | 71.1   | 124.5     | - 37.9 | 74.5      | 19.2      | - 5.1   | 72.1    |
| 9   | Julianenhavn (Yul.)              | 70.8   | 35.6      | - 13.8 | 60.7      | 314.0     | - 2.4   | 69.4    |
| 10  | Fort Rae (F.R.)                  | 69.0   | 290.9     | + 24.1 | 62.8      | 243.9     | - 2.7   | 69.7    |
| 11  | Point Barrow (P.B.)              | 68.6   | 241.2     | + 33.0 | 71.3      | 203.3     | - 2.6   | 69.5    |
| 12  | Tromsø (Tr.)                     | 67.1   | 116.7     | - 30.8 | 69.7      | 18.9      | + 0.3   | 66.7    |
| 13  | Chelyuskin (Chel.)               | 66.3   | 176.5     | - 3.2  | 77.7      | 104.3     | - 6.3   | 73.3    |
| 14  | Petsamo (Pet.)                   | 64.9   | 125.8     | - 27.6 | 63.5      | 31.2      | 0.0     | 67.0    |
| 15  | Matochkin Shar (M.Sh.)           | 64.8   | 146.5     | - 22.4 | 73.3      | 56.4      | - 1.2   | 68.2    |
| 16  | College, Fairbanks (K.F.)        | 64.5   | 255.4     | + 27.0 | 64.9      | 212.2     | + 2.0   | 65.0    |
| 17  | Sodankylä (Sod.)                 | 63.8   | 120.0     | - 26.7 | 67.4      | 26.6      | + 2.4   | 64.6    |
| 18  | Dickson Island (Dik.)            | 63.0   | 161.5     | - 12.8 | 73.5      | 80.4      | - 1.0   | 68.0    |
| 19  | Lerwick (Ler.)                   | 62.5   | 88.6      | - 23.6 | 60.1      | 358.8     | + 6.2   | 60.8    |
| 20  | Kandalaksha (Kan.)               | 62.5   | 124.2     | - 25.0 | 67.1      | 32.4      | -       | -       |
| 21  | Dombas (Dom.)                    | 62.4   | 100.2     | - 23.6 | 62.1      | 350.9     | + 6.5   | 61.5    |
| 22  | Minuk (Min.)                     | 61.8   | 301.0     | + 17.2 | 54.6      | 246.7     | + 3.8   | 63.2    |
| 23  | Uellen (Uel.)                    | 61.8   | 235.9     | + 24.5 | 66.2      | 190.2     | + 4.4   | 62.6    |
| 24  | Sitka (Si.)                      | 60.0   | 275.4     | + 21.4 | 57.0      | 224.7     | + 5.5   | 61.5    |
| 25  | Fakdalemuir (Esk.)               | 58.5   | 82.9      | - 20.4 | 55.3      | 356.8     | +10.5   | 56.5    |
| 26  | Lovo (Lov.)                      | 58.1   | 105.8     | - 22.1 | 59.4      | 17.8      | + 9.7   | 57.3    |
| 27  | Slutsk (Sl.)                     | 56.0   | 116.3     | - 20.6 | 59.9      | 30.5      | 10.2    | 56.8    |
| 28  | Rude Skov (R.S.)                 | 55.8   | 98.5      | - 20.6 | 55.8      | 12.4      | 12.7    | 54.7    |
| 29  | Agin-court (Azh.)                | 55.0   | 347.0     | + 3.8  | 43.8      | 280.7     | 11.5    | 55.5    |
| 30  | Abinger (Ab.)                    | 55.0   | 83.3      | - 18.4 | 51.2      | 359.6     | 14.8    | 52.2    |
| 31  | de Bil (D.B.)                    | 53.8   | 89.6      | - 18.9 | 52.1      | 5.2       | 15.4    | 51.6    |
| 32  | Srednikan (Sred.)                | 53.2   | 210.5     | + 12.7 | 62.6      | 152.3     | 9.5     | 57.5    |
| 33  | Moscow (Mos.)                    | 52.2   | 120.3     | - 17.0 | 55.5      | 37.3      | 14.3    | 52.7    |
| 34  | Paris - Val Joyeux (V.Zh.)       | 51.3   | 84.5      | - 17.5 | 48.8      | 2.0       | 19.7    | 47.3    |
| 35  | Yakutsk (Yak.)                   | 51.0   | 193.8     | + 5.8  | 62.0      | 129.7     | 10.8    | 56.2    |
| 36  | Svidler (Sv.)                    | 50.6   | 104.6     | - 18.3 | 50.1      | 21.2      | 17.8    | 49.2    |
| 37  | Cheltenham (Chelt.)              | 50.1   | 350.5     | + 2.4  | 38.7      | 283.2     | 16.5    | 50.5    |
| 38  | Kazan' (Kaz.)                    | 49.3   | 130.4     | - 15.7 | 55.8      | 48.8      | 15.9    | 51.1    |
| 39  | Sverdlovsk (Sver.)               | 48.8   | 140.7     | - 13.3 | 56.7      | 61.1      | 15.9    | 51.1    |
| 40  | Zuy (Irkutsk) (Ir.)              | 41.0   | 174.4     | - 1.8  | 52.5      | 104.0     | 18.6    | 48.4    |
| 41  | San Fernando (S.Fer.)            | 41.0   | 71.3      | - 13.6 | 36.5      | 353.8     | 24.8    | 42.2    |
| 42  | Tucson (Tuk.)                    | 40.4   | 312.2     | + 10.1 | 32.2      | 249.2     | 24.6    | 42.4    |
| 43  | South Sakhalin (Toyohara) (Toy.) | 36.9   | 203.5     | + 6.7  | 47.0      | 142.8     | 22.4    | 44.6    |
| 44  | Tbilisi (Tb.)                    | 36.7   | 122.1     | - 13.2 | 42.1      | 44.7      | 29.7    | 37.3    |
| 45  | Maytun (Mt.)                     | 32.4   | 198.3     | + 4.9  | 43.2      | 132.3     | 27.2    | 39.8    |
| 46  | Tashkent (Tash.)                 | 32.4   | 143.7     | - 9.0  | 41.3      | 69.3      | 31.6    | 35.4    |
| 47  | San Juan (S.Zh.)                 | 29.9   | 3.2       | - 0.7  | 18.4      | 293.9     | 37.3    | 29.7    |
| 48  | Teoloyucan (Teo.)                | 29.6   | 327.0     | + 6.6  | 19.8      | 260.8     | 35.6    | 31.4    |
| 49  | Helwan (Khel.)                   | 27.2   | 106.4     | - 12.7 | 29.9      | 31.3      | 40.0    | 27.0    |
| 50  | Kakioka (Kak.)                   | 26.0   | 206.0     | + 6.2  | 36.2      | 140.2     | 35.4    | 31.6    |
| 51  | Honolulu (Hon.)                  | 21.1   | 266.5     | + 12.3 | 21.3      | 201.9     | 45.4    | 21.6    |
| 52  | Zo-se (Z.Z.)                     | 20.0   | 189.1     | + 2.1  | 31.3      | 121.0     | 42.7    | 21.1    |
| 53  | Hong Kong (G.K.)                 | 11.0   | 182.9     | + 0.6  | 22.4      | 114.0     | 49.7    | 17.3    |
| 54  | Alibag (Bombay) (Bom.)           | 9.5    | 143.6     | - 7.2  | 18.6      | 72.9      | 54.8    | 12.2    |
| 55  | Manila (Antipolo) (An.)          | 3.3    | 189.8     | + 2.0  | 14.6      | 121.2     | 57.8    | 9.2     |
| 56  | Huancayo (Khuan.)                | - 0.6  | 353.8     | + 1.3  | - 12.0    | 284.7     | -       | -       |
| 57  | Elizabethville (Yel.)            | - 12.7 | 94.0      | + 11.7 | - 11.7    | 27.5      | -       | -       |
| 58  | Apia (Ap.)                       | - 16.0 | 260.2     | + 11.7 | - 13.8    | 188.2     | -       | -       |
| 59  | Batavia (Bat.)                   | - 18.0 | 175.6     | - 0.9  | - 6.6     | 106.8     | -       | -       |
| 60  | Pilar (Pil.)                     | - 20.2 | 4.6       | - 1.1  | - 31.7    | 296.1     | -       | -       |
| 61  | Mauritius (Mav.)                 | - 26.6 | 122.4     | - 10.3 | - 20.1    | 57.6      | -       | -       |
| 62  | Cape Town (K.T.)                 | - 32.7 | 79.9      | - 13.7 | - 33.9    | 18.5      | -       | -       |
| 63  | Watheroo (Wat.)                  | - 41.8 | 185.6     | + 1.3  | - 30.3    | 115.9     | -       | -       |
| 64  | Toolangi (Tul.)                  | 46.7   | 220.8     | + 9.5  | - 37.5    | 145.5     | -       | -       |
| 65  | Amberley (Amb.)                  | 47.7   | 252.5     | + 15.1 | 43.5      | 172.7     | -       | -       |
| 66  | South Orkney Islands (Ork.)      | 50.0   | 18.0      | - 7.2  | - 60.8    | 315.1     | -       | -       |



error involved in this assumption is not greater than  $\pm 2$  hours, since in the majority of cases the storms begin simultaneously with an accuracy of 1 hour. The calculation of  $D_{st}$  was performed by the method proposed in his day by Moos. The hourly values of the magnetic elements were entered for each storm on a separate line, and the resultant Table was averaged by columns corresponding to the hours of the time, reckoned from the beginning of the storm. This averaging eliminates the irregular fluctuations and the systematic  $S_D$ -variations, provided only that the beginnings of the storms are distributed with sufficient regularity among the hours of the day. The distribution of the 65 storms selected by me revealed a marked predominance of storms beginning in the morning hours. In view of this fact, I excluded 11 storms beginning a 4-7<sup>h</sup> Universal Time from those selected by me. The final list of the 54 storms, used as the basis for the calculations, is given in Table 2.

The  $D_{st}$ -variations of the three elements were calculated for 34 hours: from the 4<sup>th</sup> hour before the onset of the storm to the 30<sup>th</sup> hour after the onset. The calculation of the  $D_{st}$  of the declination, or of the Y-component, showed that neither in the high nor in the low latitudes was it possible to discern any regularity in the variations of these elements during the course of the storm. Since the previous literature also contained references to the absence of distinct  $D_{st}$ -variations of the declination, the data on the accumulation were not included in the consideration, and I assumed that the horizontal component of the field strength of  $D_{st}$  lies roughly (at least in the low and middle latitudes) in the plane of the magnetic meridian.

The  $D_{st}$ -variations of the H and Z components for the individual observatories are shown in Figs. 12 and 13. The time indicated on the diagram is the time reckoned from the onset of the storm; the observatories are located in the order of decreasing geographic latitudes. Our attention is struck by the mobility and irregularity of many curves, which is particularly marked on comparison of Figs. 12 and 13 with the well-known  $D_{st}$  graphs of Chapman (Bibl. 40). An explanation of the presence of random fluctuations on the graphs of Figs. 12 and 13 might in all probability be

Table 2

## List of Moderate and Great Magnetic Storms for 1931-1934

| No. of Storm | Date    | Onset | No. of Storm | Date    | Onset | No. of Storm | Date   | Onset | No. of Storm | Date   | Onset |
|--------------|---------|-------|--------------|---------|-------|--------------|--------|-------|--------------|--------|-------|
|              | 1931 r  |       |              |         |       |              |        |       | 43           | 5/VIII | 4 h   |
| 1            | 16/I    | 3 h   | 14           | 3/II    | 2 h   | 29           | 15/X   | 4 h   | 44           | 8/IX   | 14    |
| 2            | 24/II   | 0     | 15           | 3/III   | 12    | 30           | 20/X   | 6     | 45           | 7/XI   | 11    |
| 3            | 1/VI    | 11    | 16           | 10/III  | 6     | 31           | 15/XI  | 13    | 46           | 9/XII  | 9     |
| 4            | 20/VIII | 1     | 17           | 28/III  | 5     | 32           | 14/XII | 7     |              |        |       |
| 5            | 30/IX   | 18    | 18           | I/IV    | 10    |              |        |       |              |        |       |
| 6            | 2/X     | 5     | 19           | 13/IV   | 8     |              |        |       |              |        |       |
| 7            | 12/X    | 9     | 20           | 22/IV   | 7     |              | 1933 r |       |              | 1934 r |       |
| 8            | 5/XI    | 13    | 21           | 23/IV   | 2     | 33           | 19/I   | 10    | 47           | I/I    | 4 h   |
| 9            | 8/XI    | 1     | 22           | 25/IV   | 10    | 34           | 19/II  | 7     | 48           | 8/II   | 13    |
| 10           | 26/XI   | 7     | 23           | 27/IV   | 10    | 35           | 20/II  | 4     | 49           | 4/III  | 10    |
| 11           | 2/XII   | 5 h   | 24           | 2/V     | 16    | 36           | 21/II  | 8     | 50           | 30/III | 15    |
|              |         |       | 25           | 4/V     | 13    | 37           | 19/III | 11    | 51           | 29/VII | 23    |
|              |         |       | 26           | 29/V    | 7     | 38           | 22/III | 21    | 52           | 24/IX  | 2     |
|              | 1932 r  |       | 27           | 26/VIII | 8     | 39           | 24/III | 0     | 53           | 7/XI   | 11    |
|              |         |       | 28           | 5/IX    | 22    | 40           | 17/IV  | 6     | 54           | 7/XII  | 16    |
| 12           | 25/I    | 5 h   |              |         |       | 41           | 12/VI  | 16    |              |        |       |
| 13           | 27/I    | 4     |              |         |       | 42           | 23/VII | 6     |              |        |       |

explained by the insufficient experimental material and the absence of any smoothing process. A consideration of  $D_{st}$  at the polar observatories ( $\Phi > 65^\circ$ ) shows that the averaging of the data for 54 storms eliminated neither the irregular part of the field nor the  $S_D$ -variations. The influence of the  $S_D$ -variations is manifested in the marked diurnal periodicity of the curves presented, which is particularly strong

for the observatories at Dickson Island, Matochkin Shar, Tromso, Petsamo, and Sodankyla. Thus, the curves presented confirm the assertion made in the preceding Chapter to the effect that the  $D_{st}$ -variations in the polar regions cannot be calculated

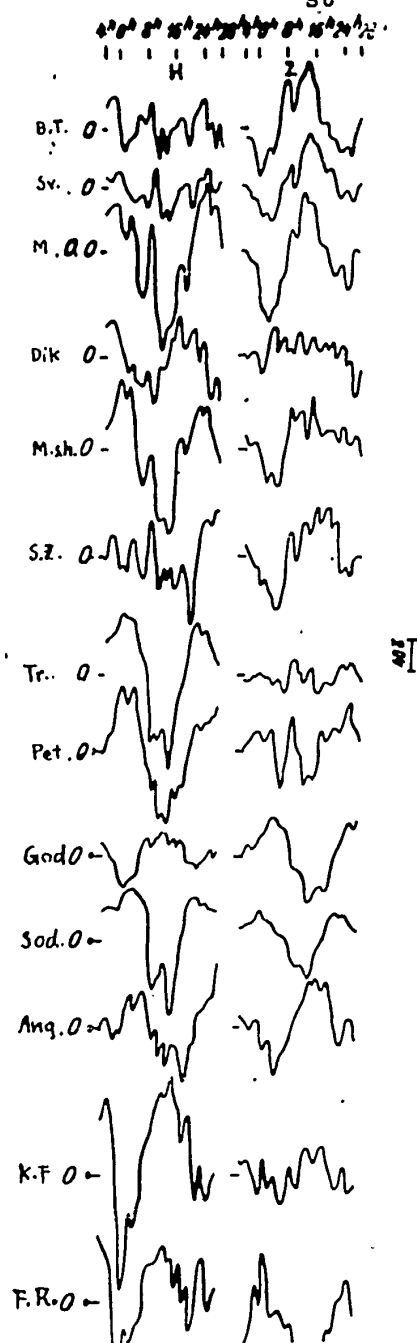


Fig.12 -  $D_{st}$ -Variations (Polar Observatories) \*

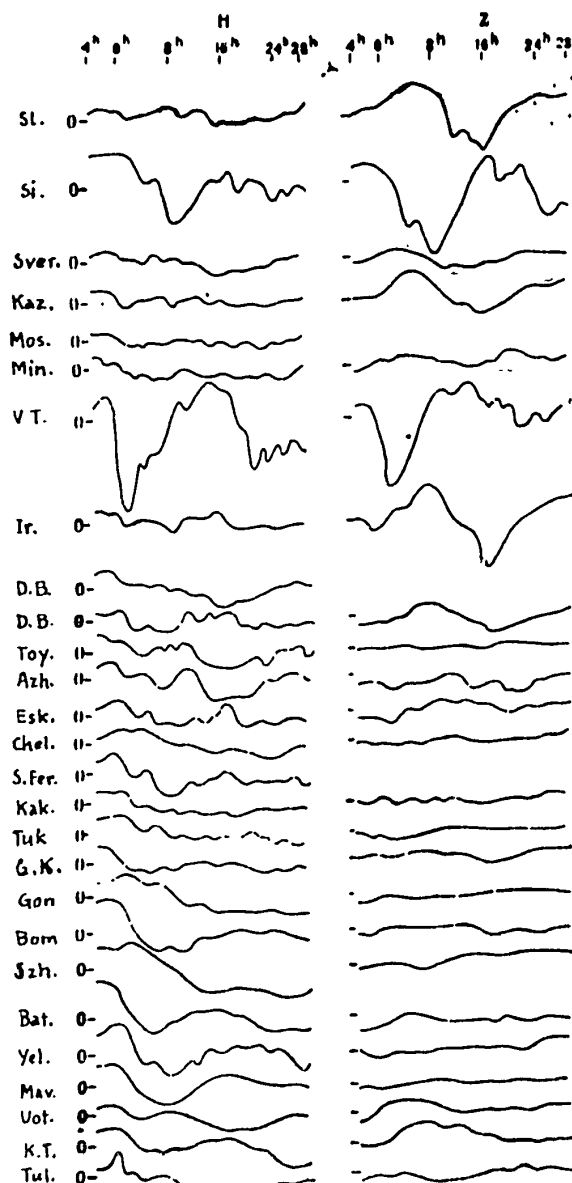


Fig.13 -  $D_{st}$ -Variations (Middle-Latitude Observatories)

\* Translator's note: For meaning of abbreviations in diagram, see Table 1.

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by the Moos-Chapman method. In the polar regions, the irregular fluctuations ( $D_1$ ), the Polar storms (P), and the  $S_D$ -variations are so great that a simple averaging of the material does not eliminate them. In view of this fact it appeared to be inadvisable to use the data of the polar observatories given in Fig.12 in calculating the potential function of the  $D_{st}$  field, characterizing the course of the disturbance over the entire earth.

A consideration of the  $D_{st}$ -variations of the H and Z components of the middle-latitude observatories ( $\Phi < 62^\circ$ ) allows us to draw the following conclusions:

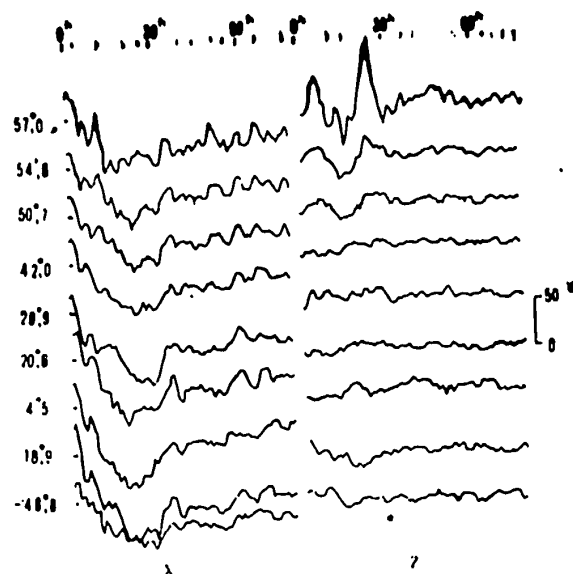


Fig.14 - The  $D_{st}$ -Variations  
(After Vestine)

1. The principal feature of  $D_{st}$  is the lowering of the H component which is well perceptible at all latitudes, and is less in value than the increase in the Z component.
2. The  $D_{st}$ -variations of the H component are so similar in the northern and southern hemispheres, both in form and in sign, that it may be assumed that the distribution of H is symmetric with respect to the geomagnetic equator. The distribution of the Z component, on the other hand,

is asymmetric with respect to the equator.

3. The  $D_{st}$ -variations of observatories lying at the same geomagnetic latitudes, on the whole, resemble one another. Thus, it may be assumed that, in first approximation, the  $D_{st}$  field depends on two arguments: the geomagnetic latitude  $\Phi$  and  $\psi$  the time elapsed from the beginning of the storm.

4. A more detailed consideration of Fig.13 shows that the  $D_{st}$ -variations of the observatories of the western hemisphere differ from those of the eastern hemisphere. A difference is also noted between the observatories of the same hemisphere (for

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example, Irkutsk-de Bilt, Cape Town-Toolangi, etc.). It follows from the examples given that the field of  $D_{st}$  likewise contains longitudinal terms which, at one time, were found in the  $S_Q$ -variations (Bibl.9).

The first phase of the storm (increases of the H component), noted by many investigators, was found to be vague on many curves of Fig.13. The possibility is not excluded that the absence of the first phase is connected with a certain inaccuracy in the determination of the time of onset of the storm, which might occur in cases when the storms begin gradually. To verify this assumption, I considered the data on the storms with sudden onsets ( $S_Q$ ). The  $D_{st}$ -variations obtained by averaging 13  $S_Q$  storms during the same interval of time, are given in Fig.14, constructed from materials furnished by Vestine (Bibl.62). Each curve of Fig.14 represents the mean for several observatories located at the corresponding latitude. A comparison of Figs.13 and 14 shows the great regularity in the distribution of the curves of Fig.14 and the presence of a distinct initial phase in them. The remaining conclusions enumerated by us with respect to the form and distribution of the  $D_{st}$ -variations are fully confirmed by Fig.14.

## Section 2. Spherical Analysis of the $D_{st}$ -Variations

In all possible types of calculation of a theoretical nature it is more convenient to operate not with the observed elements of the magnetic field H, D, Z, but with rectangular components. The geomagnetic components of the field of variations,  $X'$ ,  $Y'$ ,  $Z'$  are connected with the variations of D, H, Z by the following relations:

$$X' = H \cos(D_0 - \psi) - H_0 \sin(D_0 - \psi) \sin 1'D', \quad (1)$$

$$Y' = H \sin(D_0 - \psi) + H_0 \cos(D_0 - \psi) \sin 1'D', \quad (2)$$

$$Z' = Z,$$

where  $D_0$  and  $H_0$  are the mean annual values of these elements, and  $\psi$  is the angle between the geographical meridian of a given place.

In our case (absence of substantial and regular fluctuations in D), the second terms of eqs.(1) and (2) were rejected and it was shown that the variations of  $Y'$

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are small in comparison with  $X'$ . Thus the rectangular components of the  $D_{st}$  field were reduced to a calculation of  $X'$  by the formula  $X' = H \cos (D_0 - \psi)$ , the variations of  $X'$  so obtained proving to be very similar to those of the H component.

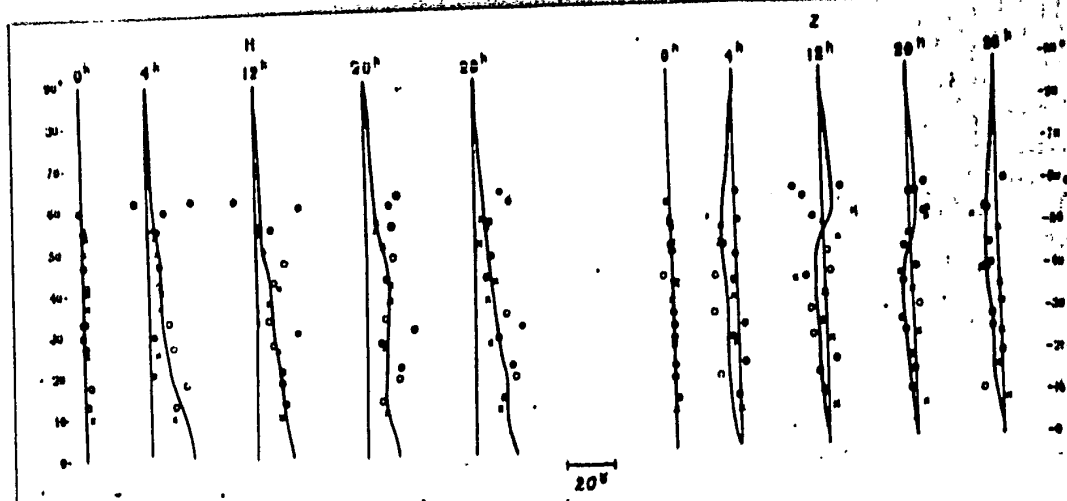


Fig.15 - Latitudinal Distribution of H and Z Components of the  $D_{st}$ -Variations  
 $\times$  = Northern hemisphere;  $\circ$  = Southern hemisphere; not taken into account  
 in calculating the mean.

Figure 15 gives graphs of the dependence of  $X'$  and Z on  $\Phi$  for a few stormtimes. The data of the southern and northern hemispheres are presented together, allowing for the symmetry of the H component and the asymmetry of the Z component. The  $D_{st}$ -variations of the individual observatories (Fig.13) were first averaged by groups in accordance with the value of  $\Phi$ , and the mean data were then entered in Fig.15. The dispersion of the points on both graphs is relatively low, and, in particular, there is absolutely no detectable systematic difference between the data of the northern and southern hemispheres,

Since we abandoned the use of the  $D_{st}$ -variations calculated from the observations for the polar observatories, the curves of  $X'$  and Z were extrapolated to the high latitudes, and, in accordance with the considerations made in Chapter II, it was assumed that, at the pole,  $X' = 0$  and  $Z \approx X'$  equiv. The comparatively simple form of the dependence of  $X'$  and Z on  $\Phi$  allowed us to use the method of spherical

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analysis for the analytic representation of the field.

Neglecting the longitude-dependence of  $D_{st}$ , the potential of the field may be represented, for a fixed instant of time  $t$ , by a series of Legendre polynomials:

$$V = R \sum_n \left[ I_n \left( \frac{R}{r} \right)^{n+1} + E_n \left( \frac{r}{R} \right)^n \right] P_n(\cos \theta), \quad (3)$$

where, as usual, the terms  $E_n$  are responsible for that part of the field due to sources external to the earth's surface, while the terms  $I_n$  are responsible for the internal part of the field.

On the earth's surface,  $r = R$ , and

$$V = R \sum_n g_n P_n(\cos \theta), \quad (4)$$

where

$$g_n = E_n + I_n. \quad (5)$$

Hence it follows that the rectangular components of the field in geomagnetic coordinates, for  $r = R$ , will be as follows:

$$X' = \frac{1}{R} \frac{\partial V}{\partial \theta} = \sum_n g_n \frac{dP_n(\cos \theta)}{d\theta}, \quad (6)$$

$$Z' = \frac{\partial V}{\partial r} = \sum_n j_n P_n(\cos \theta), \quad (7)$$

where

$$j_n = nE_n - (n+1)I_n. \quad (8)$$

The identity  $Y = 0$  completely corresponds to the absence of systematic  $D_{st}$ -variations noted by us in the D and Y elements. The calculation of the potential of the  $D_{st}$  field was performed independently for 56 moderate storms (analysis I) and for 13 storms with sudden onset (analysis II). The method of calculation in both cases was one and the same. To find the coefficients of  $g_n$  on the basis of the graphs of the dependence  $X'(\Phi)$  (the heavy curve in Fig.15), I calculated the curves  $F(\theta) = X' \sin \theta$  which, in turn, were represented by Fourier series in  $\theta$

$$F(\theta) = \sum_k \beta_k^x \sin k\theta. \quad (9)$$

On the basis of the coefficients  $\beta_k$ , I used the Schuster-Schmidt formulas (Bibl.6, 9) for calculating the constants  $g_n$ , whose values are given in Tables 3

and 4. The constants  $j_n$  (also see Tables 3 and 4) were found by the Schuster formula by direct expansion of  $Z$  into a Fourier series in  $\theta$

$$Z = \sum_k \beta_k^z \sin k\theta. \quad (10)$$

Table 3

Analysis I

| $\tau$ | $g_1^Y$ | $g_2^Y$ | $g_3^Y$ | $j_1^Y$ | $j_2^Y$ | $j_3^Y$ |
|--------|---------|---------|---------|---------|---------|---------|
| 4 h    | 24.93   | -1.23   | 3.19    | 6.70    | -5.11   | 0.23    |
| 12     | 21.74   | -3.56   | 0.79    | 1.36    | -6.21   | 1.79    |
| 20     | 23.14   | -0.61   | -0.14   | 0.84    | -4.65   | 2.38    |
| 28     | 27.80   | -3.16   | 0.16    | 6.44    | -4.49   | 0.63    |

The coefficients of the expansion of the potential into series of spherical harmonics were repeatedly calculated by the Schuster method (Bibl.6, 9, 15), but nevertheless the application of this method requires certain explanations. The Schuster method is based on the replacement in the expression

$$\begin{aligned} & \sum_n \sum_m (g_n^m \cos m\lambda + h_n^m \sin m\lambda) = \\ & = \sum_m \left[ \cos m\lambda \sum_{n=m}^q g_n^m P_n^m + \sin m\lambda \sum_{n=m}^q h_n^m P_n^m \right] \end{aligned} \quad (11)$$

of the series

$$k^m(\theta) = \sum_n g_n^m P_n^m \quad \text{и} \quad l^m(\theta) = \sum_n h_n^m P_n^m$$

by the series

$$k^m(\theta) = \sum_s a_s \cos s\theta; \quad l^m(\theta) = \sum_s a_s \cos s\theta \quad (12)$$

or

$$k^m(\theta) = \sum_s \beta_s \sin s\theta; \quad l^m(\theta) = \sum_s b_s \sin s\theta \quad (13)$$



and on the calculation of  $g$  and  $h$  in terms of  $\alpha_s, a_s$  or  $\beta_s, b_s$ . For  $m = 2t$ , Schuster recommends using eq.(13), and for  $m = 2t + 1$ , eq.(12). In these cases, the equations connecting  $g, h, c, \alpha$  or  $\beta, b$ , are rather simple and convenient for calculation.

Table 4

## Analysis II

| $\tau$ | $g_1^Y$ | $g_2^Y$ | $g_3^Y$ | $j_1^Y$ | $j_2^Y$ | $j_3^Y$ |
|--------|---------|---------|---------|---------|---------|---------|
| 1 h    | -33.3   | -0.54   | 1.15    | -0.98   | -6.19   | -       |
| 10     | 44.1    | -2.05   | -0.63   | 3.20    | -15.5   | -6.6    |
| 20     | 81.8    | -0.90   | -1.55   | -1.20   | -35.80  | 6.6     |
| 30     | 69.3    | -0.66   | -1.32   | 15.8    | -12.84  | -0.7    |
| 40     | 47.8    | -0.54   | -2.01   | 10.7    | -11.41  | 2.46    |
| 60     | 32.8    | -1.63   | -1.24   | 9.36    | - 9.92  | 1.10    |

However, the representation of the function  $k^m(\theta)$  ( $m$  being odd) known in the interval from 0 to  $\pi$ , by the series  $\sum \beta_s \sin s\theta$  imposes on it the conditions of asymmetry with respect to  $\theta = \pi$  and of its vanishing at the points  $\theta = 0$  and  $\theta = \pi$ . If the empirical function being studied satisfies these conditions, then the application of the Schuster method is theoretically irreproachable, and in practice assures high accuracy in the computation of the coefficients  $g, h$ . If, however,  $k^m(\theta)$  differs substantially from 0 at the poles, then the use of the Schuster method distorts the distribution of the function in the polar caps and may introduce substantial errors. It follows that an application of this method to the calculation of the coefficients of an expansion in spherical harmonics of the Z component of the earth's permanent magnetic field, of the  $D_{st}$  field, or of the noncyclic variations (i.e., of functions in the representation of which the terms  $P_1, P_3$ , etc., play the principal role) is not completely successful, and, in any case, should be accompanied by an estimate of the error to be expected. Accordingly, the calculation of the co-

efficients  $j_n$  in eq.(7) for one instant of time ( $\tau = 12$  hours) were calculated both by the Schuster method and the method of least squares. The good agreement between the results of the two methods (discrepancy of the order of 1%) and also a comparison of the initial curves of  $Z(\theta)$  with those calculated by eq.(7) (cf. Table 5) showed that we could calculate  $j_n$  by the Schuster method without great danger. Table 5 shows that the deviation of the calculated curve of  $Z(\theta)$  from the empirical curve is significant only in the polar regions, where, all the same, we do not know the true distribution of the field.

It follows from the symmetry of  $X'$  and the asymmetry of  $Z$  with respect to the equator ( $\theta = \pi/2$ ) that, in the expression for the potential, eq.(4) must contain only odd polynomials.

The numerical values of the constants  $g$  and  $j$  in analyses I and II (Tables 3, 4) differ somewhat but are still in good agreement. In both cases, the first harmonic has the greatest weight, having a coefficient  $g_1$  of the order of 20-30  $\gamma$  in the first case and 50-80  $\gamma$  in the second. The larger values of the coefficients in analysis II may possibly explained by the fact that almost all the storms making up the 13  $S_c$ -storms selected were great storms, while most of the storms among the 54 storms of analysis I belong to the category of moderate storms. The coefficients  $g_1$  calculated for  $D_m$  by McNish and for  $D_{st}$  by Chapman and Whitehead (Bibl.40), lie within these same limits (30-50  $\gamma$ ). The sign of  $g_1$  is everywhere positive, except for the first hour in analysis II, corresponding to the first phase of a magnetic storm. The sign of the coefficient of the third harmonic  $g_3$  is negative, while the values of  $g_5$  include both positive and negative quantities. Of the coefficients representing the expansion of  $Z$ , the greatest is  $j_3$ , characterizing the stable negative values.

It will be seen from a comparison of the coefficients  $g$  and  $j$  at different instants of time that the storm reaches its maximum development at the end of the first day or the beginning of the second. The calculation by eqs.(5) and (8) of the coefficients of the internal and external fields of  $E$  and  $I$  separately gave the re-

sults shown in Table 6. In all cases, except for one, the absolute value of the external field is greater than that of the internal field.

Table 5

$D_{st}$ -Variations of the Z Components, in

|     | $\tau = 4 \text{ h}$ |            | $\tau = 20 \text{ h}$ |            |
|-----|----------------------|------------|-----------------------|------------|
|     | Observed             | Calculated | Observed              | Calculated |
|     |                      |            |                       |            |
| 70° | 3                    | 4          | -2                    | -2         |
| 50  | 7                    | 8          | -2                    | -3         |
| 30  | 5                    | 4          | 5                     | 7          |
| 10  | 3                    | 3          | 2                     | 3          |

The ratio  $I/E$  (Table 7) is very stable for the first harmonic (mean value  $0.40 \pm 0.07$  and  $0.39 \pm 0.10$ ). Within wide limits, the ratio fluctuates for the third harmonic ( $-0.17 \pm 0.12$  and  $-0.61 \pm 0.18$ ) and is not very regular for the fifth

Table 6

| $\tau$ | $E_1^Y$ | $E_3^Y$ | $E_5^Y$ | $I_1^Y$ | $I_3^Y$ | $I_5^Y$ |
|--------|---------|---------|---------|---------|---------|---------|
| 4 h    | 18.8    | -1.4    | 1.8     | 6.1     | 0.2     | 1.4     |
| 12     | 14.9    | -1.8    | 0.6     | 6.8     | 0.2     | 0.2     |
| 20     | 15.7    | -1.0    | 0.2     | 7.4     | 0.4     | -0.3    |
| 28     | 20.7    | -1.6    | 0.2     | 7.1     | 0.0     | 0.0     |
| 1      | -22.2   | -1.2    | -       | -11.1   | 0.7     | -       |
| 10     | 30.5    | -3.4    | -       | 13.6    | 1.3     | -       |
| 20     | 54.1    | -5.6    | -       | 27.7    | 4.8     | -       |
| 30     | 51.4    | -2.2    | -       | 17.8    | 1.7     | -       |
| 40     | 35.4    | -1.9    | -       | 12.4    | 1.4     | -       |
| 60     | 25.0    | -2.3    | -       | 4.5     | 0.7     | -       |

harmonic. The value of  $I_5/E_5$  for  $\tau = 20$  hours in absolute magnitude is  $> 1$  and differs in sign from the corresponding ratio for other instants of time. This increase of the scatter of  $1/E$  for  $P_3$ , and especially for  $P_5$ , finds its natural explanation in the fact that the absolute magnitude of these harmonics is considerably smaller, and consequently, the relative errors are larger, than with the first harmonic.

The systematic change of sign of the ratio is a point of interest

$$\frac{I_1}{E_1} > 0; \quad \frac{I_3}{E_3} < 0; \quad \frac{I_5}{E_5} > 0,$$

and the very good agreement of the results obtained in analyses I and II will be noted. Table 7 also gives other data known in the literature on the separation of the perturbation field into an external and an internal part, which are likewise in good agreement with our results. The mean values of 0.39 and 0.40 obtained by us for  $g_1$ ,  $D_{st}$  agree exactly with the ratio calculated by McNish for  $D_m$ . If, furthermore, we bear in mind that  $I_1/E_1$  for  $D_{st}$  (according to the data by Chapman and Whitehead) varies within the range of 0.36-0.42 and, for the noncyclic variations, is equal to 0.30 (McNish) or 0.28 (Dolginov), then it may be considered as proved that the external part of the first harmonic is equal, on the average, to 0.30-0.40 of the value of the external part. The negative value of  $I_3/E_3$  is confirmed by the data of McNish for  $D_m$ , and in part by the data of Chapman and Whitehead for  $D_{st}$ . The numerical values of  $1/E$  will be discussed in greater detail below, in Chapter X, devoted to the discussion of the inductive origin of the internal part of the fields of variations.

### Section 3. Ionospheric System of Currents of the $D_{st}$ -Variations

It was stated above (Chapter II) that two versions of the explanation for the external part of the  $D_{st}$ -variations were proposed: one based on an ionospheric system of currents, the other on an extra-ionospheric ring current. We used the data of our analysis to calculate both these proposed current systems.

Assume that the magnetic field whose potential on the earth's surface is repre-

sented by the series of spherical functions

$$V = R \sum_n \sum_m (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m,$$

is caused by a current layer located on a sphere having a radius of  $a$  ( $a > R$ ). Then

Table 7

|                                       | $\tau$ | $\frac{I_1}{E_1}$ | $\frac{I_3}{E_3}$ | $\frac{I_5}{E_5}$ | $\frac{I_7}{E_7}$ |
|---------------------------------------|--------|-------------------|-------------------|-------------------|-------------------|
| $D_{st}$ Analysis I                   | 4      | 0,32              | -0,16             | 0,78              |                   |
|                                       | 12     | 0,46              | -0,11             | 0,33              |                   |
|                                       | 20     | 0,47              | -0,40             | -1,50             |                   |
|                                       | 28     | 0,34              | 0,00              | 0,00              |                   |
| Mean . . . . .                        |        | 0,40              | -0,17             |                   |                   |
| $D_{st}$ Analysis II                  | 1      | 0,50              | -0,88             |                   |                   |
|                                       | 10     | 0,45              | -0,38             |                   |                   |
|                                       | 20     | 0,51              | -0,86             |                   |                   |
|                                       | 30     | 0,35              | -0,77             |                   |                   |
|                                       | 40     | 0,35              | -0,75             |                   |                   |
|                                       | 60     | 0,18              | -0,34             |                   |                   |
| Mean . . . . .                        |        | 0,39              | -0,61             |                   |                   |
| $D_{st}$ acc.to Chapman and Whitehead | 1      | -5/-11            | -0,5/-0,5         | -1/0              |                   |
|                                       | 3      | -2/-6             | -1/1              | -2,5/-1,5         |                   |
|                                       | 6      | 3/11              | -2/2              | -3,0/-1,0         |                   |
|                                       | 12     | 10/26             | 1/1               | -1/0              |                   |
|                                       | 18     | 10/28             | 0/0               | -1/0              |                   |
|                                       | 24     | 11/26             | -2/-1             | 0/-0,5            |                   |
|                                       | 30     | 9/24              | 0/1               | -1/-1             |                   |
|                                       | 36     | 9/23              | 1/-1              | 0/-1              |                   |
|                                       | 42     | 7/21              | 0/1               | -1/-1             |                   |
|                                       | 48     | 7/20              | 1/-1              | -0,5/-0,5         |                   |
| $D_m$ acc.to McNish . .               |        | 0,39              | -1,12             | 0,27              | -0,80             |
| nch acc.to Dolginov                   |        | 0,28              | 0,20              | 0,86              | 1,46              |
| nch                                   |        | 0,23              |                   |                   |                   |
| acc.to McNish 1923                    |        | 0,37              |                   |                   |                   |
| acc.to McNish 1926                    |        |                   |                   |                   |                   |

Note. nch = Non cyclic variations

the distribution of the current function in the layer can be calculated by the Bidlingmayer formula

$$I = -\frac{10}{4\pi} \sum_n \sum_m \frac{2n+1}{n+1} \left(\frac{a}{R}\right)^n (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m \frac{A}{cM^2}.$$

In our case

$$I = -\frac{10}{4\pi} \sum_n \left(\frac{R+h}{R}\right)^n E_n P_n \frac{A}{cM^2}. \quad (14)$$

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Since the most probable region of concentration of the currents responsible for the magnetic disturbances is the  $F_2$  layer of the ionosphere, we assumed  $h = 300$  km and calculated, from the coefficients  $E_n$  of analysis I, the current density for  $\tau$  equal to 4, 12, 20, 28 hours.

As an example, Fig.16 represents the current systems for  $\tau$  equal to 12 and 28 hours. Since the potential of the  $D_{st}$  field is represented only by zonal harmonics,

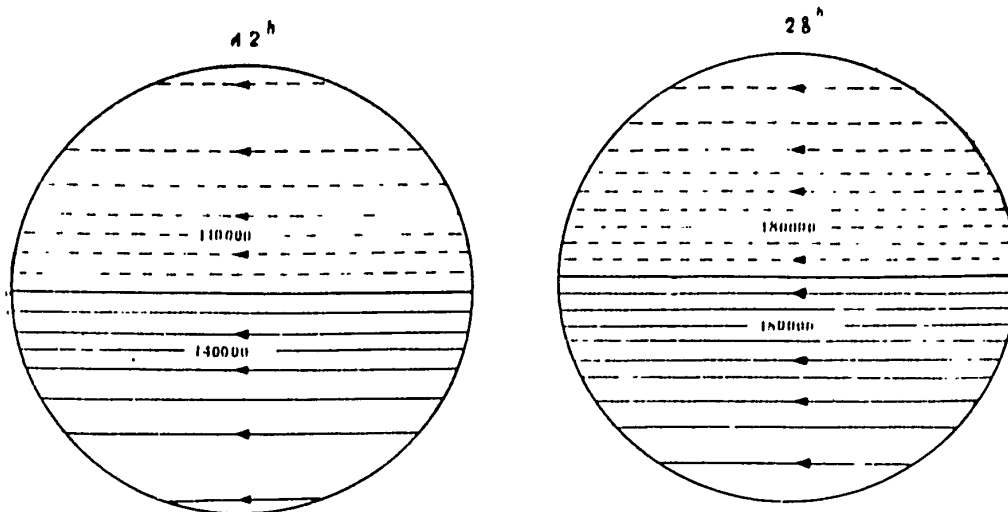


Fig.16 - Electric Currents of the  $D_{st}$ -Variations; a Current of 20,000 amp Flows Between Two Adjacent Lines

\_\_\_\_\_ Positive values of current function; - - - Negative values

it is natural that the lines of current shown in the diagram should be parallel circles. Since only the odd polynomials ( $P_1, P_3, P_5$ ) entered the expression for the potential, it follows that the configuration and intensity of the currents in the northern and southern hemispheres are identical, but that the sign of the current function is different. In the northern hemisphere,  $V > 0$  and  $I < 0$  (the lines of current are given by broken lines) while in the southern hemisphere  $V < 0$  and  $I > 0$  (solid current lines). The Chapman current system (Fig.4a) does not allow for the change in sign of the current function on crossing the equator, in view of which fact, the current systems in Figs.4a and Fig.16 differ in their outward

forms\*.

The lines of currents in Fig.16 are so drawn that a current of 20,000 amp flows between two adjacent lines. The intensity of the currents in the two systems is about the same. The total value of the current flowing from east to west between the pole and the equator is equal to 180,000 (for the system constructed by us). The current density  $\rho = \frac{\partial I}{\partial s} = 0.0002$  amp/cm. The direction of the current in Fig.16 is determined according to the rule that current flows around  $I_{\min}$  clockwise and around  $I_{\max}$  counterclockwise. Thus in both hemispheres the current flows westerly during the main phase of a storm.

A comparison of the current systems (Figs.4a and 16) will show the increased density of the current lines in the polar regions in the Chapman current system, corresponding to the intensification postulated by him for the  $D_{st}$  field in the high latitudes. This densification of the current lines is absent from the systems constructed by us. Conversely, a certain densification of the lines in the equatorial regions can be noted, which expresses the well-known fact that the amplitude of  $D_{st}$  increases in the low latitudes. The current density varies from  $\rho = 0.0001$  amp/cm at latitudes  $\phi$  from 50 to 60°, to  $\rho = 0.0003$  amp/cm near the equator. A comparison of the current calculated for 12 and 28 hours shows that during a storm the configuration of the current system hardly changes and that only its intensity varies. Only in the prolongation of the first phase of the storm ( $\tau = 1$  hour in analysis II) is the direction of the currents opposite (from west to east). The systems of  $D_{st}$  curves in Fig.16 correspond to a mean decrease of  $H$  in the temperate latitudes, by 40-50 γ. During certain storms, this decrease reaches 1000 γ, so that the intensity of the currents equivalent to these storms should increase to  $3-4 \times 10^6$  amp, and the density of the current should be  $\rho = 0.0004$  amp/cm.

\* The figure of the  $D_{st}$ -currents given in Mitra's monograph (Bibl.50), also shows that the current function is of opposite sign in different hemispheres.

#### Section 4. The Equatorial Current Ring

Leaving the discussion of the question as to the actual existence of the resultant system for later (cf. Chapter VIII), let us now turn to a calculation of the equatorial current ring, which presents an alternate explanation for the  $D_{st}$ -variations of the magnetic elements. As is generally known, the potential of the magnetic field of a linear ring current whose center lies on the axis  $\theta = 0$ , may be represented by the following series of spherical functions:

$$V_p = 2\pi i \left[ 1 - \cos \theta_0 + (1 - \cos^2 \theta_0) \sum_n \frac{1}{n} P_n(\cos \theta) P'_n(\cos \theta_0) \left( \frac{R}{a} \right)^n \right].$$

Here  $i$  denotes the current strength in the ring,  $\theta_0$  and  $a$  are the polar distance and radius of the ring, respectively, while  $\theta$  and  $\theta_0$  are spherical coordinates of the point  $P$ . If the ring lies in the plane  $\theta = 90^\circ$  (the plane of the equator), then  $\cos \theta_0 = 0$ , and

$$V_p = 2\pi i \left[ 1 + \sum_n \frac{1}{n} P_n(\cos \theta) P'_n(0) \left( \frac{R}{a} \right)^n \right]. \quad (15)$$

Confining ourselves to the first three terms of the sum and substituting numerical values of  $P'_n(0)$  in eq. (15), we have

$$V_p = 2\pi i \left[ 1 + \frac{R}{a} P_1 - \frac{3}{2} \left( \frac{R}{a} \right)^3 P_3 + \frac{15}{8} \left( \frac{R}{a} \right)^5 P_5 \right]. \quad (15')$$

Let us likewise confine ourselves to three terms in the expression for the external part of the  $D_{st}$  potential. For the earth's surface ( $r = R$ ), we have

$$V_e = R [E_1 P_1 + E_3 P_3 + E_5 P_5]. \quad (16)$$

Equating the potential  $V_e$ , calculated from the observations, to the potential of the magnetic field of the ring, an equation will be obtained by which the param-



eters of the ring can be evaluated. Equating eqs.(15') and (16), we have \*

$$\left. \begin{aligned} RE_1 &= 2\pi i \frac{R}{a} \\ RE_3 &= -2\pi i \frac{3}{2} \left(\frac{R}{a}\right)^3 \\ RE_5 &= 2\pi i \frac{15}{8} \left(\frac{R}{a}\right)^5 \end{aligned} \right\} \quad (17)$$

Combining eqs.(17) pairwise, we obtain the following three equations: :

$$\left(\frac{a}{R}\right)^2 = -\frac{3}{2} \frac{E_1}{E_3}; \quad \left(\frac{a}{R}\right)^2 = -\frac{5}{4} \frac{E_3}{E_5}; \quad \left(\frac{a}{R}\right)^4 = \frac{15}{8} \frac{E_1}{E_5}, \quad (18)$$

which lead to the numerical values of  $a/R$  given in Table 8. As will be seen from the Table, the values of  $a/R$  fluctuate within relatively narrow limits. The calculations of  $a/R$  on the basis of  $E_3/E_5$  in  $E_1/E_5$  indicate the systematic increase of the radius of the ring during the development of a storm. But the data for  $E_1/E_3$ , which should be most trustworthy of all, do not display this increase. The mean value  $a = 3.8R \pm 0.8R$  is in good agreement with the views of Chapman, Forbush, and Kalinin that have been discussed above. Thus both theoretical arguments and empirical data from the field of terrestrial magnetism and cosmic rays lead to a magnitude of the ring of the order of 3-5 earth radii. It goes without saying that the ring may be considerably larger than this during individual storms, but all the same Stoermer's hypothesis of a ring with a radius of several hundred earth-radii must be rejected. The current strength in the ring corresponding to the  $D_{st}$ -variation of 56 moderate storms is equal to

$$i = \frac{aE_1}{2\pi} = 7 \times 10^6 \text{ A},$$

\* The term  $2\pi i$  in eq.(15') denotes a part of the potential that is the same for the entire surface of the earth. There is no analogous term in eq.(16), since eq.(16) determines the field potential with accuracy to a constant.

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and that corresponding to 13  $S_c$  storms, equal to  $i = 20 \times 10^5$  amp. This estimate, too, is in good agreement with the ideas of Chapman and Ferraro on the current ring.

Table 8

|             | $\tau$  | $\frac{E_1}{E_3}$ | $\frac{E_3}{E_5}$ | $\frac{E_1}{E_5}$ | Mean          |
|-------------|---------|-------------------|-------------------|-------------------|---------------|
| Analysis I  | 4 Hours | 4.5               | -                 | 2.1               | 3.3           |
|             | 12      | 3.6               | 2.0               | 2.7               | 2.8           |
|             | 20      | 5.0               | 2.5               | 3.6               | 3.7           |
|             | 28      | 4.5               | 3.3               | 3.8               | 3.9           |
|             | Mean    | 4.4               | 2.6               | 3.1               | $3.4 \pm 0.8$ |
| Analysis II | 1 Hour  | -                 | -                 | -                 | -             |
|             | 10      | 3.8               | -                 | -                 | -             |
|             | 20      | 3.8               | -                 | -                 | -             |
|             | 30      | 5.9               | -                 | -                 | -             |
|             | 40      | 5.2               | -                 | -                 | -             |
|             | 60      | 4.0               | -                 | -                 | -             |
|             | Mean    | $4.6 \pm 1.0$     | -                 | -                 | -             |

## CHAPTER IV

## CALCULATION OF ELECTRIC CURRENTS BY THE METHOD OF SURFACE INTEGRALS

Section 1. The Vestine Method of Separating the Observed Field into an External and an Internal Part

Spherical analysis, applied by us in the preceding Chapter to the study of the field of the  $D_{st}$ -variations, was long the only method for calculating the potential from the magnetic elements observed at a number of points of the earth's surface. It has been repeatedly used with great success in the representation of the permanent field and the  $S_q$  variation, and has allowed the solution of a number of major problems of the nature and structure of these fields. It has also been used in considering the secular and annual variations, and, as we have seen above, of certain parts of the field of variations:  $D_{st}$ ,  $D_m$ , and  $nch$ . But the use of spherical analysis is limited by the requirement that the field studied must possess spherical symmetry and that it can be successfully represented by the first few terms of the series. If, however, the field has a rather complex structure and requires a large number of terms for its representation, then the labor needed in calculating the coefficients is immeasurably increased, and the series so obtained ceases to be convenient for various practical or theoretical applications. Accordingly, the spherical analysis of the  $S_D$ -variations, which characterize a complex geographical distribution, would seem a priori to be doomed to fail, and Chapman, Vestine, and other authors who have studied  $S_D$ , have abstained from any analytic representation of the field at all. In 1941, Vestine (Bibl.57a, b) proposed a new method of mathematical

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analysis which, according to the author's idea, was to replace spherical analysis in the case of rather complex fields. This method, based on the representation of the potential of the field by the aid of surface integrals, allows a separation of the potential into a part of internal origin and one of external origin, from the components of the field as observed on the surface of the sphere. Since it imposes no restrictions whatever on the configuration of the field, I decided to apply this method to the calculation of the potential of the  $S_D$ -variations. For the purpose of our work, however, as for many questions of geomagnetism, it is necessary not only to separate the field into an external and an internal part, but also to calculate the electric currents whose field is equivalent to the observed field. The calculations performed by us showed that this problem, too, is successfully solved by the aid of surface integrals.

Since the method of surface integrals is here used in geomagnetic practice for the first time\*, its mathematical foundations and practical methods will be discussed in the present Chapter, while the description of the calculations of the currents of the  $S_D$ -variations will be reserved for the next Chapter.

The theory of the method is very simple. Let the volume  $v$  be surrounded by a closed surface  $S$ , and let there be, both within and without the surface  $S$ , sources exciting the magnetic field. If  $U$  and  $V$  are functions with continuous first derivatives in the region  $v$  and on the surface  $S$ , and continuous second derivatives in the volume  $v$ , then Green's fundamental theorem indicates that

$$\int_v (U\Delta V - V\Delta U)dv = \int_S \left( U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n} \right) ds, \quad (1)$$

\* In Vestine's note (Bibl.57b) the calculation of the potential of the geomagnetic field at one instant of time is given as an example, and the work by Vestine and Davies (Bibl.61) on the interpretation of magnetic anomalies contains several formulas based on the solution of two-dimensional problems by the aid of surface integrals.

where  $n$  denotes the direction of the external normal. Let us assume that  $U = 1/r$  ( $r$  = distance from a fixed point  $P$  outside  $S$  to the variable point  $M$ , in  $v$  or on  $S$ ) and

$$V = V_e + V_i, \quad (2)$$

where  $V_e$  is the potential of the field of the sources external with respect to  $S$ , and  $V_i$  is the potential of the field of sources internal with respect to  $S$ ; we then have

$$\int_v \left( \frac{1}{r} \Delta V - V \Delta \frac{1}{r} \right) dv = \int_s \left( \frac{1}{r} \frac{\partial V}{\partial n} - V \frac{\partial \frac{1}{r}}{\partial n} \right) ds. \quad (1')$$

For any point of the volume  $v$

$$\Delta \frac{1}{r} = 0, \Delta V_e = 0 \text{ and } \Delta V_i = -4\pi\mu_i$$

( $\mu_i$  = density of the internal magnetic masses). Consequently, the left side of eq.(1') gives

$$\int_v \frac{1}{r} \Delta V dv = -4\pi \int_v \frac{\mu_i}{r} dv = -4\pi V_i;$$

and the potential at the external point  $P_e$  of the field of internal sources yields

$$V_i = -\frac{1}{4\pi} \int_s \left( \frac{1}{r} \frac{\partial V}{\partial n} - V \frac{\partial \frac{1}{r}}{\partial n} \right) ds. \quad (3)$$

By similar reasoning we get the result that the potential at the internal point  $P_i$  of the field of external sources reads as follows:

$$V_e = \frac{1}{4\pi} \int_s \left( \frac{1}{r} \frac{\partial V}{\partial n} - V \frac{\partial \frac{1}{r}}{\partial n} \right) ds. \quad (4)$$

Let the points  $P_e$  and  $P_i$  be located on the external and internal normals to  $S$  passing through the point of the surface  $P$ , where both  $P_e \rightarrow P$  and  $P_i \rightarrow P$ .

Then, taking  $\frac{1}{4\pi} \int \frac{1}{r} \frac{\partial V}{\partial n} ds$  as the potential of a single layer of the density

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$\rho = -1/4 \frac{\partial V}{\partial n}$ , and  $\frac{1}{4\pi} \int_S V \frac{\partial \frac{1}{r}}{\partial n} ds$  as the potential of a double layer of the density  $\rho = \frac{1}{4\pi} V$ , and allowing for the known properties of the potentials of single and double layers, we get the result that

$$\begin{aligned} V_i|_{P_+} &= \int_S \frac{\sigma}{r} ds + \left[ \int_S \rho \frac{\partial \frac{1}{r}}{\partial n} ds + 2\pi\rho_P \right] = \\ &= -\frac{1}{4\pi} \int_S \frac{1}{r} \frac{\partial V}{\partial n} ds + \frac{1}{4\pi} \int_S V \frac{\partial \frac{1}{r}}{\partial n} ds + \frac{1}{2} V_P, \end{aligned} \quad (5)$$

$$\begin{aligned} V_e|_{P_-} &= -\int_S \frac{\sigma}{r} ds - \left[ \int_S \rho \frac{\partial \frac{1}{r}}{\partial n} ds - 2\pi\rho_P \right] = \\ &= \frac{1}{4\pi} \int_S \frac{1}{r} \frac{\partial V}{\partial n} ds - \frac{1}{4\pi} \int_S V \frac{\partial \frac{1}{r}}{\partial n} ds + \frac{1}{2} V_P, \end{aligned} \quad (6)$$

where  $P_+$  and  $P_-$  denotes that the approach to the point was from the side of the external and internal normals. Hence,

$$V_e - V_i|_P = \frac{1}{2\pi} \int_S \left( \frac{1}{r} \frac{\partial V}{\partial n} - V \frac{\partial \frac{1}{r}}{\partial n} \right) ds. \quad (7)$$

Knowing the distribution of the surface  $S$  of the total potential  $V$  and its normal derivative  $\partial V / \partial n$ ,  $V_e - V_i$  may be calculated for any point of the surface; then, by combining eqs.(2) and (7), the value of  $V_e$  and  $V_i$  can be separately determined.

It must be noted, however, that the potential  $V_0$  of any uniform double layer of a density of  $\rho_0$ , located on the closed surface  $S$ , equals zero outside the surface and  $-4\pi\rho_0$  inside it. For this reason, if, to the postulated double layer with a density of  $1/4\pi V$ , we add a layer of uniform density  $\rho_0 = -V_0/4\pi$ , then eq.(3) will remain without change, while eq.(4) will take the form

$$V_e = \frac{1}{4\pi} \int_S \left[ \frac{1}{r} \frac{\partial V}{\partial n} - (V - V_0) \frac{\partial \frac{1}{r}}{\partial n} \right] ds + V_0, \quad (8)$$

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whence

$$V_e|_{P_-} = \frac{1}{4\pi} \int_S \left[ \frac{1}{r} \frac{\partial V}{\partial n} - (V - V_0) \frac{\partial \frac{1}{r}}{\partial n} \right] ds + \frac{V - V_0}{2} + V_0. \quad (9)$$

Instead of eq.(5) we will now have

$$V_i|_{P_+} = -\frac{1}{4\pi} \int_S \left[ \frac{1}{r} \frac{\partial V}{\partial n} - (V - V_0) \frac{\partial \frac{1}{r}}{\partial n} \right] ds + \frac{V - V_0}{2} \quad (10)$$

and, consequently,

$$V_e - V_i = \frac{1}{2\pi} \int_S \left[ \frac{1}{r} \frac{\partial V}{\partial n} - (V - V_0) \frac{\partial \frac{1}{r}}{\partial n} \right] ds + V_0. \quad (11)$$

It follows from eqs.(9) - (11) that, if the potential  $V$  on the surface  $S$  is known only with an accuracy to an additive constant, then the values of  $V_e - V_i$  and  $V_e$  may be found only with an accuracy to a constant, while the values of  $V_i$  will be calculated exactly. Besides the above general formulas, Vestine also gave formulas applicable to cases when the surface  $S$  is a sphere or a plane. In spherical coordinates with the pole coinciding with the point  $P$  (cf. Fig. 17a), the distance between the points  $P$  and  $M$  ( $r_0 \theta$ ) is equal to

$$r = 2R \sin \frac{\theta}{2} = 2R \sin \psi, \quad \psi = \frac{\theta}{2},$$

$$\frac{\partial \frac{1}{r}}{\partial n} = -\frac{1}{r^2} \cos \alpha = -\frac{\sin \psi}{r^2} = -\frac{1}{4R^2 \sin \psi}.$$

Denoting  $\partial V / \partial n$  by  $Z$ , let us now transform eq.(7) into

$$V_e - V_i = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} (V + 2RZ) \cos \psi \, d\psi \, d\varphi \quad (12)$$

or

$$V_e - V_i = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} (V - V_0 + 2RZ) \cos \psi \, d\psi \, d\varphi, \quad (13)$$

if the value of  $V$  on the sphere is known only with an accuracy to the constant  $V_0$ .

It was eq.(12) that I used to separate the potential of the  $S_D$ -variations into an external and an internal part.

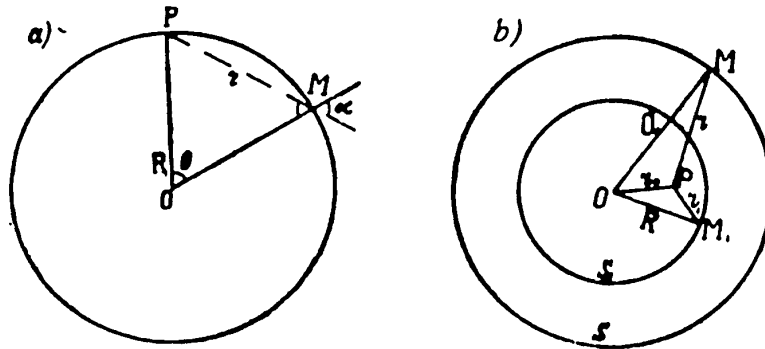


Fig.17

The solution of the two-dimensional problem leads to still simpler formulas. In cylindrical coordinates  $r, \varphi, z$ , whose origin is placed at the point P,  $PM = r$ , and

$$V_e - V_i = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} \left( \frac{1}{r} Z - V \frac{\partial}{\partial z} \frac{1}{r} \right) r dr d\varphi = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} Z dr d\varphi, \quad (14)$$

i.e., to find the difference of the external and internal potentials, it is sufficient to know the values on the plane of the  $Z$ -component of the field.

## Section 2. Practical Methods of Calculating the External and Internal Potentials

The fundamental difficulty in performing practical calculations of the difference  $V_e - V_i$  by eq.(13) is that it is given in coordinates connected with the position of the point P, for which we seek the value of  $V_e - V_i$ . The transformation of the equation to any fixed coordinate at all (for instance, geographic or geomagnetic) by means of the usual formulas for the transformation of coordinates, leads to a complex expression inconvenient for mass calculations. In view of this fact I adopted the following technique:

If we denote by  $\bar{V}$  and  $\bar{Z}$  the mean values of  $V$  and  $Z$  on the circle of latitude  $\psi = \text{const}$ , then eq.(12) is transformed into



$$V_e - V_i = \int_0^{\frac{\pi}{2}} (2R\bar{Z} + \bar{V}) \cos \psi d\psi. \quad (15)$$

In order to calculate the integral of eq.(15), we must find, for each point P on the surface of the sphere, the mean values  $\bar{Z}$  and  $\bar{V}$  along the circles of latitude (P being taken as the pole of the coordinate system). For this purpose, the formulas of transition from one system of spherical coordinates (fixed pole) to another

(with the pole at P) were used for preparing overlays on which the lines  $\psi = \text{const}$  were plotted. The formulas for calculating the overlays were obtained from the solution of the spherical triangles NDQ and PDQ (Fig.18). The figure uses the following notation: N, pole of the fixed system of coordinates. In this system, P( $\theta_0, \Lambda_0$ ). In coordinates with the pole P, the point Q is determined by the polar distance  $\theta$  and

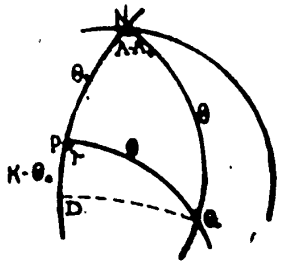


Fig.18

the longitude  $\lambda$ . On dropping from Q a perpendicular to the prolongation of the arc NP and denoting the arc ND by k, we have

$$\begin{aligned} \operatorname{tg} k &= \operatorname{tg} \theta \cos (\Lambda - \Lambda_0), \\ \operatorname{tg} \lambda &= \frac{\operatorname{tg} QD}{\sin PD}, \\ \operatorname{tg} QD &= \sin k \operatorname{tg} (\Lambda - \Lambda_0), \end{aligned} \quad (16)$$

whence

$$\operatorname{tg} \lambda = \frac{\operatorname{tg} (\Lambda - \Lambda_0)}{\sin (k - \theta_0)}. \quad (17)$$

From  $\Delta PQD$  it follows that

$$\operatorname{ctg} \theta = \operatorname{ctg} (k - \theta_0) \cos \lambda. \quad (18)$$

By combining eqs.(17) and (18), we get

$$\operatorname{ctg}^2 \theta = \frac{\cos^2 (k - \theta_0)}{\operatorname{tg}^2 (\Lambda - \Lambda_0) \sin^2 k + \sin^2 (k - \theta_0)}. \quad (19)$$

Equations (17) and (19) give an expression for the coordinates of an arbitrary

point Q in the system with the pole P, in terms of the coordinates of the points P and Q in the system with the pole N. By eq.(19) we calculated the curves of  $\psi = 0/2 = \text{const}$  for various values of  $\theta_0$ , and  $\lambda_0$  and plotted them on the coordinate net  $(\theta_0 = 20^\circ)$

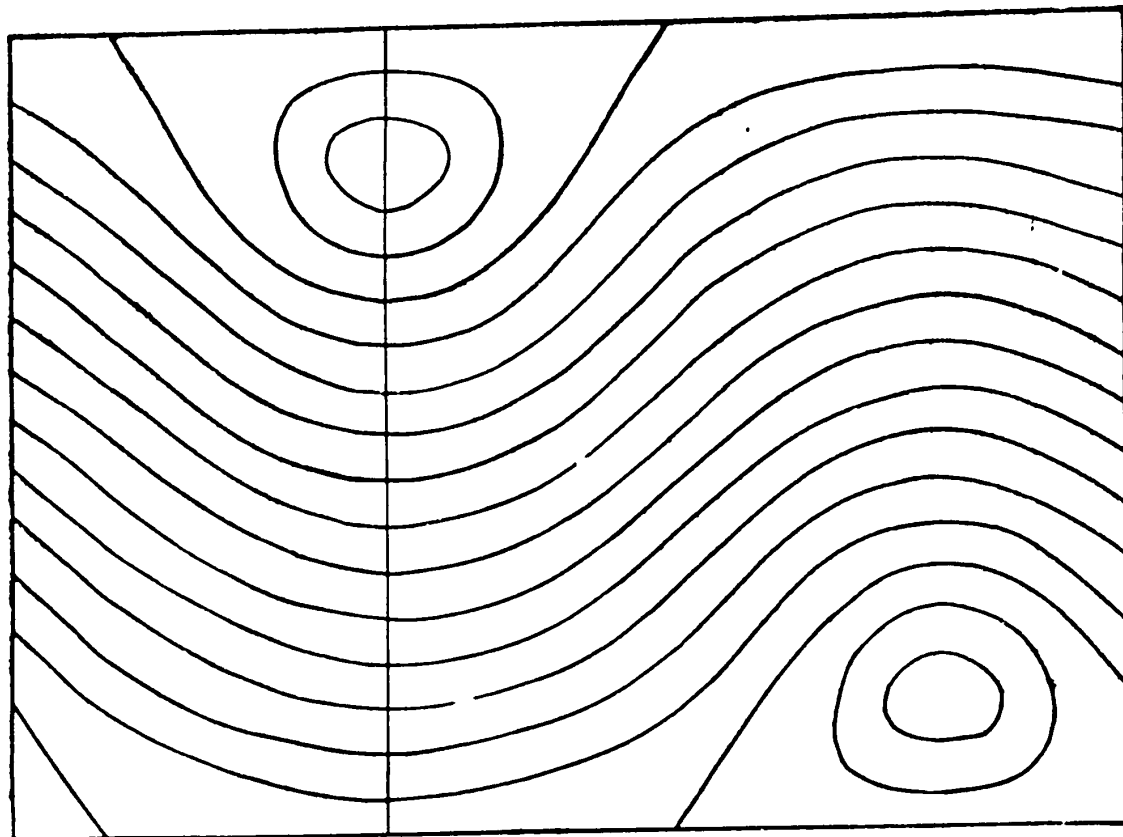


Fig.19 - The Overlay

$\theta, \lambda$ . By the aid of the overlays so obtained, the process of calculating  $V_e$  and  $V_i$  at the point  $P(\theta_0, \lambda_0)$  reduces to the following:

1. From the components observed on the earth's surface, we must calculate (for instance, by integration of the X component) the potential  $V$  for a number of points covering the entire earth with a uniform net.
2. Plot the value of the potential and the Z component on the coordinate net  $\theta, \lambda$ . For brevity we will, in the following, call a coordinate net with plotted values of any element a cartogram.
3. Placing on the cartograms of  $Z$  and  $V$  the overlay traced on transparent paper on the same scale and calculated for  $\theta_0, \lambda_0$ , take off the values of  $V$  and  $Z$  along

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each line of  $\psi$ , equal to  $\psi_1$ ,  $\psi_2$ , etc. and average those values of  $V$  and  $Z$ .

4. Calculate for each value of  $\psi$  the binomial  $(2R\bar{Z} + \bar{V})$ , and, by means of num-

Table 9

SD Variations, Z-component (in  $\gamma$ )

|     | 0   | 2   | 4   | 6   | 8   | 10  | 12  | 14  | 16  | 18  | 20  | 22  | 24  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 90  | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 80  | 0   | 24  | 49  | 74  | 99  | 124 | 149 | 174 | 199 | 224 | 249 | 274 | 299 |
| 70  | 33  | 43  | 49  | 51  | 50  | 47  | 42  | 36  | 29  | 21  | 13  | 5   | -3  |
| 60  | 50  | 60  | 66  | 65  | 63  | 59  | 54  | 48  | 41  | 33  | 25  | 17  | 9   |
| 50  | 84  | 134 | 160 | 160 | 150 | 130 | 100 | 70  | 40  | 10  | -20 | -47 | -84 |
| 40  | -3  | 3   | -55 | -60 | -60 | -50 | -30 | -10 | 20  | 50  | 80  | 107 | 144 |
| 30  | -10 | -42 | -40 | -28 | -14 | 9   | 23  | 36  | 54  | 77  | 100 | 123 | 146 |
| 20  | -12 | -16 | -16 | -15 | -8  | -3  | 0   | 12  | 30  | 48  | 67  | 86  | 105 |
| 10  | -4  | -7  | -4  | -7  | -3  | 0   | 3   | 7   | 15  | 27  | 43  | 60  | 77  |
| 0   | 0   | -2  | -2  | -4  | -3  | -2  | -1  | 0   | 4   | 10  | 18  | 29  | 40  |
| -10 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -20 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -30 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -40 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -50 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -60 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -70 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -80 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |
| -90 | 0   | 0   | 0   | -2  | -2  | -1  | 0   | 2   | 6   | 12  | 20  | 31  | 42  |

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erical or graphical integration, obtain the difference  $V_e - V_i$  from eq.(15).

5. Knowing  $V_p = V_e + V_i$  and  $V_e - V_i$ , find  $V_e$  and  $V_i$  separately. As an example, Fig.19 and Tables 9 and 10 give cartograms of the  $Z$  and  $V$  of the  $S_D$ -variations and an overlay for  $P(\theta_0)$  with  $\theta_0 = 20^\circ$ . The geomagnetic coordinates have been taken as the fixed coordinate system; the overlay and cartogram are given in cylindrical projection, while the isolines of  $\psi$  are drawn at intervals of  $5^\circ$ . For one and the same values of  $\theta_0$ , but different values of  $\Lambda_0$ , one overlay can be used, by shifting it in proper manner on the cartogram. With preliminarily prepared cartograms and

overlays, the calculation of  $V_e - V_i$  for each point is not so laborious.

### Section 3. Calculation of the Electric Currents by the Integral Method

For calculating the currents whose field correspond to the external and internal parts of the observed magnetic field, the method of integral equations was selected. As is commonly known, the solution of potential problems is one of the

Table 10

$S_D$ -Variations  $V \times 10^{-3}$  CGS

|     | 2   | 4   | 6   | 8   | 10  | 12  | 14  | 16  | 18  | 20  | 22  | 24  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 90  | 0   | 10  | 10  | 0   | 10  | 10  | 10  | 10  | 10  | 10  | 10  | 10  |
| 80  | 28  | 32  | 25  | 1   | 12  | -11 | -24 | -37 | -20 | -18 | 13  | 28  |
| 70  | 34  | 64  | 49  | 0   | 14  | -14 | -41 | -78 | -42 | -35 | 1   | 32  |
| 60  | 76  | 90  | 76  | 4   | 24  | -18 | -57 | -95 | -64 | -52 | -22 | 38  |
| 50  | 72  | 80  | 48  | 15  | 24  | -26 | -30 | -50 | -24 | -43 | -37 | 15  |
| 40  | 23  | 4   | -24 | -37 | -26 | -13 | 1   | 5   | 20  | 18  | 14  | 5   |
| 30  | 3   | -30 | -49 | -45 | -32 | -12 | -11 | -19 | -45 | -47 | -37 | -14 |
| 20  | 7   | -34 | -52 | -47 | -32 | -12 | 12  | 21  | 48  | 50  | 38  | 15  |
| 10  | -7  | -34 | -50 | -44 | -30 | -12 | 10  | 20  | 45  | 49  | 37  | 15  |
| 0   | -6  | -31 | -46 | -40 | -27 | -10 | 8   | 18  | 40  | 44  | 34  | 15  |
| -10 | -5  | -28 | -40 | -36 | -22 | -9  | 7   | 15  | 35  | 40  | 31  | 14  |
| -20 | -5  | -24 | -35 | -31 | -18 | -8  | 6   | 12  | 29  | 34  | 27  | 12  |
| -30 | -5  | -21 | -30 | -27 | -14 | -4  | 5   | 8   | 24  | 29  | 23  | 10  |
| -40 | -4  | -18 | -26 | -23 | -10 | -2  | 4   | 7   | 20  | 24  | 19  | 9   |
| -50 | -2  | -15 | -22 | -18 | -8  | 1   | 3   | 6   | 16  | 20  | 15  | 8   |
| -60 | -2  | -11 | -17 | -14 | -6  | 0   | 3   | 5   | 13  | 15  | 10  | 7   |
| -70 | -2  | -8  | -12 | -9  | -3  | 0   | 2   | 4   | 10  | 11  | 8   | 6   |
| -80 | -2  | -4  | -5  | -4  | -2  | 0   | 2   | 3   | 6   | 7   | 5   | 4   |
| -90 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 90  | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 80  | -2  | 4   | 5   | 4   | 2   | 0   | -1  | -2  | -5  | -6  | -3  | 0   |
| 70  | -2  | 8   | 12  | 9   | 3   | 0   | -2  | -4  | -10 | -11 | -6  | 2   |
| 60  | 2   | 11  | 17  | 14  | 6   | 0   | -3  | -5  | -13 | -15 | -10 | 2   |
| 50  | -2  | 15  | 22  | 18  | 8   | 1   | -3  | -6  | -16 | -20 | -15 | 4   |
| 40  | 4   | 18  | 26  | 23  | 10  | 2   | -4  | -7  | -20 | -24 | -18 | 8   |
| 30  | -5  | 21  | 30  | 27  | 14  | 4   | -5  | -8  | -24 | -29 | -23 | 11  |
| 20  | 5   | 24  | 35  | 31  | 18  | 8   | -6  | -12 | -29 | -34 | -27 | 12  |
| 10  | -5  | 28  | 40  | 36  | 22  | 9   | -7  | -15 | -35 | -40 | -31 | 14  |
| 0   | 6   | 31  | 46  | 40  | 27  | 10  | -8  | -18 | -40 | -44 | -34 | 15  |
| -10 | -7  | 34  | 50  | 44  | 30  | 12  | -10 | -20 | -45 | -49 | -37 | 15  |
| -20 | 7   | 34  | 52  | 47  | 32  | 12  | -12 | -21 | -48 | -50 | -38 | 15  |
| -30 | -3  | -30 | -49 | -45 | -32 | -12 | -11 | -19 | -45 | -47 | -37 | -14 |
| -40 | -23 | -4  | -24 | -37 | -26 | -13 | -1  | -5  | -29 | -18 | -14 | -5  |
| -50 | -72 | -80 | -48 | -15 | -24 | -26 | -30 | -50 | -24 | -43 | -37 | -15 |
| -60 | -76 | -90 | -76 | -4  | -24 | -18 | -57 | -95 | -64 | -52 | -22 | -38 |
| -70 | -34 | -64 | -49 | 0   | -14 | -14 | -41 | -78 | -42 | -35 | 1   | -32 |
| -80 | -28 | -32 | -25 | -1  | -2  | -11 | -24 | -37 | -20 | -18 | 13  | -28 |
| -90 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|     | 0   | 2   | 4   | 6   | 8   | 10  | 12  | 14  | 16  | 18  | 20  | 22  |

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classical fields of application of integral equations. The internal Dirichlet problem for the sphere is reduced to the Fredholm equation of the second kind:

$$-W_{IP} = 2\pi v_P + \int_S \frac{v_M \cos \alpha}{r^3} ds, \quad (20)$$

where  $v$  denotes the density of the double layer located on the sphere;  $W_1$  are values of the potential assigned for the surface of the sphere ( $\Delta W_1 = 0$  inside the sphere); and the values of  $r$  and  $\alpha$  are the same as in Section 1. Unfortunately, the conditions of our problem do not lead directly to this easily studied equation. The specific peculiarity of our problem resides in the fact that we know the function  $V_e$  on the surface  $S_1$  of a sphere of the radius  $R$  (on the earth's surface), satisfying the Laplace equation inside the sphere  $S_1$ , while we desire to obtain distributions of the current function on the surface  $S$  of a sphere of a radius  $a$ , if  $a > R$  (cf. Fig. 17b). Since the magnetic potential of a current layer with a current density of  $v$  is equivalent to the potential of an inhomogeneous magnetic layer of a density of  $v$ , it follows that we can replace derivation of the current function by derivation of the density of the double layer.

Let  $M(\theta, \varphi)$  be a variable point of the sphere  $S$ ; and let  $M_1(\theta_1, \varphi)$  on  $S_1$  and  $P(\theta_0, \varphi_0, r_0)$  be a certain point inside the sphere  $S_1$  or on its surface. Then,

$$V_M = \int_S v_M \frac{\partial}{\partial n} \frac{1}{r} ds, \quad (21)$$

where  $r = \overline{PM}$ . Since the values on the surface  $S_1$  are assigned, it follows that the expression  $V_P$  for  $r_0 \leq R$  may be found by the aid of the Poisson integral

$$V_P = \frac{R^2 - r_0^2}{4\pi R} \int_{S_1} \frac{V}{r_1^3} ds, \quad (22)$$

where  $r_1 = \overline{PM_1}$ . By equating the right sides of eqs. (21) and (22), we get the Fredholm integral equation of the first kind:

$$\frac{R^2 - r_0^2}{4\pi R} \int_{S_1} \frac{V}{r_1^3} ds = \int_S v_M \frac{\partial}{\partial n} \frac{1}{r} ds. \quad (23)$$

for  $r_0 = R$

$$V_{M_1} = \int_S v(\theta, \varphi) \frac{\partial}{\partial n} \frac{1}{r} ds. \quad (24)$$

It is generally known that the solution of Fredholm equations of the first kind in the general form is very complex and requires the core of the equation and the free term to satisfy certain conditions. This compels us to dispense with its solution. In order to reduce the determination of the density to the solution of eq.(20), the function  $V_e$ , which is harmonic within the sphere, must be extrapolated, by some method, to the external space. But this is extremely difficult, since  $V_e$  in the external space is an irregular function. I therefore decided to take another path. Namely, knowing the function  $V_e$  at any point within the sphere  $S_1$ , the fictitious density  $v$  is calculated for certain surfaces  $\rho \leq R$ , and then the values of  $v$  are extrapolated to  $\rho = a$ . This procedure is legitimate in principle, since  $v$  is a function regular throughout the entire space. In order to show that this method assures the accuracy necessary for many geophysical problems, we present two examples.

I. Let, on the sphere  $R$ , the potential of the external sources be assigned as

$$V = RgP_2^1(\cos \theta) \cos \varphi \left(\frac{\rho}{R}\right)^2.$$

Everywhere, for  $\rho \leq R$ , we have  $\Delta V = 0$ , and therefore  $V$  may be analytically continued on any  $\rho < R$ . For

$$\rho = R_1 \quad V = gRP_2^1 \cos \varphi \left(\frac{R_1}{R}\right)^2,$$

$$\rho = R_2 \quad V = gRP_2^1 \cos \varphi \left(\frac{R_2}{R}\right)^2,$$

$$\rho = R_3 \quad V = gRP_2^1 \cos \varphi \left(\frac{R_3}{R}\right)^2.$$

Assume that we are on the sphere  $R_3$ .

Then: 1) we know the field of  $V$  on the sphere  $R_3$ ; 2) we know that it is of external origin; and 3) we do not know at what  $\rho$  ( $\rho > R_3$ ) its sources are located. It may therefore be assumed that the field is located on the sphere  $R_2$ ,  $R_1$ ,  $R$ , etc, so that the current density can be calculated by the usual formulas. If the potential

on the sphere  $R_3$  is represented by the function

$$V = R_3 \gamma_n^m P_n^m \frac{\cos m\varphi}{\sin m\varphi},$$

then the current density on the surface of  $\rho$  ( $\rho > R_3$ ) will be

$$i = -\frac{10R_3}{4\pi} \frac{2n+1}{n+1} \left(\frac{\rho}{R_3}\right)^n P_n^m \frac{\cos m\varphi}{\sin m\varphi} \gamma_n^m.$$

In our case  $\frac{2n+1}{n+1} = \frac{5}{3}$  and  $R_3 \gamma = gR(R_3)^2$ , whence

$$i = -\frac{25}{6\pi} \frac{\rho^3}{R} P_2^1 g \cos \varphi.$$

Denoting  $-\frac{25}{6\pi} P_2^1 \cos \varphi g$  by  $B$ , we have

$$\text{for } \rho = R_2 \quad i = B \frac{R_2^2}{R}; \quad \text{at } R_2 = 0.96R \quad i = 0.92B;$$

$$\text{for } \rho = R_1 \quad i = B \frac{R_1^2}{R}; \quad \text{at } R_1 = 0.98R \quad i = 0.96B;$$

$$\text{for } \rho = R \quad i = BR; \quad \text{at } R = 1.00R \quad i = 1.00B.$$

But in reality the current flows along the sphere  $\rho > R$ , where we do not know the value of  $V_e$ . Let us find  $i$  for  $\rho > R$  by simple graphic extrapolation (Fig. 20). Then, for  $a = 1.02R$ , we have  $i = 1.04B$ . Check: from the value of  $V$  on the sphere  $R$  we have, for  $a = 1.02R$ :

$$i = -\frac{25}{6\pi} P_2^1 \cos \varphi (1.02)^2 = 1.04B.$$

II. On a sphere of a radius of  $R$ , the potential

$$V = RgP_{10} \cos \varphi.$$

is known.

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On the sphere

$$\rho = R_1 \quad V = gRP_{10}^1 \cos \varphi \left(\frac{R_1}{R}\right)^{10},$$

$$\rho = R_2 \quad V = gRP_{10}^1 \cos \varphi \left(\frac{R_2}{R}\right)^{10},$$

$$\rho = R_3 \quad V = gRP_{10}^1 \cos \varphi \left(\frac{R_3}{R}\right)^{10}.$$

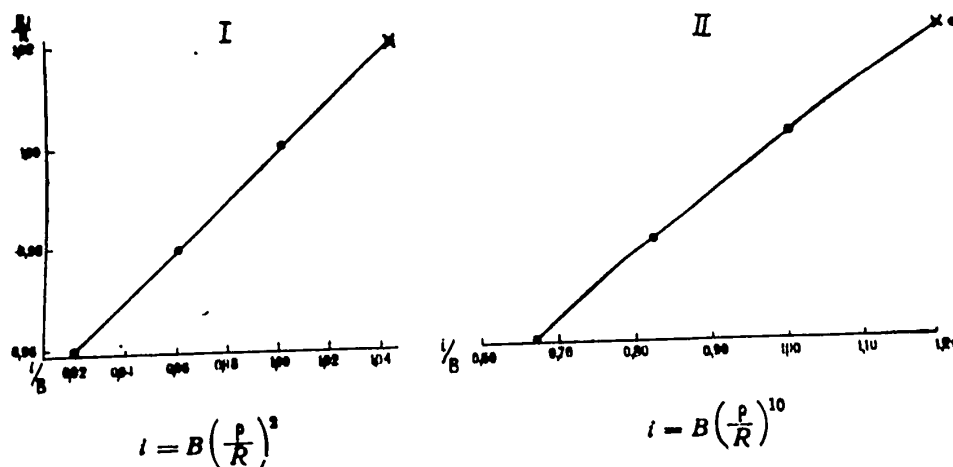


Fig.20 - Calculation of Current Density by Extrapolation

(. Calculated Values; x, Extrapolated Values)

Starting out from the values of  $V$  on the sphere  $R_3$ , we have

$$i = -\frac{10R_3}{4\pi} \frac{21}{11} \left(\frac{\rho}{R_3}\right)^{10} P_{10}^1 \cos \varphi g \left(\frac{R_3}{R}\right)^9 = B \frac{\rho^{10}}{R^{10}},$$

where  $B = -\frac{210}{44} g R P_{10}^1 \cos \varphi$ .

For  $\rho = 0.96R$   $i = 0.67B$ ,

For  $\rho = 0.98R$   $i = 0.82B$ ,

For  $\rho = 1.00R$   $i = 1.00B$ .

Extrapolating on the graph of Fig.20 for  $\rho = 1.02R$  will give  $i = 1.20B$ . Accord-  
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ing to the formulas of spherical analysis,  $i = 1.22B$ , i.e., the error is 2%.

#### Section 4. Finding the Current Density from an Assigned Potential on the Sphere.

##### Extrapolation of the Potential

After having thus reduced the solution of our problem to the classical problem of finding the density of a double spherical sheet from values of the potential assigned on the sphere, we must select a practically convenient method of solving eq.(20). Equation (20) is a special case of the equation

$$V_P = 2\pi v_P + \lambda \int_S v_M \frac{\cos \alpha}{r^2} ds \quad (25)$$

for  $\lambda = +1$ .

The usual method of solving an integral equation of the second kind by the aid of resolvents is inapplicable in this case, since the series expressing the resolvent becomes divergent at  $\lambda = +1$ . Two methods of solving eq.(25) are known in the literature. The first was given by Neumann in 1875, and the second by Bogolyubov and Krylov in 1926. Neumann's method (Bibl.14) reduces to the following: Since

$$2\pi = \int_S \frac{\cos \alpha}{r^2} ds,$$

then

$$2\pi v_P = \int_S \frac{v_P \cos \alpha}{r^2} ds$$

and

$$V_P = \int_S (v_M - v_P) \frac{\cos \alpha}{r^2} ds + 4\pi v_P.$$

This transformation is necessary in order to replace the integral  $\int_S v_M \frac{\cos \alpha}{r^2} ds$ ,

which has a singularity at the point M, coinciding with P, by a convergent integral that has no singularities as  $M \rightarrow P$ .

The equation

$$v_p = \frac{\lambda}{4\pi} \int_S \int (v_p - v_M) \frac{\cos \alpha}{r} ds + \frac{V_p}{4\pi} \quad (26)$$

is solved by the aid of the series

$$v_p = \frac{1}{4\pi} [U_{0P} + \lambda U_{1P} + \lambda^2 U_{2P} + \dots + \lambda^n U_{nP}], \quad (27)$$

which is convergent for  $\lambda = +1$ .

The functions  $U_n$  are determined successively through the recurrence formulas:

$$\left. \begin{aligned} U_{0P} &= V_p \\ U_{1P} &= \int_S (U_{0P} - U_{0M}) \frac{\cos \alpha}{r^2} ds \\ &\dots \dots \dots \\ U_{nP} &= \int_S (U_{n-1,P} - U_{n-1,M}) \frac{\cos \alpha}{r^2} ds \end{aligned} \right\} \quad (28)$$

This method is convenient since the value of the  $n^{\text{th}}$  term depends on the first  $(n-1)$  terms and does not depend on the  $(n+1)^{\text{th}}$  and subsequent terms. Thus, in order to pass from the  $n^{\text{th}}$  approximations to the  $(n+1)^{\text{th}}$  approximation, it is necessary to add, to the sum of  $n$  terms, an  $(n+1)^{\text{th}}$  term without changing the first  $n$  terms.

On transforming eq.(28) into a form convenient for our calculations, we have

$$U_{1P} = \frac{1}{4\pi} \int_S (V_p - V_M) \frac{\cos \alpha}{r^2} R^2 \sin \theta d\theta d\lambda. \quad (29)$$

Denoting the angle between the radius vector of the points M and P by  $\delta$ , we have

$$r = 2R \sin \frac{\delta}{2}, \quad \alpha = \frac{\pi}{2} - \frac{\delta}{2},$$

where  $\cos \delta = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos (\lambda - \lambda_0)$ , if the coordinates of P are  $\theta_0, \lambda_0$  and the coordinates of M are  $\theta, \lambda$ .

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Whence

$$U_P = \frac{1}{16\pi} \int_0^{2\pi} \int_0^\pi (V_P - V_M) \operatorname{cosec} \frac{\delta}{2} \sin \theta \, d\theta \, d\lambda. \quad (30)$$

We remark that, for  $M \rightarrow P$ ,  $\operatorname{cosec} \delta/2 \rightarrow \infty$ , but since  $V_P - V_M \rightarrow 0$ , it is easy to show that  $(V_P - V_M) \operatorname{cosec} \frac{\delta}{2}$  remains finite.

If, in calculating  $U_1$ , we take for each point  $P$  its own system of coordinates with the pole  $P$ , then  $\delta \equiv \theta$  and

$$U_{1P} = \frac{1}{8\pi} \int_0^{2\pi} \int_0^\pi (V_P - V_M) \cos \frac{\theta}{2} \, d\theta \, d\lambda. \quad (31)$$

Introducing  $\theta/2 = \psi$  and  $\bar{V}_P = \bar{V}_M = \frac{1}{2} \int_0^{2\pi} (V_P - V_M) \, d\lambda$  (the mean value of  $V_P - V_M$  from the parallel circle), we have

$$U_{1P} = \frac{1}{2} \int_0^{\frac{\pi}{2}} (\bar{V}_P - \bar{V}_M) \cos \psi \, d\psi \quad (32)$$

and in general

$$\begin{aligned} U_{nP} &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (U_{n-1,P} - U_{n-1,M}) \cos \psi \, d\psi \\ &= \frac{1}{2} U_{n-1,P} - \frac{1}{2} \int_0^{\frac{\pi}{2}} \bar{U}_{n-1,M} \cos \psi \, d\psi. \end{aligned} \quad (33)$$

Thus in zero approximation, the function of current density is  $v_P = -1/4\pi V_P$  and, in first approximation,

$$v_P = -\frac{1}{4\pi} \left[ \frac{3}{2} V_P - \frac{1}{2} \int_0^{\frac{\pi}{2}} \bar{V}_P \cos \psi \, d\psi \right] \quad (34)$$

etc.

These formulas very clearly show that, in zero approximation, the current density is equal to the potential with an accuracy to a constant coefficient. Therefore, for a rough estimate of the configuration of a current system, it is sufficient to construct the isolines of potential. In first approximation, as will be clear from

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eq.(34), the value of the current density is determined by the inequality:

$$\left| \frac{1}{4\pi} V_p \right| < |v_p| < \left| \frac{1}{4\pi} \frac{3}{2} V_p \right|. \quad (35)$$

Equation (34), serving to calculate the current density, allows us again to use the overlays described in the preceding paragraph.

Let us now turn to the question of extrapolating the values of  $V_e$  for  $\rho < R$ . Since we obtain  $V_e$  on the earth's surface not in an analytical but in a numerical form, the most convenient method of extrapolating  $V_e$  within the sphere is the Poisson integral. If the variable point of the surface of the sphere is  $M(R, \theta, \varphi)$  and the internal point at which we desire to calculate the potential is  $P(\rho, \theta_0, \varphi_0)$ , then

$$U_p = \frac{R^2 - \rho^2}{4\pi R} \int_S U_M \frac{ds}{r^3}, \quad (36)$$

where  $U_M$  denotes the surface value of the potential. For convenience in practical computation, we will perform certain transformations.

Since

$$r^2 = R^2 + \rho^2 - 2R\rho \cos \delta,$$

where  $\delta$  is the angle between OM and OP, then

$$U_p = \frac{R(R^2 - \rho^2)}{4\pi} \int_0^{2\pi} \int_0^\pi \frac{U_M \sin \theta d\theta d\varphi}{(R^2 + \rho^2 - 2R\rho \cos \delta)^{3/2}}. \quad (37)$$

If we again place the pole of the coordinate system at P, then

$$U_p = \frac{R(R^2 - \rho^2)}{4\pi R^3} \int_0^\pi \frac{\sin \theta}{\left(1 + \frac{\rho^2}{R^2} - 2\frac{\rho}{R} \cos \theta\right)^{3/2}} \int_0^{2\pi} U_M d\varphi d\theta = \int_0^\pi \bar{U}(\theta) K(\theta) d\theta, \quad (38)$$

where  $\bar{U} = \frac{1}{2\pi} \int_0^{2\pi} U_M d\varphi$  denotes the mean value of  $U_M$  for the circle of latitude

and

$$K(\theta) = \frac{1}{2} \left(1 - \frac{\rho^2}{R^2}\right) \frac{\sin \theta}{\left(1 + \frac{\rho^2}{R^2} - 2 \frac{\rho}{R} \cos \theta\right)^{1/2}}. \quad (39)$$

Thus the extrapolation of  $V_e$  to any spherical surface  $\rho < R$  may be performed by means of the same overlays that were used in calculating the difference  $V_e - V_1$ .

#### Section 5. Practical Methods. Conclusions as to the Suitability of the Method

In accordance with the above, the calculation of the external electric currents from geomagnetic elements known at the earth's surface reduces to the following steps:

- 1) Calculation of the potential  $V_e$  at the earth's surface;
- 2) Calculation of the potential  $V_e$  on spheres of radii  $\rho_1$  and  $\rho_2$ ;
- 3) Calculation of the current density  $v$  for three spheres of radii  $\rho_1$ ,  $\rho_2$  and  $R$ ;
- 4) Calculation of the current density for a sphere of radius  $a$  ( $a - R$  is the height of the currents above the earth's surface) by means of graphic extrapolation of  $v_{\rho_2}$ ,  $v_{\rho_1}$ , and  $v_R$ .

We succeeded in turning all three laborious operations (calculation of  $\int_0^{\pi/2} (2R\bar{Z} + \bar{V}) \cos \psi d\psi$ ,  $\int_0^{\pi} \bar{U}(\theta) K(\theta) d\theta$  and  $\int_0^{\pi/2} \bar{U}_{n-1} \cos \alpha d\alpha$ ) into operations of a single type which could easily be performed by the aid of overlays. The total volume of computation work in this case is comparable with the work for spherical analysis.

In this way, we calculated the electric currents responsible for the  $S_D$ -variations. The experience in actual calculations showed the method to be completely applicable to the study of magnetic fields with a complex geographic distribution.

These equations permit calculating the electric currents responsible for the external part of the potential. Formulas interpreting the internal part of the potential may be obtained by an entirely analogous method. The basic method in this case will be the solution of the external problem of Dirichlet by means of the

Fredholm equation

$$V_p = 2\pi v_p - \int_s \frac{v_M \cos \alpha}{r^2} ds \quad (40)$$

$V_p$  in this equation denotes the values, known on the surface of the sphere, of the potential satisfying the Laplace equation outside the sphere. It must be taken into account here that the corresponding homogeneous equation

$$2\pi v = \int_s \frac{v \cos \alpha}{r^2} ds \quad (41)$$

has a solution other than zero ( $v = \text{const}$ ) in view of which certain complications arise in the solution of the external problem of Dirichlet. In courses on mathematical physics, however, it is shown that this problem can be solved for any distribution of  $V_p$ . It is true, of course, that the values of  $v$  will be found with an accuracy to a constant. Thus, in both cases (finding the external currents responsible for  $V_e$  and the internal currents responsible for  $V_i$ ), the problems are not solved with complete accuracy. In the former case,  $V_e$  is determined from the observed distribution of the geomagnetic components with an accuracy to a constant, while in the latter case the current density  $v_i$  is found from the calculated distribution  $V_i$  with an accuracy to a constant. But these limitations are negligible in considering the variations with time or space of the geomagnetic field. In studying the variable magnetic field or magnetic anomalies, we may reconcile ourselves to the fact that no uniform system of currents related to the permanent part of the potential is being calculated.

The extrapolation of the values of  $V_i$  known on the surface of the sphere  $R$  to external space may likewise be accomplished by means of the Poisson integral which, in this case, will be of the form

$$U_{p_e} = \frac{\rho'^3 - R^2}{4\pi R} \int_s \frac{\overline{U}_i}{r'^3} ds, \quad (42)$$

where  $\rho'$  is the radius vector of the point  $P_e$ , and  $r'$  is the distance between the point  $P_e$  and a variable point on the sphere. Thus the practical methods of calculating the internal currents may likewise be of entirely of the same type.

A consideration of the question of accuracy (for more details, cf. Section 5, Chapter V), has shown that the error of any operation of computation may be made as small as desired, and thus the accuracy of the results obtained is completely determined by the accuracy and completeness of the initial empirical data. Exactly as with spherical analysis, the integral method requires a knowledge of the geomagnetic components over the entire surface of the sphere. If the data do not cover the entire earth and are absent over large areas, then the formal calculation of the potentials and the calculations of the currents still remains possible, but the values so obtained will represent the phenomenon well for regions with abundant data and, possibly, represent it poorly for regions for which there are no data. A shortcoming of the method is the fact that it does not yield a compact analytic expression for the potential and for the current function: the final results are obtained in graphical or tabular form. Nevertheless, the fundamental problems (separation of the parts of external and internal origin from the observed field and calculation of the sources of the field, whether electric currents or magnetic masses) are solved by means of surface integrals. The method described above is therefore completely capable of replacing spherical analyses in cases of complex fields, in the consideration of a number of problems of geomagnetism, and possibly of other branches of geophysics as well. For example it would seem advisable to use this method in studying magnetic disturbances, the secular march representing a rather local phenomenon, a field of magnetic anomalies, etc.

A certain limitation of the method must also be pointed out. The extrapolation of the current density  $v$  gives an accuracy which is sufficient for practical purposes if the current-carrying layer is not too far from the surface on which the distribution of potential is known. In the opposite case, however, an extrapolation may

lead to great errors. Thus, in the second example of Section 3 of the present Chapter, the extrapolation of  $v$  to a sphere with a radius of  $1.20R$  gives an error of the order of 10%. If the sources of the magnetic field, however, are far enough from the surface at which the observations are made, then the field will not be characterized by great complexity of geographic distribution, and, consequently, there will be nothing to prevent the use of spherical analysis for studying the field. But in all cases of complex geophysical fields (magnetic anomalies, magnetic storms, and a number of others), which do not allow the use of spherical analysis, it may be assumed that the sources of the field are located not more than  $0.1R - 0.2R$  away from the earth's surface, at which the extrapolation required by the method of surface integrals is entirely allowable.



## CHAPTER V

The  $S_D$ -VariationsSection 1. Basic Data

As stated above in Chapter II, the second portion of the field of magnetic storms, the disturbed diurnal variations  $S_D$ , is rather well represented by the difference between the diurnal march on disturbed days ( $S_d$ ) and on quiet days ( $S_q$ ). The differences  $S_D = S_d - S_q$ , averaged over the year, were calculated for the 61 stations enumerated in Table 1. Since the  $S_D$ -variations vary markedly with the 11-year cycle of solar activity (Chapter VII), the observations for the Second International Polar Year (II MPG) were used as in the study of  $D_{st}$ , in order to assure uniformity of the starting data for most of the observatories. The list of quiet and disturbed days for these years, established by the International Association for Terrestrial Magnetism and Electricity on the basis of the magnetic characteristics of a worldwide net of observatories, is given in another paper (Jib1.40). The characteristics of magnetic activity on these days show that the disturbed days were days of moderate and great magnetic disturbances, while the quiet days were completely quiet. Thus the difference  $S_d - S_q$  can be completely characterized by the additional diurnal fluctuations which, on stormy days, are superimposed on the normal  $S_q$  variations. From seven observatories which are of great interest because of their geographic location, we did not have the data for the Second International Polar Year at our disposition. In view of this, we used the 1944 observations for three of them (Yakutsk, Tbilisi, and Tashkent). A comparison of the  $S_D$ -variations for 1944 and 1933 for a

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number of observatories showed that both form and amplitude of the  $S_D$  in these years closely resembled each other. The data of four observatories (Chelyuskin, Uelen, Kakioka, and Apia) relate to 1935, but to "reduce" them to the Second International Polar Year, the amplitudes of  $S_D$  were decreased in accordance with the change in the  $S_D$ -variations from 1933 to 1935 at the other observatories. To reduce the first two of these four stations, we used the series of observations at Tikhaya Bay and Dickson Island, and to reduce the last two we used Watheroo.

It is well known that the field of  $S_D$  displays more symmetry with respect to the geomagnetic coordinates than to the geographic. In view of this, we calculated the geomagnetic component  $X'$ ,  $Y'$ ,  $Z$  for each observatory from the observations of the variations of  $H$ ,  $Z$ , and  $D$ . A summary of these components is given in Fig.1 for the 44<sup>th</sup> observatory listed in Table 1. The time on the diagram is local geomagnetic time. The local geomagnetic time  $t_M$  was derived in 1936 by McNish (Bibl.9) by analogy to local mean solar time. Without allowing for small seasonal fluctuations, it is assumed that

$$t_M = T + \Lambda - 69^\circ;$$

$T - 69^\circ$  denotes the time of the zero geomagnetic meridian. In the middle latitudes,  $\Lambda - 69^\circ$  differs little from the geographic longitude of the locality  $\lambda$ , and therefore the difference between the local geomagnetic and geographic times is slight.

Before proceeding to calculating the potential of the field of  $S_D$ -variations and the construction of the current systems corresponding to it, it seems advisable to make a qualitative examination of the collected material with the object of elucidating certain questions of the morphology of the field. The most substantial of these questions are as follows: 1) dependence of  $S_D$  on local and universal time; 2) geographic distribution of  $S_D$  and selection of a system of coordinates convenient for the execution of the computational work; 3) choice of a working hypothesis as to the sources of the  $S_D$ -variations.

Section 2 of the present Chapter is devoted to the first two of these questions.

Section 3 is devoted to the third question, while an exposition of the results of the calculation of the potential and electric currents of  $S_D$  is given in the next four Sections.

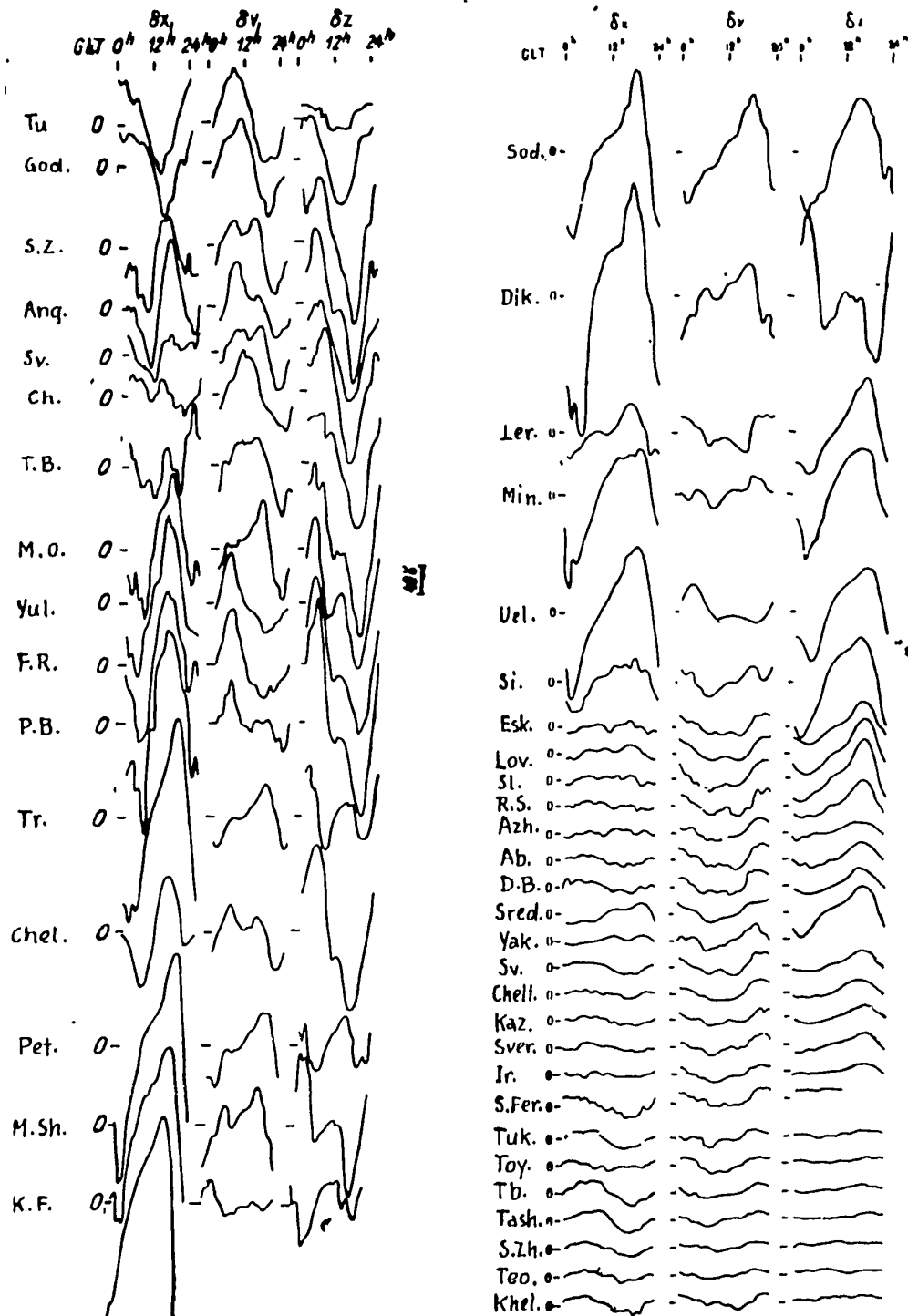


Fig.21 -  $S_D$ -Variations. The Observatories are Arranged in Order of Decreasing Geomagnetic Latitude ( $\phi$ )

## Section 2. Dependence of the $S_D$ -Variations on Local and Universal Time.

### The $S_D$ -Variations in the Polar Regions

The graph in Fig.21 discloses the rather evident dependence of  $S_D$  on  $t_M$  and  $\Phi$ . It still seems necessary to verify whether the position of the principal (or, perhaps, of the secondary) extreme values is tied to Universal Time, since the question of the influence of Universal Time on the diurnal variations has been repeatedly raised in the literature. K.K.Fedchenko, studying the diurnal variations of declination (for all days) has shown that the location of the diurnal maximum of  $D$  in the high latitudes is governed by Universal Time. A.P.Nikol'skiy (Bibl.26) makes analogous assertions with respect to the maximum of disturbance. According to him, there are two independent maxima in the diurnal march of the disturbance, one of which occurs in the evening hours of local time, and the other toward 17<sup>h</sup> of Universal Time (noon at the magnetic axis pole). Since the correlation between the  $S_D$ -variations and the diurnal march of disturbance ( $S_a$ ) is a priori very probable, and since in addition, in the high latitudes the  $S_D$ -variations are very close to the diurnal march for all days, these assertions force us to admit the possibility of the existence of two waves in the  $S_D$ -variations as well. The times of the principal and secondary maxima and minima of the  $Z$  component of the observatories of the northern hemisphere are noted on the diagram in Fig.22. Figure 22 shows the rather regular distribution of the extreme values as a function of the local geomagnetic time, while their distribution by Universal Time is completely random. The presence of two maxima in the  $S_D$ -variations is also found in the polar zone ( $\Phi = 63 - 67^\circ$ ), over which, according to present ideas, the electric currents causing the strong disturbance of the high latitude must flow. For other latitudes, both at the center of the polar cap and in the middle latitude belt, the existence of two maxima is not characteristic. Thus Figs.21 and 22 compel us to consider that the  $S_D$ -variations over the entire earth are governed primarily by local time. The Universal Time either has no influence at all on the distribution of  $S_D$  or exerts such an insignificant influence that it cannot

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be detected without a special workup.

The existence of  $S_D$ -variations at the magnetic axis pole appears to be somewhat in contradiction with this conclusion. At the magnetic axis pole, the concept of local geomagnetic time loses its ordinary meaning (just as the concept of local time at the geographic pole has a special meaning, since the altitude of the sun does not vary in the course of the day). It would seem that, if a diurnal periodicity exists at all at this boreal pole, it would have to be due only to Universal Time. In fact, in the zone near the pole (cf. the data of the Thule Observatory) the  $S_D$ -variations of the X and Y components are rather distinct, and only the variations of the Z component are equal to zero. But this contradiction is merely an apparent one. It is not hard to show that the existence of diurnal variations of the horizontal component at the pole ( $\theta = 0$ ) may be explained even without assuming the dependence of the field on Universal Time. Indeed, the potential  $V$  of the diurnal variations in the general case may be represented as a sum of Tesseral harmonics\*:

$$P_n^m(\cos \theta) \frac{\cos mt}{\sin mt}, \quad m \neq 0.$$

From the definition

$$P_n^m(\cos \theta) = \sin^m \theta \frac{d^m P_n(\cos \theta)}{d(\cos \theta)^m}$$

it follows that, for  $\theta = 0$ ,

$$P_n^m \equiv 0 \text{ and } V(0) = 0 \quad (m > 0)$$

Since

$$X = \frac{1}{r} \frac{\partial V}{\partial \theta} = \frac{1}{r} \left[ m \sin^{m-1} \theta \cos \theta \frac{d^m P_n}{d(\cos \theta)^m} + \sin^{m+1} \theta \frac{d^{m+1} P_n}{d(\cos \theta)^{m+1}} \right] \frac{\cos mt}{\sin mt},$$

$$Y = -\frac{1}{r \sin \theta} \frac{\partial V}{\partial t} = \pm \frac{m}{r} \sin^{m-1} \theta \frac{d^m P_n}{d(\cos \theta)^m} \frac{\sin mt}{\cos mt},$$

$$Z = \frac{\partial V}{\partial r} = \sin^m \theta \frac{d^m P_n}{d(\cos \theta)^m},$$

\* This reasoning is equally correct for geographic and geomagnetic coordinates.

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then  $Z(0) \equiv 0$  for  $m = 1$ , but  $X(0) \neq 0$  and  $Y(0) \neq 0$ , i.e.,  $Y$  and  $X$  depend on  $\sin t$  and  $\cos t$ , and the vector diagram in the horizontal plane of diurnal variations must

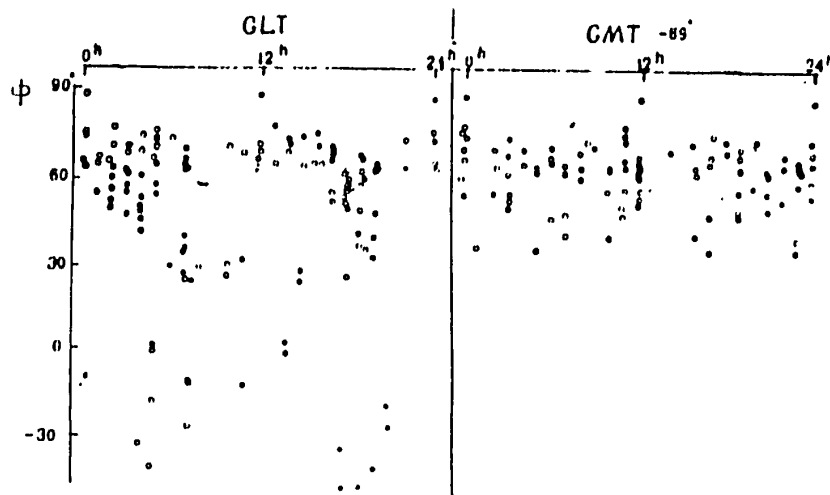


Fig.22 - Distribution of the Extremes of the ( $Z$  component of the  $S_D$ -Variations) by Hours of the Day. O Time of Occurrence of Minimum;

● time of Occurrence of Maximum

have a circular form at the pole. The actual data on  $S_D$  at the poles are in complete agreement with these arguments. As a matter of fact,  $S_D Z$  is very small at Thule, while the vector diagram of  $S_D$  in the  $XY$  plane for Thule, as for Cape Evans (an observatory near the south geomagnetic pole, cf. Bibl.40), is of circular form.

Here it is found that the vector of the horizontal component of  $S_D$  rotates with the variation of the azimuth of the sun (at the pole, the latitude of the sun does not vary throughout the course of the day, while its azimuth does vary). Thus, if viewed from the sun, the distribution of the vectors of the field of  $S_D$  remains constant, as is also the case for the  $S_Q$ -variations.

Returning to a consideration of the graphs in Fig.21, we note that the dependence of  $S_D$  on  $\phi$  in its general features may be described in the following way: The auroral zone ( $\phi = 65 - 69^\circ$ ) is characterized by small amplitudes of  $X'$  and by an unstable two-wave form of  $Z$ . This is in agreement with the hypothesis that a linear current flows along the zone at the height of the ionosphere (or that the lines  $\bar{S}\bar{T}\bar{A}\bar{T}$

current of the surface current system are more closely spaced). North of the zone, Z has the form of a single wave with a minimum at noon, the greatest amplitude being

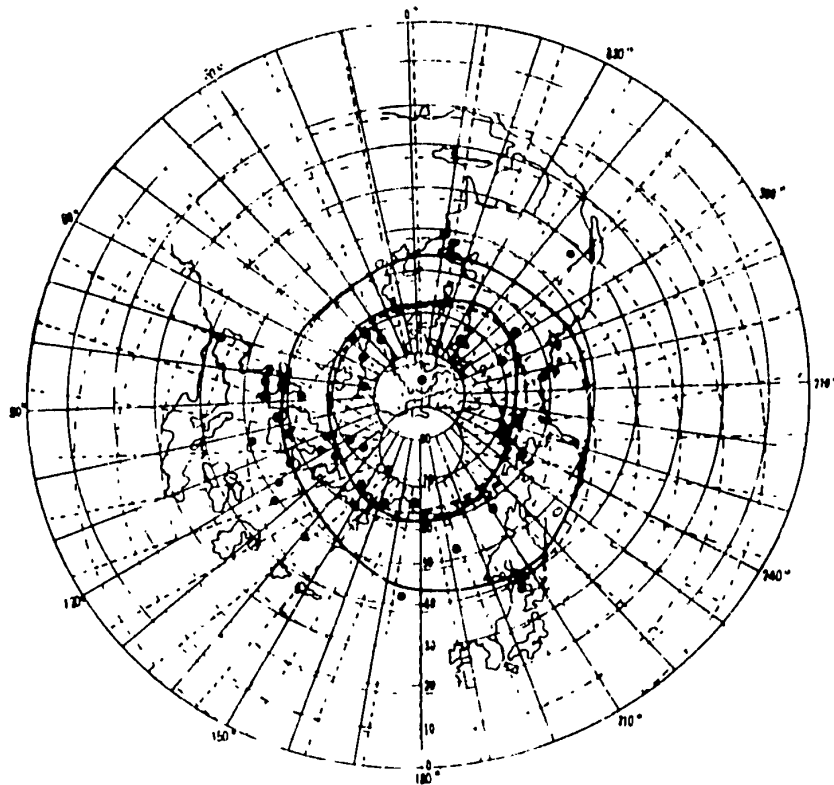


Fig.23 - The Auroral Zone and the line of Centers of the Middle-Latitude Eddies of  $S_D$ -Variations (—— Position of Zone According to Vestine.

Coordinate Nets: —— Geomagnetic; ----- Geographic)

reached at latitudes  $75^\circ$ , north of which the amplitude again decreases. North of the zone,  $X'$  decreases in amplitude up to  $\phi = 75^\circ$  where the phase is reversed. South of the zone, Z retains its form down to the very equator and only gradually decreases in amplitude, while  $X'$  decreases sharply in amplitude, resulting in still another phase reversal at latitude  $40$  to  $50^\circ$ .

However, there are a number of cases (Angmassalik, Chelyuskin, Sodankyla, etc.) which indicate a longitudinal asymmetry in the incidence of the variations. Vestine has pointed out that the asymmetry is considerably decreased if, instead of  $\phi$  as the argument in the geographical distribution of  $S_D$  we take  $\eta$ , which is the distance to the zone of the hypothetical linear current. He determined the position of this zone

from the data of the magnetic variations and compared it with the maximum isochasm (the Vestine zone is shown by the broken line in Fig.23).

The data collected by us for the Second International Polar Year have allowed the position of the zone to be pin-pointed (the solid line in Fig.23), bringing it somewhat further south on the territory of the USSR. To determine the position of this zone, maps of the isoamplitude of  $S_D X'$  were prepared, and the zone indicated in Fig.23 is the isoline of maximum amplitude of  $S_D X'$ . The second line in the diagram, lying between  $40^\circ$  and  $50^\circ$ , indicates the latitude at which the  $S_D$ -variations of  $X'$  change their phase\*.

In columns 3 and 9 of Table 1 for each observatory, the values of  $\eta$  expressed in degrees and the values of  $\phi'$  are given ( $\phi'$  is the geomagnetic latitude corrected for the deviation of the zone from the circle of latitude  $\phi_0 = 67^\circ$ ;  $\phi' = 67^\circ - \eta^\circ$ ). The arrangement of the graphs of  $S_D$  in accordance with  $\phi'$  (Fig.24) leads to an almost complete destruction of the anomalies in the distribution of  $S_D$ , especially in the high latitudes. In the low latitudes, the replacement of  $\phi$  by  $\phi'$  is insubstantial, and a consideration of the middle and low latitude portions of Figs.21 and 24 shows the same absence of any clearly anomalous observatories.

In this connection, the complete normality of the  $S_D$  at Huancayo and other observatories of the equatorial region is of great interest in this connection. Since the  $S_q$ - and  $L$ -variations of Huancayo are abnormally great, the absence of anomalies in  $S_D$  and  $D_{st}$  is an indication that the current systems of the disturbances and of the quiet variations are located in different layers of the ionosphere.

The close relation between  $S_D$  and  $\phi'$  stands out most vividly on the graphs in Fig.25, which give the "meridian" of  $X'$ ,  $Y'$ ,  $Z$  corresponding to 0, 6, 12, and 18<sup>h</sup> local geomagnetic time. The dispersion of the points, mapping the relation  $X'(\phi')$  and  $Z(\phi')$ , is small. The dispersion of the curve of  $Y'(\phi')$  is considerably

\* As we will see later, the centers of the middle latitude eddies of  $S_D$ -variations are located at this latitude.

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greater. This indicates that  $Y'$ , like  $D$ , depends to a greater extent on the value of  $D_0$ .

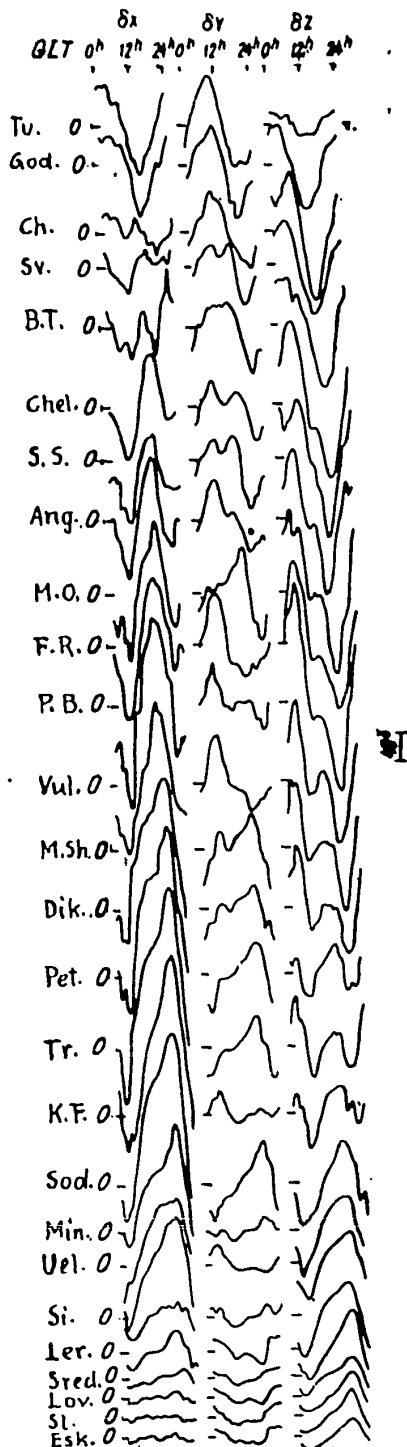


Fig. 24 - The  $S_D$ -Variations; Observatories are Arranged in Order of Decreasing Corrected Geomagnetic Latitude ( $\Phi'$ )

A comparison of Fig. 25 with the corresponding graphs of Vestine (Bibl. 58) discloses certain differences. The most substantial of these is that, according to Vestine  $X'$  in the auroral zone has extreme values at 6 and 18<sup>h</sup>, and  $Z$  at 0 and 12<sup>h</sup>, while according to our data the extreme values of  $X'$  are shifted to 4 and 16<sup>h</sup>, respectively.

In concluding this discussion of the geographic incidence, let us describe the distribution of the field in the southern hemisphere. The data collected by us show a similarity in the behavior of  $X'$ ,  $Y'$  and  $Z'$  in the low-latitude portions of the northern and southern hemispheres. The values  $Z'$  and  $Y'$  are opposite in sign in the different hemispheres and are equal to zero on the equator. The value  $X'$  is symmetric in the two hemispheres and has its maximum amplitude on the equator. Unfortunately we lack observations of high-latitude stations of the southern hemisphere for the Second International Polar Year, which would allow us to judge whether this symmetry in the two hemispheres persists at all latitudes. The literature contains only one attempt to determine the zone of magnetic activity from the data of the  $S_D$ -variations in the southern hemisphere (the work by Vestine and Snyder (Bibl. 63), but the results of this work do not

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seem trustworthy. The authors found that the polar zone in the southern hemisphere also has an elliptical form with its focus at the magnetic axis pole, except that it

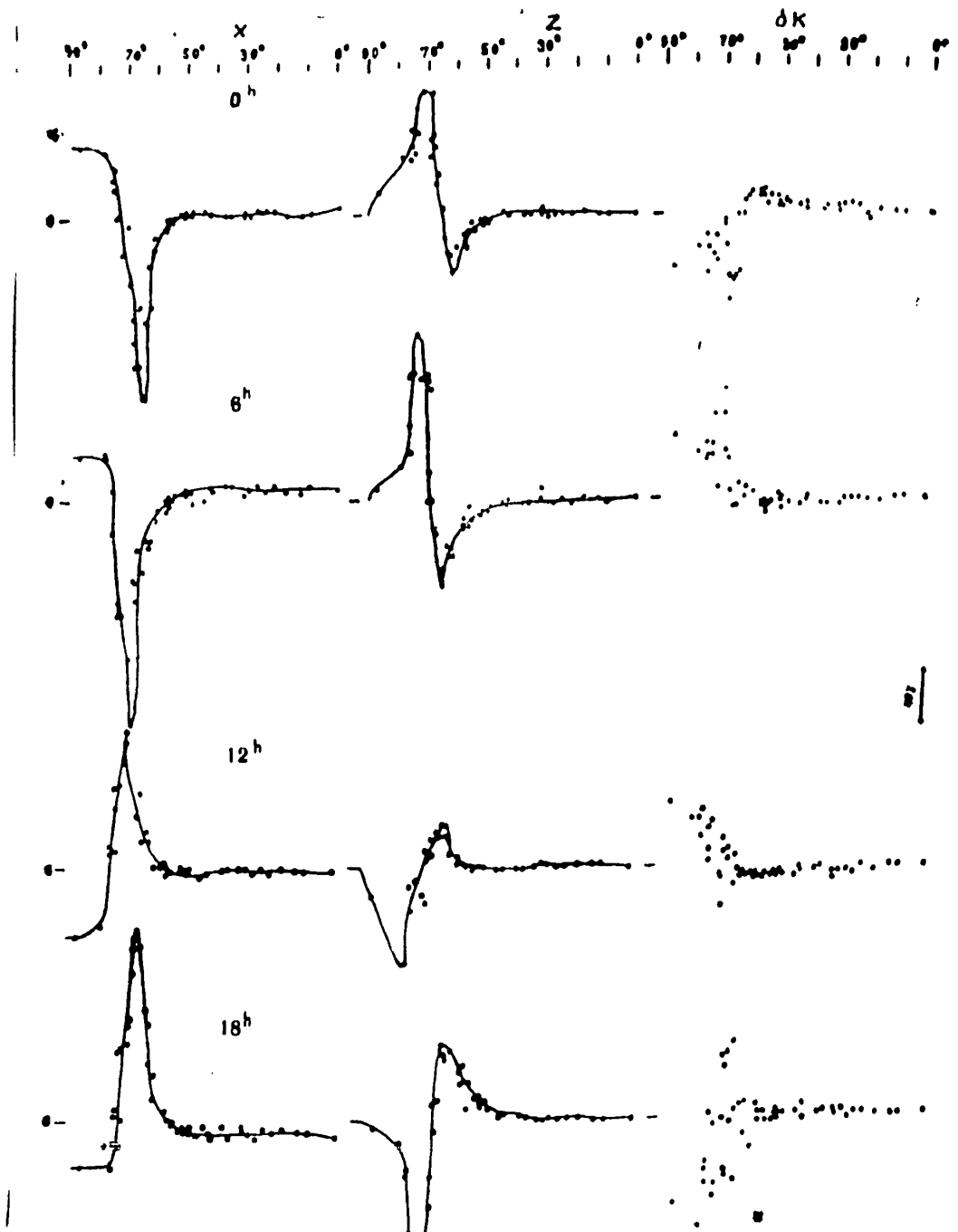


Fig.25 - Latitudinal Distribution of  $S_D$ -Variations

is not located symmetrically to the northern zone but is elongated on the side opposite the south geographic pole. This conclusion is based on extremely scanty and nonuniform material - observations of 1882, 1903, 1911, 1933, 1941 are used. Since

$S_D$  varies considerably (cf. Chapter VII) with the 11-year cycle, the use of these observations without an appropriate reduction might give erroneous results. In view of the impossibility, at the present time, of making the position of the southern zone of magnetic activity more precise, we used only the data of the northern hemisphere in the analytic representation of the field, considering that, although only in rough approximation, the potential of  $S_D$  is the same in magnitude, but opposite in sign in the northern and southern hemispheres.

These arguments on the geographic distribution of the  $S_D$ -variations compel us to consider that, in calculating the potential, the field of  $S_D$  may be assumed to depend on two arguments, the geomagnetic latitude  $\phi$  and the geomagnetic time  $t_M$ . In this case, however, the longitudinal terms will be rather great. The longitudinal asymmetry will be considerably less if the corrected value  $\phi'$  is taken instead of  $\phi$  as the first argument, i.e.,  $S_D = S_D(\phi', t_M)$ . The replacement of  $\phi$  by  $\phi'$  corresponds to the replacement of the actual auroral zone by an arbitrary circular zone.

### Section 3. Selection of the Type of the Current System

The selection of the method of calculating the currents responsible for the magnetic variations depends to a large extent on our a priori opinion as to the form of the current system. It is well known that the problem of finding the distribution of the currents from an assigned geomagnetic field on a sphere is in practice many-valued; for this reason all investigators desiring to evaluate the intensity and configuration of the currents have adopted in advance some hypothesis with respect to their distribution. We have already shown in Chapter II that, from this point of view, all investigators of magnetic storms are divided into two groups. The works of the first group assume that the magnetic disturbances are due to linear currents or relatively narrow current belts in the auroral zone. The second group embraces the works of Chapman and his colleagues, who consider that the currents of magnetic storms encompass the earth as a whole, forming a spherical current layer. To check these hypotheses, we calculated the geographical distribution of the fields of linear cur-

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rents and compared them with the actually observed distribution of the field of  $S_D$ -variations. Figure 26 shows the latitudinal distribution of the X-component of the

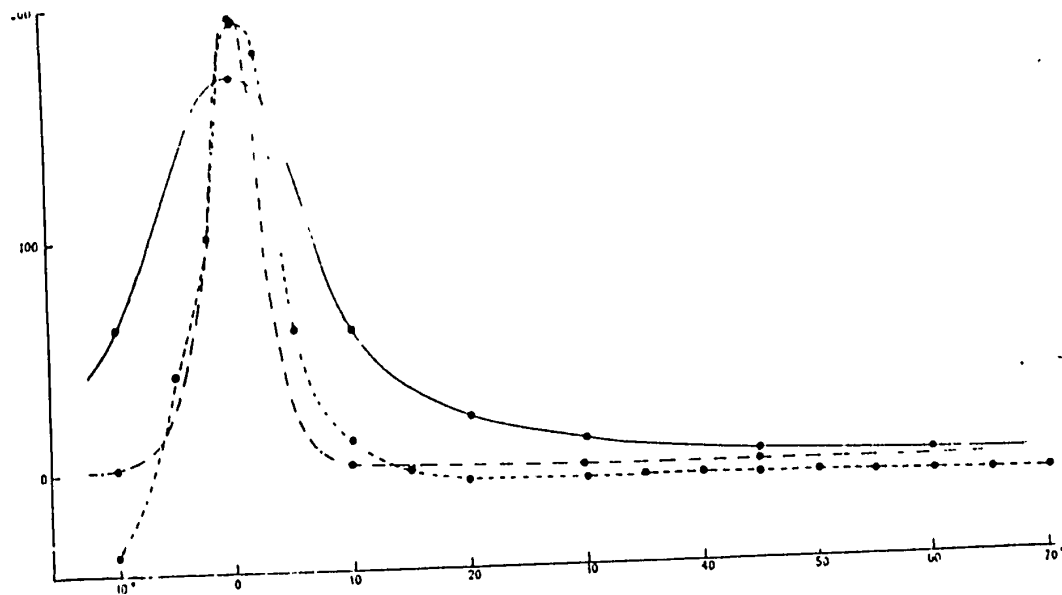


Fig. 26 - Comparison of  $S_D$ -Variations with the Field of the Linear Electric Current. Along the horizontal, the distance in degrees to the zone of linear current (the auroral zone) is plotted, and along the verticals, the field intensity in gammas (---, amplitude of night minimum of X component of  $S_D$ -variations; —, X component of field of vertical current (according to Gnevyshev); -.-.-, X component of field of horizontal current (according to Birkeland))

field of vertical current (according to Gnevyshev), and of the horizontal current (according to Birkeland). In reading the figure, the vertical linear current is conceived as flowing in the direction (V) from infinity to a height of 30 km, while the horizontal current flows at a height of 100 km in a direction perpendicular to the plane of the paper. The graph of the X' component of the  $S_D$ -variations is distinguished from the two theoretical curves by its asymmetry with respect to OZ and by its negative values of X' at a certain distance from OZ. Obviously the theoretical curves will be unable to approximate sufficiently well to the observed curve, no

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matter what parameters are selected. The same non-correspondence is observed between the curves of latitudinal distribution of the field of linear currents and the  $S_D$ -variations of other components and at other hours of the day.

The hypothesis of a spherical inhomogeneous current layer is more general and, as shown by sample calculations of Chapman, it is able to explain the complex structure of the field of  $S_D$ . The above arguments forced us likewise to adopt the hypothesis of a spherical current layer, and, without making any further assumptions with respect to the configuration of the current lines, to calculate them from the observed geomagnetic elements, using the method of surface integrals.

#### Section 4. Calculation of External and Internal Potential

The values of the potential  $V$ , corresponding to the  $S_D$ -variations observed on the earth's surface were obtained by integration of the  $X'$  component. It follows from the symmetry of the  $X'$  and the asymmetry of  $Z$  with respect to the equator (cf. supra) that the potential should be antisymmetric and should vanish at the equator.

The values of  $V(0, t_M)$  (for the northern hemisphere) were calculated from the graphs of Fig. 25 by the formula

$$V = - \int_0^{\frac{\pi}{2}} X' d\theta + V_{\theta=\frac{\pi}{2}} = - \int_0^{\frac{\pi}{2}} X' d\theta.$$

The calculations were made for 216 points at intervals of 2 hours in longitude and  $5^\circ$  in latitude. The integration along various meridians led to the following values of  $V_0 = V = 0^\circ$ :

|                                |   |    |    |    |     |     |    |    |    |    |     |    |
|--------------------------------|---|----|----|----|-----|-----|----|----|----|----|-----|----|
| Time, hours . . . . .          | 0 | 2  | 4  | 6  | 8   | 10  | 12 | 14 | 16 | 18 | 20  | 22 |
| $V_0 \times 10^{-3}$ . . . . . | 2 | 14 | 14 | 12 | -13 | -19 | 10 | 20 | 3  | 0  | -13 | 2  |

Since, as already noted, the potential of the diurnal variations not depending on Universal Time, must assume zero values at the pole, the values of  $V_0$  from the annexed table would have to serve as an indication of the value of the calculation errors. But in reality the values of  $V_0$  exceed the mean error of analysis, equal

to  $3 \times 10^{-3}$  CGS (for more details on the accuracy of analysis, see later). The existence of a potential-free part in the field of  $S_D$ -variations might be an explanation for this discrepancy. The existence of a potential-free part (11) in the permanent field of the earth and of the  $S_Q$ -variations has been detected by a number of authors. But since such a part requires the existence of vertical electric currents  $10^3$  to  $10^4$  times higher than the currents that can be observed by the methods of atmospheric electricity, the reality of the N field has always been subject to doubt. The vertical currents necessary to explain the N field of  $S_D$ -variations, calculated by the formula  $i = \oint H ds$ , should not exceed  $0.5 \times 10^{-14}$  CGS and should, as shown by calculations, be concentrated in the polar latitudes. The observations of the vertical current in Franc Josef Land by Scholtz and at Chelyuskia by Gerasimenko (Bibl.10) have shown that the mean value of the currents fluctuates about  $2-3 \times 10^{-16}$  CGS, while individual values often reach  $1 \times 10^{-15}$  CGS, which is only 5 times less than the values calculated for the N part of the  $S_D$ -variations.

Without discussing the question as to the reality and possible causes of the vertical currents (to which an extensive literature is devoted), we may say that it would be extremely desirable to repeat the observations in high latitudes on stormy days, which would help to solve the problem finally.

Since the magnitude of the potential-free field (if it exists) is small in comparison with the potential part, it proved possible in first approximation to neglect it, by assuming that integration along all meridians leads to one and the same value of  $V_0$ , equal to zero. The errors of closure (i.e., of the values of  $V_0$  obtained by integration) were distributed proportionally over all the latitudes, so as not to violate the condition that the algebraic sum of the 24 values of  $V$  along each parallel of latitude should be equal to zero. This last condition is dictated by the fact that the  $S_D$ -variations should be represented by deviations from the mean diurnal values of the magnetic elements. The values of the potential  $V$  obtained in the result of the above corrections are given in the cartogram of Table 10. This carto-

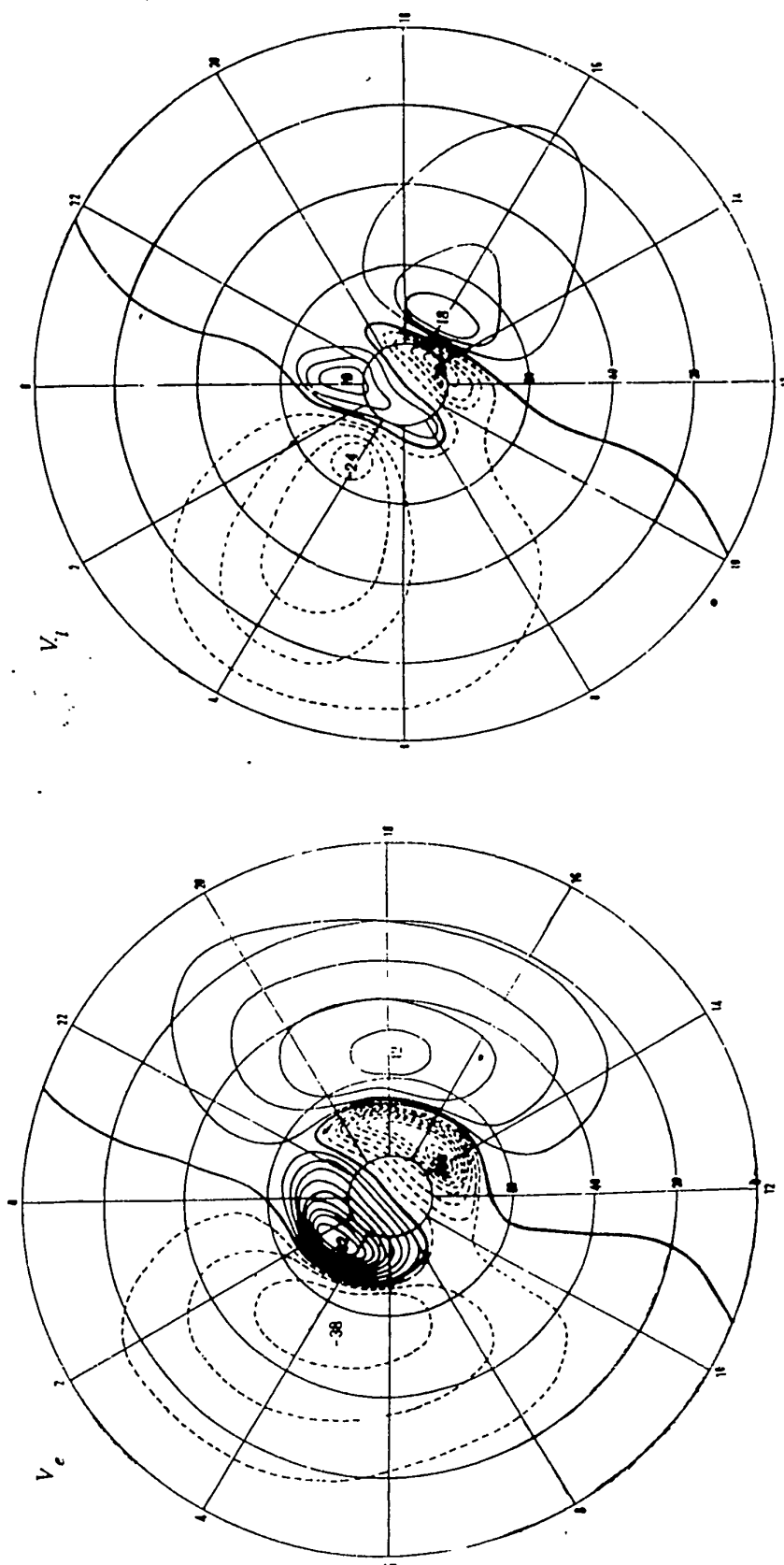


Fig.27 - Isolines of External ( $V_e$ ) and Internal ( $V_i$ ) Part of the Potential of the Variations  
 ( — — — Negative Values of Potential; — Positive values. Coordinate Net, geomagnetic  
 latitude and Geomagnetic Time. Units of Potential,  $10^3$  CGSN. Isolines drawn at Intervals  
 of  $10^4$  CGSN.)

gram and the analogous one (Table 9) for the Z component (prepared from the graphs of Fig.25) served as the starting material for calculating the external and internal parts of the potential,  $V_e$  and  $V_i$ . The calculations were performed by the method described in the preceding Chapter; the values of  $V_e$  and  $V_i$  were found for 108 points, at intervals of  $10^\circ$  in latitude and 2 hours in longitude. Figures 27 and 28 are

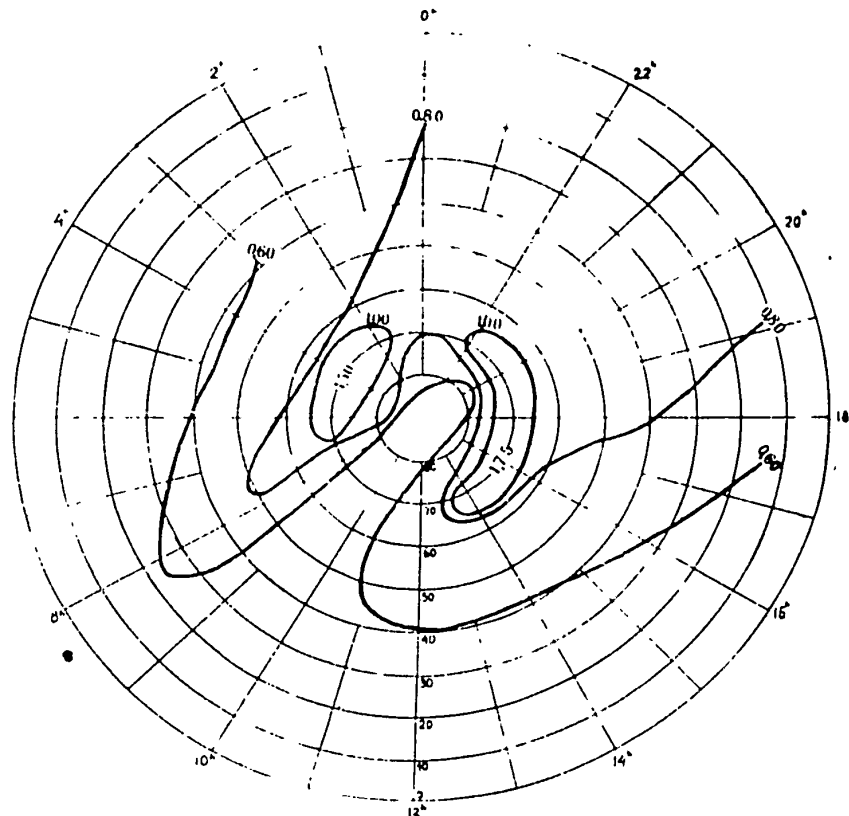


Fig.28 - The  $S_D$ -Variations. Ratio of External Part of Potential to Observed Potential ( $V_e/V$ )

maps of the potentials  $V_e$ ,  $V_i$  and the ratio  $V_e/V$ . The isolines of  $V_e$  and  $V_i$  are drawn at intervals of  $10^4$  CGS. As will be seen from the diagram, the values of the potentials for  $V_e$  range from  $92 \times 10^3$  CGS to  $-90 \times 10^3$  CGS, and for  $V_i$  from  $-29 \times 10^3$  CGS to  $19 \times 10^3$  CGS, and the signs of the potentials are different on the morning and evening sides of the earth. The distributions of the isolines of  $V_e$  and those of  $V_i$  differ considerably. It is true that the distributions of both  $V_i$  and  $V_e$  are characterized by four extremes (two polar and two middle latitude), but the form of



the isolines on the  $V_e$  map is more elongated in a longitudinal direction. There is a shift in longitude which is particularly great for the morning polar maximum. The regularity of the distribution of  $V_e$  and  $V_i$  compels us to consider the results obtained as plausible.

In absolute magnitude,  $V_e$  considerably exceeds  $V_i$ , confirming the proposition that the principal sources of the field are located outside the earth's surface. The value of  $V_e/V$  varies over a wide range, from 0.55 to 1.75. For three points,  $V_e/V > 1$  (1.50, 1.11, and 1.75). A regularity in the latitudinal distribution of  $V_e/V$  is noted: the mean value of  $V_e/V$  for the entire earth is 0.83; for the low-latitude belt ( $\phi \leq 50^\circ$ )  $V_e/V = 0.79$ ; and for the high-latitude cap ( $\phi > 50^\circ$ ),  $V_e/V = 0.89$ . The figures so obtained differ considerably from the value  $V_e/V = 0.60$ , adopted by Chapman, by analogy with the  $S_q$ -variations. It is not possible at the present time to give a trustworthy explanation of the latitudinal dependence found for  $V_e/V$ . Two hypotheses may, however, be advanced: 1) The variation in  $V_e/V$  with the latitude indicates the unequal conductivity of the earth at different latitudes; 2) it is possible that the height of the current layer is different at different latitudes, which would seem to be entirely plausible in view of our present knowledge as to the heights of the ionospheric layers.

#### Section 5. Discussion of the Accuracy of the Method

Let us now dwell on the question of the accuracy of the integral method of representing the field. First of all let us evaluate the error in the calculation of  $V$ . On replacing integration by summation, we have  $V = R\Delta\theta \sum X$ , where the  $X$ , for simplicity, are denoted by  $X'$ .

The error in  $V$  is evaluated as follows:

$$\delta V = R\Delta\theta \sum \delta X,$$

where  $\delta X$  is the error in  $X$ .

For  $\Delta\theta = 5^\circ$ ,  $R = 0.4 \lambda 10^3$  and  $\delta\lambda = 1, 5$ , and  $10 \gamma$ , we find that the maximum

error accumulated up to the pole  $\delta V_{\max}$  is  $12 \times 10^3$ ,  $12 \times 10^4$ , and  $60 \times 10^4$  CGS, respectively. The mean error  $\delta V = 1/n \delta V_{\max}$  is  $0.5 \times 10^3$ ,  $3 \times 10^3$ , and  $6 \times 10^3$  CGS. At latitude  $55^\circ$ , the maximum error for  $\delta X = 2\gamma$  will be  $\delta V_{\max} = 14 \times 10^3$  and the mean error  $\delta V = 1 \times 10^3$  CGS. If the accuracy of  $X$  is lower in the polar cap ( $\phi \geq 55^\circ$ ), for instance  $\delta X = 10\gamma$ , then the errors  $\delta V_{\max} = 6 \times 10^4$  and  $\delta V = 3 \times 10^3$  will accumulate up to the pole. Judging from the graphs of Fig. 2b, the accuracy of the observed data of  $X$  in the middle latitudes is actually of the order of  $2\gamma$ , while in the polar latitudes it is of the order of  $10\gamma$ ; thus the accuracy of  $V$  observed can be evaluated as  $1 \times 10^3$  CGS in the middle latitudes and  $3 \times 10^3$  CGS at the pole. Since the observed value of the potential reaches  $100 \times 10^3$ , the error would appear to be allowable.

The error accumulated in the calculation of  $V_e - V_i$  may be evaluated as follows: On replacing the integral expression eq. (12, IV) by the summation expression, we have

$$V_e - V_i = \frac{1}{2\pi} \Delta\psi \Delta\theta \sum \sum (2RZ + V) \cos \psi.$$

For  $\Delta\psi = \Delta\theta = 5^\circ$ ,  $\cos \psi = 0.5$ , we have

$$\delta(V_e - V_i) = \frac{0.5 \times 10^{-2}}{6.3} \sum \sum (2R\delta Z + \delta V).$$

The accuracy of observation of  $Z$  is lower than that of  $X$ ; the calculations have been made under the assumption of a  $\delta Z$  equal to 2, 5, 10, and  $20\gamma$  (Table 11).

In this calculation we assumed a mean error  $\delta V$  for the entire earth, since in calculating  $V_e - V_i$  the values of  $V$  for the entire earth enter the integrand expression. The number of terms in the expression  $\delta(V_e - V_i)$  is 2600. The small table presented shows that 1) the error  $\delta(V_e - V_i)$  is due more to the inaccuracy of  $Z$  than to the inaccuracy of  $X$ ; 2) the mean error  $\delta(V_e - V_i)$  is one or two orders smaller than the error of the observed  $V$ , even under the least favorable assumptions as to  $\delta Z$ . Thus the practical accuracy of  $V_e$  is the same as the accuracy of  $V$ .

In the calculations presented we did not take into consideration the errors of the mathematical operations themselves (integration, etc.), since these may be performed with a very high accuracy, much higher than the accuracy of the initial data.

Table 11 \*

|                                  |                    |                    |                     |                     |
|----------------------------------|--------------------|--------------------|---------------------|---------------------|
| $\delta Z$ . . . . .             | $2 \times 10^{-5}$ | $5 \times 10^{-5}$ | $10 \times 10^{-5}$ | $20 \times 10^{-5}$ |
| $2R\delta Z$ . . . . .           | $2.5 \times 10^4$  | $0.5 \times 10^4$  | $13 \times 10^4$    | $26 \times 10^4$    |
| $\delta V$ . . . . .             | $0.3 \times 10^4$  | $0.3 \times 10^4$  | $0.3 \times 10^4$   | $0.3 \times 10^4$   |
| $\delta(V_e - V_i)_{\max}$ . . . | $3 \times 10^4$    | $7 \times 10^4$    | $13 \times 10^4$    | $26 \times 10^4$    |
| $\delta(V_e - V_i)_{cp}$ . . .   | 24                 | 56                 | 104                 | 208                 |

These evaluations of the errors indicate that the integral method is able to provide adequate accuracy. In practice its accuracy is completely determined by the accuracy of the observed experimental material. By the accuracy of the observed data, of course, we mean not only the accuracy of the observations themselves, but also the stability and representative nature of the mean data and the distribution of observation points over the earth's surface.

#### Section 6. The Current System of $S_D$ -Variations

From the values of  $V_e$  calculated by the integral method, a distribution of the current density in a spherical layer of radius  $a = 1.05 R$  ( $0.05 R = 313$  km) was constructed, corresponding to the height of the  $F_2$  layer of the ionosphere, to which it is most probable that the currents of the magnetic disturbances can be referred. The current system so obtained (Fig. 29) like the above-described Chapman system, consists of four current eddies, of which the two more intense are located on the morning and evening sides of the polar cap, and the other two in the middle latitudes. The signs of the current functions are different: the polar evening and middle-latitude morning eddies have a positive sign for the current function, the polar

\* All values given in Table 11 are in the CGSM system.

morning and middle-latitude evening eddies a negative sign. The centers of the polar current eddies are located on the 2 and 15 hour meridians, those of the middle-latitude eddies on the 4 and 10 hour meridians. The evening and morning eddies are unequal in intensity: the morning polar eddy is more intense than the evening eddy, while the evening middle-latitude eddy is more intense than the morning eddy. In the zone  $\varphi = 67 - 70^\circ$  (the auroral zone) the current lines are closely spaced, giving us the right to liken this part of the current system to the linear current flowing eastward on the evening side of the earth and westward on the morning side. It is this crowding of the lines of force that is responsible for the specific peculiarity of the course of magnetic and ionospheric phenomena in the zone.

The picture so constructed for the currents allows us to interpret in the following manner the pattern of geographic distribution of  $S_D$  described by us in section 2. In each hemisphere, there are four characteristic types of  $S_D$ -variations:

I. Circumpolar type, characterizing the daily minimum in the  $\lambda'$  and  $\lambda$  components; the amplitude of  $\lambda$  is very small. This type corresponds to the center of the polar cap, over which the currents flow in the uniform layer in the direction of the 20 - 3 hour meridian.

II. Polar type, observed between the zone of close spacing of the current lines at the latitude  $67 - 70^\circ$ , and the latitude of the centers of the polar eddies ( $\varphi = 75^\circ$ ). It is characterized by the afternoon maximum in  $\lambda'$  and by the daytime minimum in  $\lambda$ . The amplitude of both components is high.

III. Middle-latitude type, observed between the auroral zone and the latitude over which the centers of the middle latitude eddies are located ( $\varphi = 55^\circ$ ). It is characterized by an evening maximum in  $\lambda'$  and  $\lambda$ .

IV. Low-latitude type (between the latitude of the centers of the middle-latitude eddies and the equator) with an evening minimum of  $\lambda'$  and an evening maximum of  $\lambda$ . The amplitudes of both components, especially of  $\lambda$ , are small.

Directly over the latitudes of the centers of the eddies ( $\varphi = 75^\circ$  and  $\varphi = 55^\circ$ )

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and under the zone of crowding of the current lines, transitional forms are observed, characterized by the change in sign of the  $X'$  variations and the maximum increase in the amplitude of  $Z$  in the former cases and by the change in sign of  $Z$  and an increase

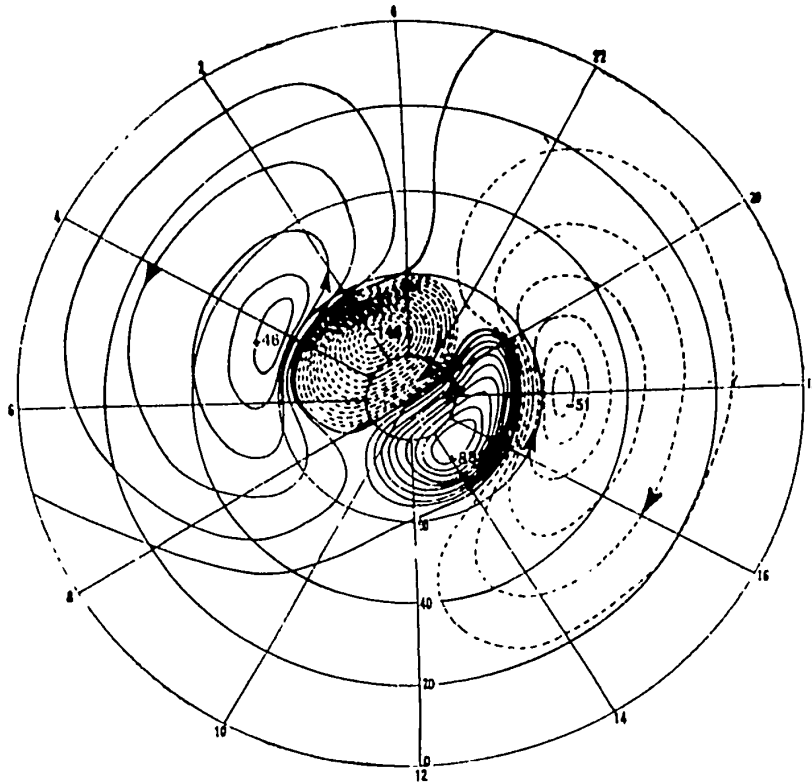


Fig.29 - Current System of  $S_D$ -Variations. Intensity of current given in thousands of amperes. A current of 10,000 amp flows between two successive lines of current. The coordinate net is the geomagnetic latitude and geomagnetic time ( ———, positive values of current functions; - - - - negative values)

in the amplitude of  $X'$  in the latter case.

The location of the centers of the middle-latitude and polar eddies at the various meridians is in full agreement with the well-known fact that the time of occurrence of the extreme values of  $S_D$  is different in the middle and polar latitudes. The unequal intensity of the morning and evening extreme values is likewise understandable if we bear in mind that the height of the evening maximum of  $S_D$  in the

middle latitudes is greater than the depth of the morning maximum. In reality, as pointed out above, the auroral zone (or the zone of linear current) is not a true parallel circle. For this reason, the boundaries of the regions corresponding to the various types of  $S_D$  will likewise deviate from the parallel.

A comparison of the system of currents of  $S_D$  calculated by us, which we shall hereafter term the  $HIIZM^*$  system) with the Chapman system (cf. Fig. 4b) discloses a number of substantial differences. First, in the Chapman sum of  $S_D$ -currents, as in his  $D_{st}$ -system, the signs of the current function are not indicated. Obviously, the difference in sign of the current eddies discovered by us must be of significance in the construction of a quantitative theory of the  $S_D$ -variations.

The second difference of the  $HIIZM$  system is the shift of the centers of the polar eddies, that of the morning eddy to  $2^h$  of geomagnetic time, that of the evening eddy to  $12 - 14^h$ , while in the Chapman system both eddies are centered symmetrically at 6 and  $13^h$ . Because of this displacement of the eddy centers, the currents in the polar cap have a direction perpendicular to the  $2 - 14$  hour meridian, which well explains the  $S_D$ -variations of the horizontal components at Thule and Godhavn, with the minimum of  $X^1$  at  $14^h$ . According to the Chapman system, a minimum of  $X^1$  at  $13^h$  and a zero value at  $12^h$  might be expected at these observatories.

The position of the morning middle-latitude eddy likewise does not agree in the two systems, and the absolute values of the intensity of the polar eddies is different. In the Chapman system, the total intensity of the current flowing through the polar cap is 450,000 amp, while in the  $HIIZM$  system it is 200,000 amp. In the Chapman system, moreover, the intensity of the morning and evening eddies is the same.

However, it does seem that all these differences (except for the difference in the signs of the current function) are due not so much to the different method of calculation as to the difference in the starting data used. An approximate evaluation of the intensity of the currents from our data gave the following results:

\* Terrestrial Magnetism Research Institute

As is commonly known, the density of a uniform current layer of sufficiently great extent is  $i = \frac{F_e}{2\pi}$ , where  $F_e$  is the field induced by this layer on the surface of the earth, perpendicular to the direction of  $i$ . Assuming that the ratio of the external field to the observed field ( $F$ ) is equal to  $k$ , we have  $i = \frac{kF}{2\pi}$ . If the width of the belt of current is  $l_{cm}$ , then the total current is  $I = \bar{i}l$ , where  $\bar{i}$  is the mean value of the flux density which, without great error, can be taken as equal to  $2/3 i_{max}$ , if parabolic distribution of the density in the flux is assumed. It follows from this that

$$I = \frac{kF^2}{2\pi^3} l \text{ CGS.} \quad (1)$$

Applying this approximate formula to the observed variations at Thule ( $X'_{max} = 60Y$ ), setting the width of the flux at  $32^\circ$ , and replacing the value of the coefficient  $k = 0.6$ , adopted by Chapman, by the value  $k = 0.9$  found by us for the polar cap, we have

$$I = \frac{0.9 \times 10 \times 10^{-5}}{6.3} \times \frac{2}{3} \times 32 \times 1.1 \times 10^7 \text{ CGS} = 22 \times 10^4 \text{ A.}$$

Thus a rough estimate of the current likewise leads to eddy intensities half as great as those given by Chapman. As for the directions of the parallel currents flowing through the polar cap, as we have already noted, it follows directly from the observations at Thule and Godhavn that the currents must be parallel to the 3 - 20 hour meridian instead of to 0 - 12 hour meridian, as is the case in the Chapman system. Thus the absence of a good agreement between the polar value of the  $S_D$ -currents in the Chapman system and the observed variations is explained simply by the inadequacy of the observational materials that were available to him. The calculations presented above show that the approximate method of estimating the intensity and direction of the currents gives very good results. Of course, the approximate method does not make it possible to separate the internal and external parts from the

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observed field, to obtain the numerical values of the potential, to determine the signs of the current function, nor to elucidate the details of the configuration of the currents, etc., but it is simply sufficient to obtain a rough picture of the currents as necessary for a qualitative discussion of various problems.

### Section 7. The Polar Part of the $S_p$ -Currents

The distribution of the  $S_p$ -currents in the auroral zone shown in Fig. 29 was also compared by us with the parameters of the current obtained under the assumption of linearity of the current. As stated in Chapter 1, the calculations of the intensity, height, and position of the linear current in the auroral zone has been carried out by a number of authors from observations of a pair of stations or of several pairs, yielding different results, depending on the materials used. Accordingly, we repeated our calculations using the same data we used in constructing the system of surface currents.

The distribution of the vectors of the magnetic field of the linear electric current is schematically represented in Fig. 30. In considering this figure we must

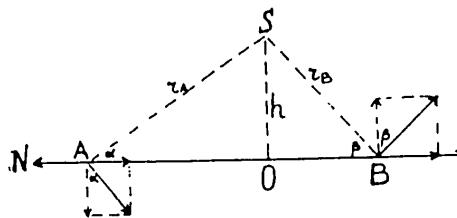


Fig. 30 - Pattern of Magnetic Field of  
Horizontal Linear Current

imagine the current to flow perpendicular to the plane of the paper in the direction from the paper toward the observer, and assume  $h$  to be the height of the current above the earth's surface, while  $A$  and  $B$  are two points at which the vectors of the magnetic field are known. On the drawing, they are located at different sides of  $O$ ,

the projection of the current onto the earth's surface. It follows from the drawing in Fig. 30 that:

$$\left. \begin{aligned} AO &= h \operatorname{ctg} \alpha \\ BO &= h \operatorname{ctg} \beta \end{aligned} \right\} \quad (2)$$

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observed field, to obtain the numerical values of the potential, to determine the signs of the current function, nor to elucidate the details of the configuration of the currents, etc., but it is simply sufficient to obtain a rough picture of the currents as necessary for a qualitative discussion of various problems.

### Section 7. The Polar Part of the $S_D$ -Currents

The distribution of the  $S_D$ -currents in the auroral zone shown in fig.29 was also compared by us with the parameters of the current obtained under the assumption of linearity of the current. As stated in Chapter 1, the calculations of the intensity, height, and position of the linear current in the auroral zone has been carried out by a number of authors from observations of a pair of stations or of several pairs, yielding different results, depending on the materials used. Accordingly, we repeated our calculations using the same data we used in constructing the system of surface currents.

The distribution of the vectors of the magnetic field of the linear electric current is schematically represented in fig.30. In considering this figure we must

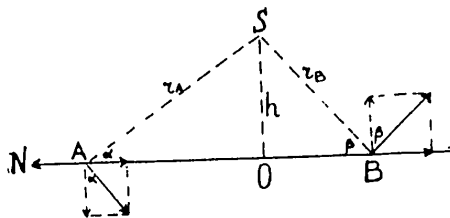


Fig.30 - Pattern of Magnetic Field of  
Horizontal Linear Current

imagine the current to flow perpendicular to the plane of the paper in the direction from the paper toward the observer, and assume  $h$  to be the height of the current above the earth's surface, while  $A$  and  $B$  are two points at which the vectors of the magnetic field are known. On the drawing, they are located at different sides of  $O$ ,

the projection of the current onto the earth's surface. It follows from the drawing in fig.30 that:

$$\left. \begin{aligned} AO &= h \operatorname{ctg} \alpha \\ BO &= h \operatorname{ctg} \beta \end{aligned} \right\} \quad (2)$$

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$$h = \frac{AB}{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta}, \quad (3)$$

$$\operatorname{tg} \alpha = \frac{X_A}{Z_A}, \quad (4)$$

$$\operatorname{tg} \beta = \frac{X_B}{Z_B}, \quad (5)$$

$$\Phi_0 = \Phi_B + h \operatorname{ctg} \beta = \Phi_A - h \operatorname{ctg} \alpha, \quad (6)$$

if  $\Phi_0$  is the latitude of point O (in our case of the auroral zone) and  $\Phi_A$  and  $\Phi_B$  are the latitudes of points A and B.

The vector of the magnetic field created by the infinite linear current  $I$  at the distance  $r$  is equal to

$$H = \frac{2I}{r}.$$

Consequently, if  $H$  is expressed in grams,  $I$  in amperes and  $r$  in kilometers, then

$$I = 5rH = 5 \frac{h}{\sin \alpha} \frac{X_A}{\sin \alpha} = 5 \frac{h}{\sin \beta} \frac{X_B}{\sin \beta}$$

or

$$I = 5hX_A(1 + \operatorname{ctg}^2 \alpha) = 5hX_B(1 + \operatorname{ctg}^2 \beta). \quad (7)$$

For the case where both points A and B are located on the same side of the projection of O, we have

$$h = \frac{AB}{\operatorname{ctg} \beta - \operatorname{ctg} \alpha}, \quad (3')$$

$$\Phi_0 = \Phi_B + h \operatorname{ctg} \beta = \Phi_A + h \operatorname{ctg} \alpha. \quad (6')$$

Equations (3), (3'), (6), (6'), and (7) allow us to calculate all the parameters of the current if we know the distance between two observatories whose observations are available to us. STAT

A consideration of Fig.23 shows that the most convenient pairs located near the

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auroral zone are as follows: I) Petsamo-Bear Islands; II) Fort Rae-Minuk; III) Point Barrow-Uellen; and IV) Tikhaya Bay-Dickson and Malochkin Shar. This selection of pairs was made so that the two stations should be on about the same geomagnetic meridian, i.e., that the direction of the hypothetical linear current should be perpendicular to the line connecting the stations.

Table 12

| Hours | I    |              |                      | II   |              |                      | III  |              |                      | IV   |              |                      |
|-------|------|--------------|----------------------|------|--------------|----------------------|------|--------------|----------------------|------|--------------|----------------------|
|       | h km | $\phi_0$     | $I \times 10^4$<br>A | h km | $\phi_0$     | $I \times 10^4$<br>A | h km | $\phi_0$     | $I \times 10^4$<br>A | h km | $\phi_0$     | $I \times 10^4$<br>A |
| 0     | 314  | $64^\circ.4$ | -17                  | 74   | $62^\circ.3$ | -2                   | 469  | $63^\circ.8$ | -18                  |      |              |                      |
| 2     | 401  | $64.5$       | -37                  | 232  | $63.7$       | -15                  | 254  | $62.9$       | -11                  |      |              |                      |
| 4     | 483  | $65.9$       | -24                  | 364  | $64.6$       | -20                  | 265  | $64.2$       | -8                   | 466  | $61^\circ.7$ | -44                  |
| 6     | 342  | $67.9$       | -12                  | 442  | $65.8$       | -16                  | 348  | $66.9$       | -17                  | 528  | $62.0$       | -13                  |
| 8     | 271  | $69.8$       | 7                    | 1082 | $68.2$       | -9                   | 350  | $67.0$       | -5                   |      |              |                      |
| 10    | 1078 | $66.3$       | 15                   | 421  | $64.6$       | 5                    | 360  | $67.2$       | 4                    |      |              |                      |
| 12    | 1270 | $70.1$       | 32                   | 580  | $67.5$       | 18                   | 645  | $67.6$       | 14                   | 293  | $66.0$       | 8                    |
| 14    | 767  | $69.0$       | 29                   | 593  | $67.0$       | 24                   | 624  | $66.6$       | 19                   | 118  | $64.2$       | 4                    |
| 16    | 561  | $67.0$       | 35                   | 442  | $66.6$       | 21                   | 585  | $64.9$       | 24                   |      |              |                      |
| 18    | 189  | $65.0$       | 13                   | 283  | $65.7$       | 14                   | 320  | $63.7$       | 16                   |      |              |                      |
| 20    |      |              |                      | 165  | $62.6$       | 5                    | 235  | $63.0$       | 10                   |      |              |                      |
| 22    |      |              |                      | 331  | $65.1$       | 3                    | -    | -            | -                    |      |              |                      |

The results of the calculations of  $h$ ,  $\phi_0$ , and  $I$  for corresponding pairs of stations given in Table 12 allow us to draw the following conclusions: The height of the current layer varies within wide limits, from 200-250 km in the night hours to 1000 km and more in the daylight hours. Since all four pairs used agree in STAT indicating an increase in height during the daytime, and since this is confirmed by the statistics of the heights of the  $F_2$  layer in the polar regions, the diurnal march of

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the height of the current may be considered to exist in reality. The most reliable height determinations appear to be those in the night and early morning hours (0-6) and in the afternoon hours (14-20) when the replacement of the surface current by the linear current is most logical. In these hours, all stations give results in agreement with each other (increase from 250 to 500 km in the period 4 to 6<sup>h</sup> and decrease from 600 to 250 km in the period from 16 to 20<sup>h</sup>), which are very close to those of ionospheric measurements. At the end of the day (20-24<sup>h</sup>) and at noontime, the values of the heights are very diverse. The calculated values often appear absurd;  $h > 1000$  km or  $h < 0$  km (cf. omissions in the height column of Table 12). The poor results during this period are entirely understandable if we bear in mind that in these hours there is no crowding of the current lines (cf. Fig. 29) which might be compared to the linear current.

Owing to the relatively great dispersion of the values of  $h$ , no systemic difference in the values calculated for different pairs of stations is found. It can only be noted that the calculation of  $h$  for the pair IV gave the worst results, which most probably can be explained by the fact that the Pikhaya Bay Observatory is located far from the zone of linear current and is in the region of action of the surface current flowing in meridional direction through the polar cap.

Thus the determination of the height of the current in the polar zone by the above-presented formulas, as rough as it may be, still does indicate that in the high latitudes the current system of  $S_D$  can likewise be referred to the level of the  $F_2$  layer, and that we did not commit a great error in adopting the height of the system  $h = 0.05 R$  for the entire earth as an average. In more detailed calculations, which would be outside the scope of the present work, the diurnal fluctuations and the latitudinal variations of the height of the current layer should also be taken into account.

The variations in the geomagnetic latitude of the linear current zone ( $\Phi_0$ ) give a still more regular picture. All stations agree in indicating an increase in

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the northward shift of the linear current during the daytime hours and a southward shift in the night hours, which is full agreement with the position of the zero current line on Fig. 29. The unexpected drop in the value of  $\Phi_0$  at 10<sup>h</sup> and the great scatter at 20-24<sup>h</sup> is explained, as in the case of the calculation of heights, by the absence of crowding of the current lines in these periods of the day. There is a notable systematic difference in the values  $\Phi_0$  I,  $\Phi_0$  II,  $\Phi_0$  III, and  $\Phi_0$  IV. Two pairs of stations, Minuk-Fort Mac and Point Barrow-Ullen give about the same values, fluctuating in the morning and evening hours about  $\Phi_0 = 66^\circ$ , which is in complete agreement with the position of the zone on Fig. 23, which we have drawn along the isoamplitudes of  $HS_D$ . As should have been expected on the basis of Fig. 23, the pair Pikhaya Bay-Katochkin Shar and Dickson indicate the southernmost position of the zone ( $\Phi_0 = 63 - 64^\circ$ ). There is a certain lack of correspondence between Fig. 23 and the values of Table 12 only for the pair Petsamo-Seer Islands ( $\Phi_0 = 65^\circ$ ) along the isoamplitudes and  $\Phi_0 = 67^\circ$  for the morning and evening hours of Table 12. Thus the calculation of the latitude of the zone of linear current on the average is in very good agreement with the position of the zero current line in the system of surface currents and allows the position of the zone to be made more precise at various longitudes.

A comparison of the intensity of the linear current with that of the surface current flowing in the belt 60-70°, indicates good agreement, both in order of magnitude and in diurnal distribution. The systematic difference in  $I_I \dots I_{III}$  again indicates the existence of the longitudinal asymmetry in the distribution of  $S_D$ , repeatedly noted by us. For obvious reasons, there is no special point in attaching any particular significance to the scattered values of  $I_{IV}$ .

The above-described parameters of the linear current, calculated by us from the  $S_D$ -variations for the Second International Polar Year, agree in part with the parameters from the calculations of Sucksdorff and (Bibl. 55) and Harang (Bibl. 44). In particular, the diurnal variations of altitude and density are about the same for

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all three studies. We did not, however, discover the existence of two branches of the current on one and the same meridian, which would follow from Harang's work. We likewise fail to find even indications of the existence of the "almost vertical" linear current calculated by Sucksdorff. On the contrary, the idea obtained by us as to the parameters of the linear current is in full agreement with the system of surface currents, which is more objective, and has been calculated without a priori assumptions as to the configuration of the current.

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## CHAPTER VI

## POLAR STORMS

Section 1. Expansion of the Field Potential and Electric Currents into Series of Cylindrical Functions

As has been stated above, during the time of a polar storm (P), the fluctuations in the magnetic elements in the high latitudes reach great amplitudes, often exceeding 1000  $\gamma$ , while in the low latitudes a polar storm manifests itself in the form of small bay-shaped disturbances. It follows from Figs. 7a and 6b that the field of a P storm for  $\phi < 55^\circ$  is so small by comparison with the field in the polar cap that, without great error, a field that vanishes at  $\theta = 50^\circ$  may be adopted, and the distribution of vectors considered only on the spherical segment  $\phi \leq 40^\circ$ . In this case, taking the spherical segment as a portion of a plane, the potential of a P storm may be represented by a series of Bessel functions. The approximation will of course be very rough and will give particularly great distortions along the edges of the regions considered, but it will still enable us to separate from the field observed on the earth's surface that part due to ionospheric sources, and to form an idea on the configuration and intensity of the currents flowing in the ionosphere. In the central part of the polar segment, we have  $\theta \leq 45^\circ$ , and it is here that the most intense fluctuations of the magnetic field are concentrated, while the distortions introduced by the replacement of the spherical surface by a plane surface are relatively small.

In view of the fact that the expansion of Bessel functions is here used for the

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first time in investigations of the variable magnetic field of the earth, we will in this section derive the necessary formulas and will devote the next section to a description of the current system obtained for the P storms.

After selecting a system of cylindrical coordinates  $r, \varphi, z$  such that its origin coincides with the magnetic axis pole, that the plane  $z = 0$  corresponds to the earth's surface, and that the positive axis  $z$  is directed toward the pole, let us represent the potential of the storm at some fixed instant of time  $T$  by the Fourier-Bessel series:

$$V = \sum_n \sum_m (\alpha_{nm}^e \cos n\varphi + \beta_{nm}^e \sin n\varphi) e^{-\frac{\lambda_n^m z}{\rho}} J_n(\lambda_n^m \frac{r}{\rho}) + \sum_n \sum_m (\alpha_{nm}^i \cos n\varphi + \beta_{nm}^i \sin n\varphi) e^{\frac{\lambda_n^m z}{\rho}} J_n(\lambda_n^m \frac{r}{\rho}). \quad (1)$$

The first half of the series converges in the half-space  $z > 0$ , below the surface of the earth, and represents the potential due to external sources ( $V_e$ ). The second half of the series converges for  $z < 0$  and represents the potential of the field due to external sources ( $V_i$ ). Here  $\lambda_n^m$  denotes the  $m^{\text{th}}$  root of the Bessel function of the  $n^{\text{th}}$  order, and eq.(1) vanishes on the surface of the cylinder of radius  $r = \rho$ .

Since we have assumed that  $V = 0$  for  $\theta \geq 40^\circ$ , the numerical value of  $\rho$  in our analysis equals the length of the segment of the meridian included between  $\theta = 0^\circ$  and  $\theta = 40^\circ$ , i.e.,  $111 \times 40$  km, or  $4.5 \times 10^3$  cm. The field intensity is

$$F = -\text{grad } V \text{ и } R = -\frac{\partial V}{\partial r}, \quad Z = -\frac{\partial V}{\partial z}, \quad (2)$$

where  $X$  denotes the component of the horizontal vector directed along the geomagnetic meridian. Since the direction toward the pole is usually considered positive in geomagnetic measurements, while  $r$  increases with increasing distance from the pole, we have, from eq.(1),

$$X = -R \frac{\partial V}{\partial r}. \quad (2')$$



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From eq.(1) we also have

$$Z = \sum_n \sum_m \frac{\lambda_n^m}{\rho} (\alpha_{nm}^e \cos n\varphi + \beta_{nm}^e \sin n\varphi) e^{z \frac{\lambda_n^m}{\rho}} J_n \left( \lambda_n^m \frac{r}{\rho} \right) - \sum_n \sum_m \frac{\lambda_n^m}{\rho} (\alpha_{nm}^i \cos n\varphi + \beta_{nm}^i \sin n\varphi) e^{-z \frac{\lambda_n^m}{\rho}} J_n \left( \lambda_n^m \frac{r}{\rho} \right). \quad (3)$$

on the earth's surface,  $z = 0$  and

$$V = \sum_n \sum_m (a_{nm} \cos n\varphi + b_{nm} \sin n\varphi) J_n \left( \lambda_n^m \frac{r}{\rho} \right), \quad (4)$$

$$Z = \sum_n \sum_m (c_{nm} \cos n\varphi + d_{nm} \sin n\varphi) J_n \left( \lambda_n^m \frac{r}{\rho} \right), \quad (5)$$

where

$$a_{nm} = \alpha_{nm}^e + \alpha_{nm}^i, \quad c_{nm} = \frac{\lambda_n^m}{\rho} (\alpha_{nm}^e - \alpha_{nm}^i),$$

$$b_{nm} = \beta_{nm}^e + \beta_{nm}^i, \quad d_{nm} = \frac{\lambda_n^m}{\rho} (\beta_{nm}^e - \beta_{nm}^i).$$

If  $\alpha_{nm}^e \dots \beta_{nm}^e$  are known, then  $\alpha_{nm}^i \dots \beta_{nm}^i$  may be calculated by the formulas

$$\alpha_{nm}^e = \frac{1}{2} \left( a_{nm} + \frac{\rho}{\lambda_n^m} c_{nm} \right); \quad \alpha_{nm}^i = \frac{1}{2} \left( a_{nm} - \frac{\rho}{\lambda_n^m} c_{nm} \right), \quad (6)$$

$$\beta_{nm}^e = \frac{1}{2} \left( b_{nm} + \frac{\rho}{\lambda_n^m} d_{nm} \right); \quad \beta_{nm}^i = \frac{1}{2} \left( b_{nm} - \frac{\rho}{\lambda_n^m} d_{nm} \right). \quad (7)$$

The values of  $c_{nm}$  and  $d_{nm}$  are easily obtained by expanding into series  $J_n$   $\left( \lambda_n^m \frac{r}{\rho} \right)$  the properly work-up data of the variations of the  $Z$  component of the geomagnetic field. For the calculation of  $a_{nm}$  and  $b_{nm}$  it is more convenient to make use of the data of the variations of the  $V$  component. From eq.(2') it follows, for  $r_0 > r$ , that

$$V_{r_0} - V_r = \int_r^{r_0} X dr = V - \int_r^{r_0} X dr + V_{r_0}.$$

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Taking  $r_0 = \rho$ , we have

$$V = - \int_r^{\rho} X dr, \quad (8)$$

since, by hypothesis,  $V_{\rho} = 0$ .

On finding, by numerical integration of the  $X'$  component, the value of  $V$  for all the region  $r < \rho$ , we may find the coefficients  $a_{nm}$  and  $b_{nm}$  of the expansion of eq. (4).

From the distribution of the potential  $V_0$  found on the surface  $z = 0$  it is not difficult to pass to the distribution of the currents responsible for it. Let the potential  $V_0$  known on  $z = 0$  be due to a plane layer of currents lying at the level  $z = -z_0$ . Denoting the value of the potential for the lower surface of the layer by  $V_-$ , and that on the upper layer by  $V_+$ , we have, under the condition that the normal derivative is continuous,

$$\frac{\partial V_+}{\partial z} = \frac{\partial V_-}{\partial z}. \quad (9)$$

Since the current layer is equivalent to a double magnetic sheet, the second equation

$$V_+ - V_- = 4\pi I, \quad (10)$$

where  $I(r, \varphi)$  is the density of the current layer, will also hold for the level  $z = -z_0$ . On expanding  $V_+$ ,  $V_-$  and  $I$  into a series of Bessel functions, we have

$$V_- = \sum_n \sum_m (\alpha_{nm}^- \cos n\varphi + \beta_{nm}^- \sin n\varphi) e^{\frac{\lambda_n^m}{\rho}(z+z_0)} J_n\left(\lambda_n^m \frac{r}{\rho}\right) \text{ as } |z| < z_0, \quad (11)$$

$$V_+ = \sum_n \sum_m (\alpha_{nm}^+ \cos n\varphi + \beta_{nm}^+ \sin n\varphi) e^{\frac{\lambda_n^m}{\rho}(z-z_0)} J_n\left(\lambda_n^m \frac{r}{\rho}\right) \text{ as } |z| > z_0, \quad (12)$$

$$I = \sum_n \sum_m (s_{nm} \cos n\varphi + \tau_{nm} \sin n\varphi) J_n\left(\lambda_n^m \frac{r}{\rho}\right) \text{ as } |z| = z_0. \quad (13)$$

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Substituting eqs.(11) - (13) in eqs.(9) and (10), and equating the coefficients of the same  $\overline{\cos n\varphi} J_n$ , we have

$$\alpha_{nm}^+ = -\alpha_{nm}^-; \quad \alpha_{nm}^+ - \alpha_{nm}^- = 4\pi s_{nm}, \quad (14)$$

$$\beta_{nm}^+ = -\beta_{nm}^-; \quad \beta_{nm}^+ - \beta_{nm}^- = 4\pi \tau_{nm}, \quad (15)$$

whence

$$s_{nm} = -\frac{1}{2\pi} \alpha_{nm}^-; \quad \tau_{nm} = -\frac{1}{2\pi} \beta_{nm}^-. \quad (16)$$

For  $z = 0$ , we have the two identical expressions:

$$V_- = \sum_n \sum_m (\alpha_{nm}^- \cos n\varphi + \beta_{nm}^- \sin n\varphi) e^{-\lambda_n^m \frac{z_0}{\rho}} J_n \left( \lambda_n^m \frac{r}{\rho} \right), \quad (17)$$

$$V_+ = \sum_n \sum_m (\alpha_{nm}^+ \cos n\varphi + \beta_{nm}^+ \sin n\varphi) J_n \left( \lambda_n^m \frac{r}{\rho} \right). \quad (18)$$

On equating them, we have

$$\left. \begin{aligned} \alpha_{nm}^- &= \alpha_{nm}^+ e^{\lambda_n^m \frac{z_0}{\rho}}; & \beta_{nm}^- &= \beta_{nm}^+ e^{\lambda_n^m \frac{z_0}{\rho}} \\ s_{nm} &= -\frac{\alpha_{nm}^+}{2\pi} e^{\lambda_n^m \frac{z_0}{\rho}}; & \tau_{nm} &= -\frac{\beta_{nm}^+}{2\pi} e^{\lambda_n^m \frac{z_0}{\rho}} \end{aligned} \right\} \quad (19)$$

where  $z_0$  is the modulus of the height of the currents postulated by us.

## Section 2. Starting Material. Results of the Analysis

As our starting material for calculating the currents responsible for polar storms, we used the data collected and worked up by Silsbee and Vestine (Bibl.54). Polar storms are so diverse in form and in intensity that the formal averaging of series of observations cannot give such good results as it gives with the  $S_q$  or  $S_D$ -variations. Nevertheless, the statistics of a large number of storms for certain

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observatories given by these authors do show that there is a definite regularity in the distribution of storms by hours of the day. The relation between the number of storms and the time of day given in Table 13 shows that the positive storms (i.e., deviation in  $H$ ,  $\Delta H > 0$ ) and the negative storms ( $\Delta H < 0$ ) are usually encountered at different times of the day, the diurnal march of the bays depending on the latitude of the point of observation. The list of observatories used in the work of Silsbee and Vestine, and the number of bays registered, are given in Table 14.

Table 13  
Diurnal March of Frequency of Bay-Shaped Disturbances  
(Number of bays in %)

| $\Phi^\circ$<br>from - to | $\Delta H$ | Hours |    |    |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---------------------------|------------|-------|----|----|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|                           |            | 1     | 2  | 3  | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 38                        | +          | 4     | 4  | 3  | 5 | 6 | 5 | 3 | 3 | 7 | 8  | 8  | 5  | 3  | 2  | 4  | 3  | 4  | 7  | 5  | 3  | 5  | 5  | 4  | 4  |
| 70 - 63                   | +          | 0     | 0  | 0  | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 2  | 3  | 2  | 1  | 1  | 1  | 0  | 0  |
|                           | -          | 14    | 11 | 10 | 9 | 8 | 6 | 4 | 3 | 2 | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 3  | 4  | 6  | 8  | 10 |
| 43 - 40                   | +          | 8     | 6  | 4  | 3 | 2 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 3  | 6  | 6  | 9  | 10 |
|                           | -          | 0     | 0  | 0  | 0 | 0 | 0 | 1 | 1 | 2 | 3  | 3  | 3  | 3  | 4  | 5  | 5  | 4  | 3  | 2  | 2  | 2  | 1  | 0  | 0  |
| 3 - 16                    | +          | 9     | 6  | 5  | 3 | 2 | 2 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 3  | 6  | 8  | 8  | 9  |
|                           | -          | 0     | 0  | 0  | 0 | 0 | 0 | 1 | 1 | 1 | 2  | 3  | 2  | 2  | 2  | 2  | 3  | 4  | 4  | 3  | 3  | 2  | 2  | 1  | 0  |

A consideration of the form and intensity of the bay-shaped fluctuations for all three elements enabled me to construct for each observatory a picture of the mean (or more exactly, idealized) bays by averaging the disturbances encountered at one and the same hours of local time. These mean bays were different for different hours of the day of the local day, but resembled each other for observatories located at the same latitude. In other words, I found that the storm field depends on the local time and the latitude. As in Chapter V, allowance was made, in averaging, for the local geomagnetic time and the geomagnetic latitude. Figure 31 gives the distribution of the field of an idealized bay for 0h Universal Time. The coordinate net on

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Table 14

| Group | Observatory     | $\phi$         | $\Lambda$     | Number of Days |
|-------|-----------------|----------------|---------------|----------------|
| 1     | Thule           | $83^{\circ}.0$ | $0^{\circ}.0$ | 204            |
| 2     | Julianahobab    | 70.8           | 35.6          | 227            |
|       | Fort Rae        | 69.0           | 290.9         | 243            |
|       | Promso          | 67.1           | 116.7         | 99             |
|       | College, Alaska | 64.5           | 255.4         | 146            |
|       | Dickson         | 63.0           | 161.5         | 103            |
| 3     | Tucson          | 40.4           | 312.2         | 191            |
|       | Ebro            | 43.9           | 79.3          | 147            |
|       | Watheroo        | -41.3          | 135.6         | 173            |
| 4     | Antipolo        | 3.3            | 139.8         | 117            |
|       | Huancayo        | -0.6           | 353.8         | 98             |
|       | Mogadiscio      | -2.7           | 114.3         | 124            |
|       | Apia            | -16.0          | 260.0         | 72             |

the map is formed by the geomagnetic parallels and meridians. On the edge of the diagram, the local geomagnetic time of the meridians corresponding to  $0^h$  Universal Time is shown. In preparing the diagram, I used not only the values of the vectors for the 13 enumerated observatories for  $0^h$ , but also for  $21^h$  and  $3^h$ , the latter values being placed on the meridians corresponding to  $21^h + \Lambda - 69^{\circ}$  and  $3^h + \Lambda - 69^{\circ}$  geomagnetic time. The horizontal component of the storm field is represented by the vector, the vertical component by the digit at the origin of the vector. It will be clear from the figure that the vectors of  $H$  are directed primarily along the geomagnetic meridians, that the vectors reach their maximum values in the zone  $\phi = 60 - 65^{\circ}$ , and that, at latitudes lower than  $50^{\circ}$  they are negligibly small. The representation of the field of a polar storm shown in Fig.31 by a series of Bessel functions was accomplished in the following manner: The data of the Z components were interpolated for

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various latitudes ( $\phi$  equal to 90, 70, 67, 64 and 50°). For each latitude I calculated the coefficients of the expansion of  $Z$  into the Fourier series:

$$Z = \sum (p_n \cos n\varphi + q_n \sin n\varphi),$$

where the argument  $\varphi$  corresponds to the geomagnetic longitude  $\Lambda$ . For a satisfactory representation of  $Z$  it proved to be sufficient to confine myself to  $n = 0.1$  and 2.

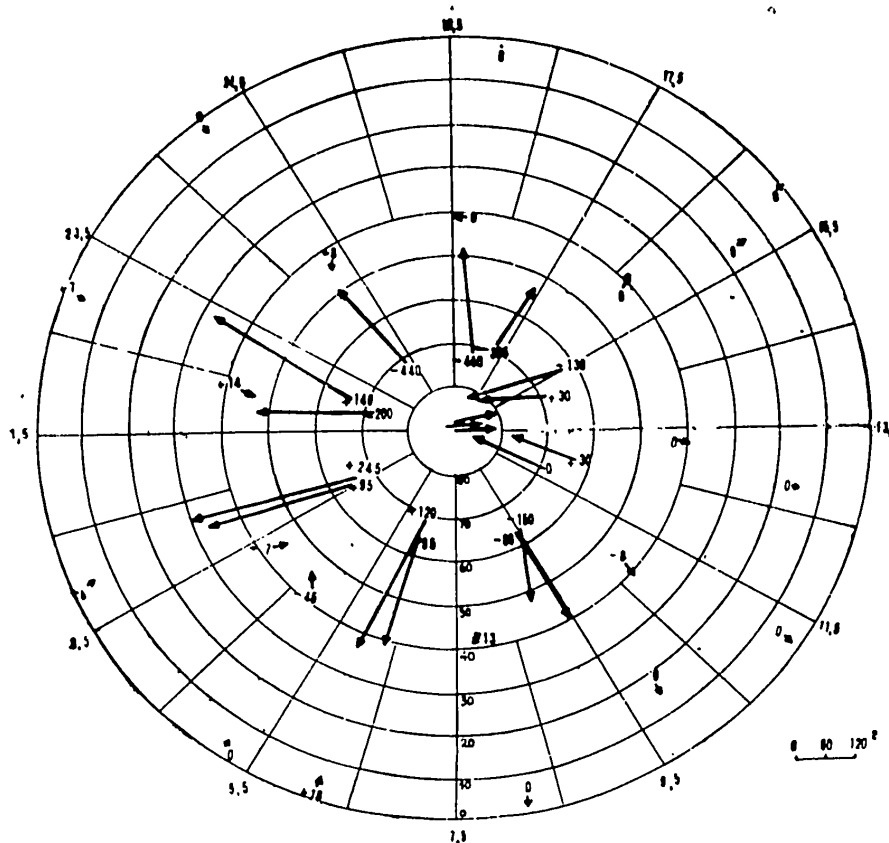


Fig.31 - Field of Polar Storm (According to Vestine). The horizontal component of the field is shown by an arrow, the vertical by numerals (in gammas). The coordinate net is the geomagnetic latitude and the geomagnetic time

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Then  $p_n$  and  $q_n$  were represented by a series of Bessel functions of the  $n^{\text{th}}$

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order

$$p_n = \sum_m c_{nm} J_n \left( \lambda_n^m \frac{r}{\rho} \right),$$

$$q_n = \sum_m d_{nm} J_n \left( \lambda_n^m \frac{r}{\rho} \right).$$

while  $c_{nm}$  and  $d_{nm}$  were found by the well-known formulas

$$c_{nm} = \frac{2 \int_0^2 r \rho_n(r) J_n \left( \lambda_n^m \frac{r}{\rho} \right)}{[J_{n-1}(\lambda_n^m)]^2},$$

$$d_{nm} = \frac{2 \int_0^2 r q_n(r) J_n \left( \lambda_n^m \frac{r}{\rho} \right)}{[J_{n-1}(\lambda_n^m)]^2}.$$

The values of the H component, as indicated above, were first integrated to obtain the value of the potential V, and then  $a_{nm}$  and  $b_{nm}$  were calculated by means of eq.(4).

Table 15 gives a summary of the coefficients  $a_{nm} \dots d_{nm}$  so obtained.

Table 15 \*

| m . . . . .     | 1    | 2    | 3     | 4     | 5     |
|-----------------|------|------|-------|-------|-------|
| $c_0$ . . . . . | -32  | -39  | 53    | 46    | -50   |
| $c_1$ . . . . . | -51  | -77  | -6    | 60    | -30   |
| $d_1$ . . . . . | 27   | -81  | -84   | 73    | -34   |
| $c_2$ . . . . . | -26  | -64  | -18   | 25    | 1     |
| $d_2$ . . . . . | -31  | -7   | 8     | -2    | -12   |
| $a_0$ . . . . . | 3.47 | 1.66 | -0.60 | 0.02  | -0.09 |
| $a_1$ . . . . . | —    | —    | —     | —     | —     |
| $b_1$ . . . . . | 3.28 | 3.66 | -1.48 | -0.25 | 1.04  |
| $a_2$ . . . . . | 1.03 | 1.36 | 0.18  | 0.09  | 0.11  |
| $b_2$ . . . . . | —    | —    | —     | —     | —     |

Equations (4) and (5) satisfactorily represent the initial observed data, as indicated in Table 16 which gives the calculated and observed values of V (in CGSM) and STAT

\* In Table 15 the values of the coefficients are given in units of  $10^{-5}$  CGS. The values of  $a_1$  and  $b_2$  do not exceed a few units of the fifth decimal place.

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of  $Z$  (in  $\gamma$ ) for four points.

The separation of the potential into portions of external and internal origin by means of eqs.(6) and (7) showed (Table 17) that the external potential  $V_e$  consid-

Table 16

| $\Phi$       | $\Lambda$   | $Z_{\text{obs}}$ | $Z_{\text{calc}}$ | $V_{\text{obs}}$   | $V_{\text{calc}}$  |
|--------------|-------------|------------------|-------------------|--------------------|--------------------|
| $30^{\circ}$ | $0^{\circ}$ | -170             | -140              | $5 \times 10^{-5}$ | $5 \times 10^{-5}$ |
| 80           | 90          | -60              | -70               | 5                  | 6                  |
| 66           | 0           | 120              | 100               | 2                  | 2                  |
| 66           | 90          | 180              | 200               | 2.5                | 2.5                |

erably exceeds the internal potential  $V_i$ . For a quantitative estimate of the ratio of the external to the internal fields it is more convenient to represent  $V$  in the form

$$V = \sum_n \sum_m \left( E_n^m e^{-\lambda_n - i(n\varphi + \gamma_n^m)} + e^{\lambda_n - i(n\varphi + \gamma_n^m)} \right) I_n^m(\lambda r), \quad (20)$$

where

$$\begin{aligned} E_n^m &= \sqrt{(\alpha_{nm}^e)^2 + (\beta_{nm}^e)^2}; \quad \text{tg } \gamma_n^m = -\frac{\beta_{nm}^e}{\alpha_{nm}^e}, \\ I_n^m &= \sqrt{(\alpha_{nm}^i)^2 + (\beta_{nm}^i)^2}; \quad \text{tg } \zeta_n^m = -\frac{\beta_{nm}^i}{\alpha_{nm}^i}. \end{aligned} \quad (21)$$

Table 17 gives the value of  $E_n^m \dots \gamma_n^m$  and also of  $f = I/E$  and  $\delta = \gamma - \zeta$ .

This table shows that in two cases  $f > 1$  and in one case  $f$  could not be calculated, because of the smallness of the initial coefficients. In Chapter X, we will show that the values obtained for  $f$  and  $\delta$  are in agreement with the hypothesis that the internal part of the field is of inductive origin. The mean value  $f = 0.36$  (STAT the same as that obtained for the polar cap from the data of the  $S_p$ -variations ( $V_i/V_e = 0.89$ ), which indicates the plausibility of these values. The value  $f = 0.86 - 0.89$  considerably exceeds the corresponding values for the  $S_q$ - and  $D_{st}$ -variations



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## CHAPTER VII

SEASONAL AND 11-YEAR VARIATIONS OF THE  $D_{st}$  AND  $S_D$  CURRENTSSection 1. The 11-Year and Seasonal Variations of the  $D_{st}$  Currents

The present Chapter is devoted to a discussion of the variations that the mean pictures of the electric currents described by us undergo with the seasons of the year, and with the 11-year cycle. It is not possible to collect the observational data for a series of years from the wide net of observatories that is necessary for the mathematical calculation of the currents. I therefore confined myself to the study of the 11-year and annual variations of the  $D_{st}$  and  $S_D$  variations from individual base stations, on the basis of which I then drew my conclusions as to the variations of the current system as a whole. As my basis I selected observatories with long series of observations whose variations are characteristic for the corresponding regions.

Let us first turn to the 11-year fluctuations of the  $D_{st}$  currents. The dependence of the degree of magnetic disturbance on the level of solar activity is widely known: the coefficient of correlation between the annual numbers of the u-measure of magnetic activity and the relative sunspot number may go as high as 0.9. Since with increasing solar activity, the number and mean intensity of the disturbances also increases, it may be expected that in the 11-year cycle the mean characteristic of the  $D_{st}$  variations will vary and, consequently, the intensity of the system of currents equivalent to it will also vary. Instead of the very laborious calculation

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in whose analysis the data from the entire earth were used. The possibility is not excluded that this discrepancy is not fortuitous, and that it indicates the anisotropy of the deep parts of the earth (for more details, see Chapter X).

Table 17 \*

| External Field |      |      |       |       |      |
|----------------|------|------|-------|-------|------|
| $m$            | 1    | 2    | 3     | 4     | 5    |
| $\alpha_0$     | 2,33 | 0,99 | -0,44 | -0,10 | 0,03 |
| $\alpha_1$     | 0,30 | 0,25 | 0,01  | -0,10 | 0,04 |
| $\beta_1$      | 1,48 | 2,09 | -0,55 | -0,25 | 0,56 |
| $\alpha_2$     | 0,63 | 0,85 | 0,12  | 0,00  | 0,06 |
| $\beta_2$      | 0,14 | 0,02 | 0,00  | 0,00  | 0,02 |
| $E_1$          | 1,51 | 1,96 | 0,55  | 0,32  | 0,57 |
| $\gamma_1$     | 284  | 277  | 89    | 92    | 274  |
| $E_2$          | 0,65 | 0,85 | 0,10  | 0,00  | 0,06 |
| $\gamma_2$     | 348  | 359  | 0     | 90    | 342  |

| Internal Field |       |       |       |       |       |
|----------------|-------|-------|-------|-------|-------|
| $m$            | 1     | 2     | 3     | 4     | 5     |
| $\alpha_0$     | 1,02  | 0,72  | -0,16 | -0,07 | -0,12 |
| $\alpha_1$     | -0,30 | -0,25 | -0,01 | 0,10  | -0,04 |
| $\beta_1$      | 1,80  | 1,57  | -0,92 | 0,00  | 0,48  |
| $\alpha_2$     | 0,40  | 0,51  | 0,06  | 0,08  | 0,06  |
| $\beta_2$      | -0,14 | -0,02 | 0,00  | 0,00  | -0,02 |
| $I_1$          | 1,82  | 1,59  | 0,92  | 0,10  | 0,50  |
| $\zeta_1$      | 260   | 261   | 91    | 0     | 265   |
| $I_2$          | 0,42  | 0,51  | 0,06  | 0,08  | 0,06  |
| $\zeta_2$      | 20    | 3     | 0     | 0     | 18    |

| Ratio of Internal and External Fields |      |      |      |      |      |
|---------------------------------------|------|------|------|------|------|
| $m$                                   | 1    | 2    | 3    | 4    | 5    |
| $f_1$                                 | 1,21 | 0,81 | 1,67 | 0,31 | 0,88 |
| $f_2$                                 | 0,65 | 0,60 | 0,60 | -    | 1,00 |
| $\delta_1$                            | 24   | 16   | -2   | 92   | 9    |
| $\delta_2$                            | -32  | -4   | 0    | 90   | -36  |

The ionospheric currents whose field is identical with the field of the idealized polar storm were calculated by eqs. (13) and (19). The disturbances of the polar ionosphere, as a rule, extend to heights of 100-30 km. Down to these same heights the lower boundaries of the aurora usually descend. This forces us to consider that the most probable height of the currents of polar storms is the region of the E layer of the ionosphere, i.e., 90-120 km. The highly local character of the course of these storms, when the form and intensity of the disturbance varies considerably over a distance of a few hundred kilometers, likewise prevents us from referring these currents to great heights. These assumptions forced us to use  $z_0 = 100$  km in eq. (19). The

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\* In table 17, the values of  $\alpha$ ,  $\beta$ ,  $E$ , and  $I$  are given in gammas and those of  $\zeta$ ,  $\delta$ , and  $\gamma$  in degrees.

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of the  $D_{st}$  variations for different years, I limited myself to the consideration of the quantity  $D_m = H_d - H_q$ . A consideration of  $D_m$  is entirely adequate for judging the geographic distribution or time fluctuations of the field of  $D_{st}$ , since, as noted above,  $D_m$  is approximately equal to the mean value of  $D_{st}H$  on the two first days of a storm.

From Fig.33, showing the relation of  $D_m$  and  $\phi$ , it will be seen that the geographic distribution of  $D_m$  in the middle latitudes ( $\phi \leq 50^\circ$ ) varies little during the course of the 11-year cycle: the curves of  $D_m(\phi)$  in the years of high activity (1938) and low activity (1933) almost parallel each other. In the high latitudes ( $\phi > 50^\circ$ ) there is a considerable change in the form of the curve of  $D_m(\phi)$ . However, as repeatedly pointed out, the  $\tilde{D}_m$  of high latitudes are due mainly to polar storms and do not characterize the  $D_{st}$  field. In view of this it can be considered that, from year to year, there is little change in the configuration of the  $D_{st}$  current system, but that there is considerable change only in its intensity.

In the preceding Chapters we have pointed out that the approximate method of evaluating the intensity of currents gives good results, close to those of the exact mathematical methods. Thus, for example, the intensity of the  $D_{st}$  current flowing west along the parallels of latitude in each hemisphere, according to the data of spherical analysis, is  $I = 180,000$  amp, while the approximate estimate, based on the value of  $D_m$  at Huancayo, gives  $I = 176,000$  amp. Starting out from this good correspondence, the current strength of  $D_{st}$  was calculated from the Huancayo observations for 1922 - 1944. As will be seen from Fig.34\* the current strength undergoes great fluctuations, from  $12 \times 10^4$  amp in years of low solar activity to  $40 \times 10^4$  and  $50 \times 10^4$  amp in years of high activity. Corresponding to this, the mean current density varied from  $1.2 \times 10^5$  to  $5.0 \times 10^5$  CGSM. Just as in the consideration of the

\* The values of  $I$  for 1919 - 1922 are calculated from the values of  $D_m$  at Watheroo, and for 1945 to 1950 from  $D_m$  at Zuy. The values of  $D_m$  at Watheroo and Zuy were multiplied by the factor 1.2 to reduce them to the values at Huancayo.

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coefficients  $s$  and  $\tau$  (in amperes) calculated under this hypothesis are given in Table 18, and the resultant system of currents is given in Fig.32. A comparison of

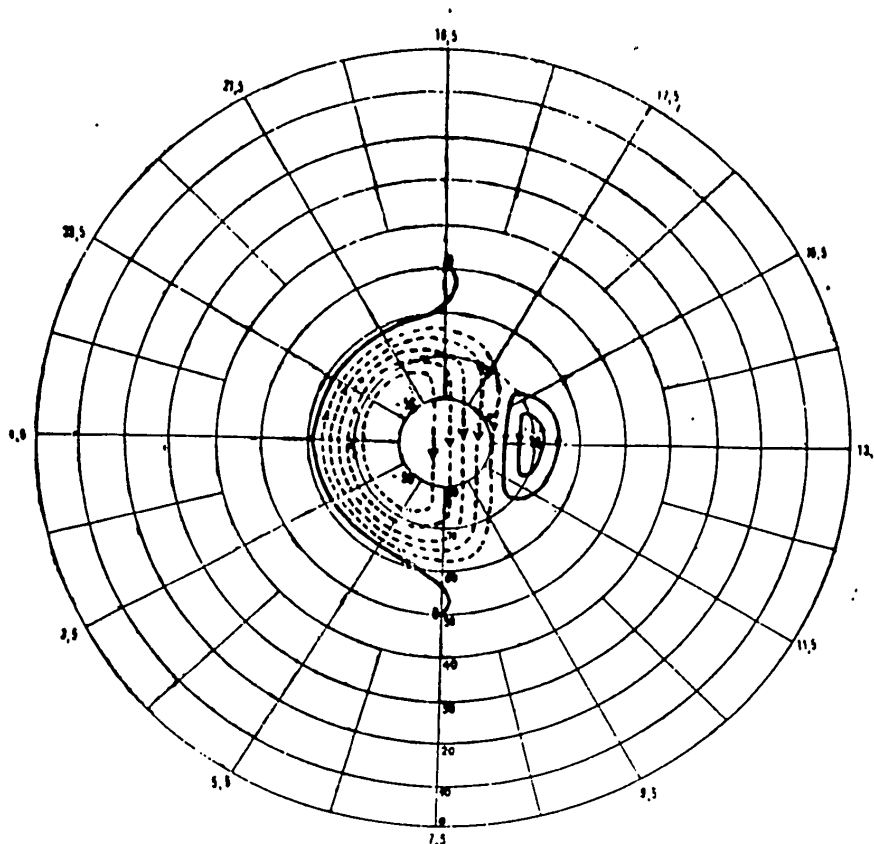


Fig.32 - Current System of Polar Storms; Intensity of Current in 10,000 Amperes. The current flowing between two adjacent lines of current is 10,000 amp. The coordinate net is the geomagnetic latitude and geomagnetic time. ( — positive values of current function; - - -negative values)

the current system of Fig.32 with the Silsbee - Vestine system, constructed on the basis of these same data, but by an approximate method, indicates their great resemblance. This is still another confirmation of the conclusion drawn by us in Chapter V to the effect that the approximate method gives a good idea of the configuration and intensity of the current lines, and can be successfully used in cases where a qualitative idea of the current system must be obtained without great expenditure of

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11-year cycle of other magnetic characteristics, a lag of the magnetic maxima behind the solar maxima is noted in good agreement with the corpuscular nature of magnetic

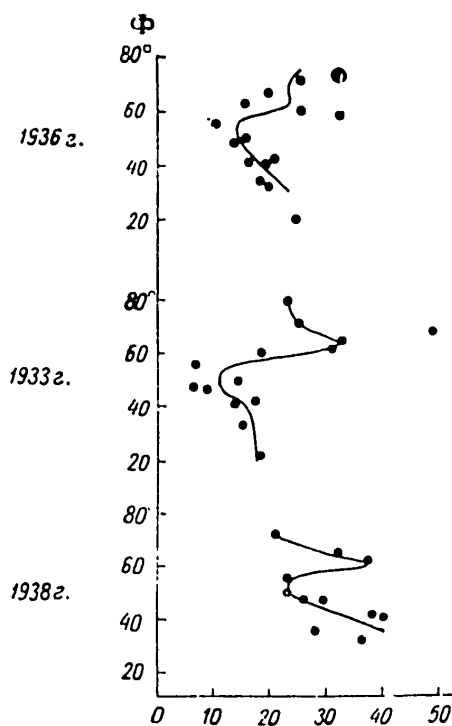


Fig.33 - Dependence of  $D_m = H_q - H_d$  (in  $\gamma$ ) on the Geomagnetic Latitude

disturbances.

In the work of Chynk (Bibl.41), cited in Chapter I, it has been established that  $D_m$  has systematic seasonal fluctuations. Besides the double wave with maxima at the epoch of the equinoxes and minima at the epoch of the solstices, which is inherent in all measures of magnetic activity, a simple sinusoidal wave with a maximum in the winter for each hemisphere and a minimum in the summer may also be separated from the annual march of  $D_m$ . At Huancayo, located close to the geomagnetic equator ( $\phi = -0.6^\circ$ ;  $\varphi = 12^\circ S$ ), the value of  $D_m$  is about equal at the December and June solstices (cf. Fig.35, which gives the values of  $D_m$  for the years 1922 - 1944). This compels the assumption that in the epoch of the solstices the

lines of zero value of the current function are not deflected far from the geomagnetic equator, in contrast to what happens in the case of the  $S_q$  variations. The intensity of the current (in  $10^4$  amperes) in the northern hemisphere, (calculated from the  $D_m H$  of Zuy Observatory) and the southern hemisphere (calculated from the  $D_m H$  of Watheroo Observatory) is shown in Table 19. The mean values for 1938 - 1944 of the intensity of the  $D_{st}$  current are given separately in Table 20 for the northern and southern hemispheres.

Thus the seasonal fluctuations of  $D_{st}$  actually do have a maximum at the epoch of the equinox and a minimum at the epoch of the solstice. The summer minimum is

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labor.

The system of currents represented in Fig.32 consists of two eddies in which the current flows in opposite directions. This explains the fact, illustrated by Figs.9 and 31, that at polar stations located in different hemispheres the storms are usually observed simultaneously but have different signs for the  $H$  component. The current eddy located on the morning side of the polar cap is considerably weaker than the evening eddy: The total current in it reaches 16,000 amp, while on the evening side it is 55,000 amp.

The system so described is completely different from the currents postulated to explain the polar storm by Birkeland, but, conversely, it does resemble the polar part of the currents of the  $S_D$ -variations. This resemblance is entirely understandable if we bear in mind the fact that the polar disturbances, which everywhere accompany worldwide storms, make the greatest contributions to the  $S_D$ -variations.

Table 18

| $m$ . . . . .      | 1                   | 2                   | 3                 | 4                  | 5                 |
|--------------------|---------------------|---------------------|-------------------|--------------------|-------------------|
| $s_0$ . . . . .    | $-32 \times 10^3 A$ | $-17 \times 10^3 A$ | $8 \times 10^3 A$ | $23 \times 10^3 A$ | $1 \times 10^3 A$ |
| $s_1$ . . . . .    | —                   | —                   | —                 | —                  | —                 |
| $\tau_1$ . . . . . | -26                 | -40                 | 11                | 5                  | 13                |
| $s_2$ . . . . .    | -11                 | -16                 | -2                | 0                  | -1                |
| $\tau_2$ . . . . . | —                   | —                   | —                 | —                  | —                 |

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deeper in both hemispheres than the winter minimum.

Table 19

| Year | a) |    |    | Year | a) |    |    | b) |    |    |
|------|----|----|----|------|----|----|----|----|----|----|
|      | c) | d) | e) |      | c) | d) | e) | d) | c) | e) |
| 1919 | 12 | 18 | 26 | 1935 | 12 | 16 | 22 |    |    |    |
| 1920 | 10 | 15 | 31 | 1936 | 17 | 19 | 28 |    |    |    |
| 1921 | 10 | 39 | 16 | 1937 | 15 | 25 | 38 |    |    |    |
| 1922 | 9  | 13 | 21 | 1938 | 35 | 34 | 49 | 45 | 26 | 51 |
| 1923 | 5  | 8  | 21 | 1939 | 26 | 45 | 48 | 29 | 39 | 44 |
| 1924 | 5  | 19 | 11 | 1940 | 18 | 25 | 64 |    |    |    |
| 1925 | 15 | 17 | 16 | 1941 | 25 | 40 | 42 | 27 | 31 | 43 |
| 1926 | 18 | 23 | 41 | 1942 | 21 | 15 | 28 | 23 | 10 | 29 |
| 1927 | 4  | 20 | 28 | 1943 | 18 | 29 | 28 | 23 | 20 | 27 |
| 1928 | 13 | 28 | 22 | 1944 | 21 | 14 | 22 | 22 | 11 | 20 |
| 1929 | 30 | 19 | 22 | 1945 |    |    |    | 17 | 11 | 20 |
| 1930 | 26 | 29 | 28 | 1946 |    |    |    | 24 | 25 | 63 |
| 1931 | 13 | 16 | 14 | 1947 |    |    |    | 42 | 30 | 56 |
| 1932 | 13 | 17 | 19 | 1948 |    |    |    | 25 | 25 | 34 |
| 1933 | 16 | 16 | 19 | 1949 |    |    |    | 38 | 35 | 44 |
| 1934 | 9  | 16 | 15 |      |    |    |    |    |    |    |

a) Watheroo; b) Zuy; c) Summer; d) Winter; e) Equinox

## Section 2. 11-Year Variation of the Middle-Latitude Part of the $S_D$ Currents

The 11-year and seasonal fluctuation of the  $S_D$  variations are considerably more complex. The mean annual  $S_D$  variations were calculated for a number of observatories for all years for which the data was available to us. As an example, the  $S_D$  variations for three observatories (Dombas, Slutsk and Huancayo) are given in Tables 21 - 23. A consideration of the materials collected by us has shown a very systematic variation of the  $S_D$  variations from year to year. In many cases these changes are expressed in the increase of the amplitudes with the increase of solar activity. But in a number of cases, changes of form and a shift in the time of the extreme values is observed (thus, for example the  $S_{DH}$  of Slutsk, the  $S_{DZ}$  of Uellen and Matochkin Shar, etc). At different latitudes, the cyclic variations of  $S_D$  proceed differently. The peculiarities noted in the variations of  $S_D$  force us to assume that the intensity, location and dimensions of the four current eddies making up the current system of  $S_D$  vary during the course of the cycle, the variations in the polar

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and middle-latitude eddies being unequal. In this Section we shall discuss the fluctuations of the middle-latitude eddies responsible for the course of the  $S_D$  variations in the belt of  $\pm 60^\circ$  geomagnetic latitude.

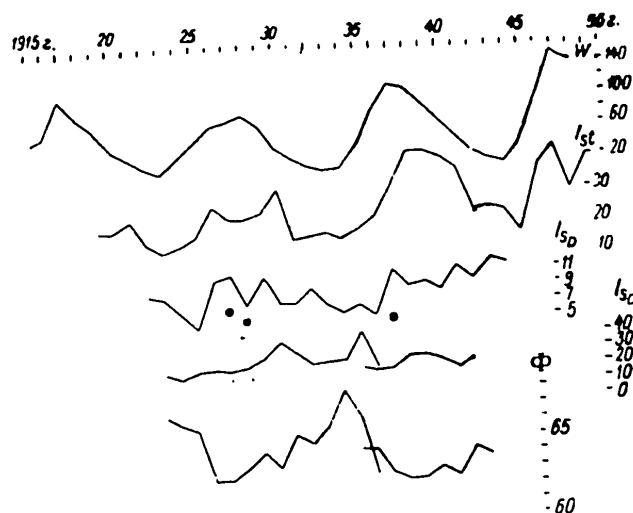


Fig.34 - Cyclical Fluctuation of Electric Currents of  $D_{st}$  and  $S_D$  Variations ( $W$  - relative sunspot number;  $I_{st}$  - intensity of  $D_{st}$  current;  $I_{SD}^1$  - intensity of middle-latitude eddy of  $S_D$  current). The dots indicate the intensity of the additional eddies.  $J_{SD}$  - intensity of polar eddy. Units of intensity  $-10^4$  amp.  $\Phi$  - geomagnetic latitude of auroral zone (from 1922 to 1936, from data of the  $S_D$  variations at Sitka observatory, from 1934 to 1943, from data of the  $S_D$  variations at Uellen Observatory)

Table 20

| Hemisphere | Equinox          | Winter           | Summer           |
|------------|------------------|------------------|------------------|
| Northern   | $36 \times 10^4$ | $28 \times 10^4$ | $23 \times 10^4$ |
| Southern   | 40               | 28               | 23               |
| Middle     | 38               | 28               | 23               |

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The variation of the  $S_D$  variations at the low latitude observatories during the 11-year cycle are small (for example, the  $S_D$  variations at Huancayo, Watheroo and Paris), and manifest themselves mainly in variations of amplitudes. In the equatorial zone, the  $S_D Z$  components differ little from zero, while the H components of the contrary are rather distinct. At the latitudes of the centers of the current eddies ( $\phi = \pm 40^\circ - 50^\circ$ ), the  $S_D$  variations of the X components are faint while those of the

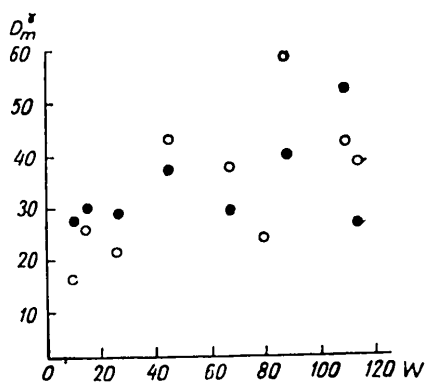


Fig.35 - Values of  $D_m = H_q - H_d$  from Huancayo Data for 1922 - 1935 (o - I, II, XI, XII months; • - V, VI, VII, VIII months; W - Relative Sunspot Numbers)

Z components are distinct. In view of this fact, it is more convenient to select the amplitude of  $S_D H$  (or X) at the equatorial stations ( $R_H$ ) and the amplitude Z at the middle-latitude stations, as our index of intensity of the  $S_D$  variations. The values of  $R_H$  and  $R_Z$  given in Table 24 for Huancayo, Watheroo, Paris, and other observatories show that there is no exact parallelism between the march of the amplitude and the annual relative sunspot numbers; but still the periods of elevated activity (1925 - 1931, 1936 - 1942) are likewise marked by an increase of amplitude at

all observatories. A certain shift of the maxima is noted (a lag of the maxima of the  $S_D$  amplitudes behind W), which is entirely absent in the  $S_q$  amplitudes (for comparison we also give in the Table the  $R(S_q H)$  for Watheroo and the u-measure of activity). Particularly characteristic in this respect is the maximum of the cycle 1923 - 1933 which occurred in solar activity and  $S_q$  variations in 1928, and in  $S_D$ , in 1930 (see the very sharp increase of  $RS_D$  at Watheroo). The march of the  $RS_D$  numbers is less smooth than the march of  $RS_q$ , the u-measure and W. The cyclical variations depends on the latitude: at Paris, the  $RS_D$  are greater than at Watheroo and Huancayo.

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The observatories located in the zone of the center of the middle-latitude eddy ( $\lambda = 40 - 50^\circ$ ), display not only considerable fluctuations in the amplitudes of  $S_D$ , but also a variation in the form. This indicates that the position of the center of the middle-latitude eddy varies in latitude from year to year.\* Thus, according to the data of the  $S_D$  at Slutsk (cf. Table 22) it is very clear that in 1924, 1931, 1933, and 1935 the center of the eddy was at the latitude of Slutsk, in 1932 and 1934 somewhat north of Slutsk, and in the remaining years south of Slutsk. In the first years of those enumerated, the  $S_{DH}$  hardly deviates from the zero line, in the following group of years the form of  $S_{DH}$  approaches the low-latitude type (with a minimum in the afternoon hours), and in the years of the last group, the form of  $S_{DH}$  is typically middle-latitude, with a clear maximum in the evening hours. The same variations in the phase of  $S_{DH}$  takes place at Sverdlovsk, Kazan', de Bilt, and Zuy. To give a more impressive idea of the fluctuations of the center of the middle-latitude eddy, Fig. 36 shows the position of the center of this eddy in 1932 - 1933, 1938 - 1939, 1941, 1944, and 1948. The position of the center in the II International Polar Year is plotted from the most complete data of all. It will be seen from the figure that it represents, like the zone of magnetic activity, an ellipse which, in very coarse approximation, is confocal with the zone of magnetic activity. This is evidence for the view that the asymmetry of  $S_D$ , which was mentioned in Chapter V, also exists in the middle latitudes, but to a lesser degree. On the territory of the USSR the line of the center of the eddy passes through Slutsk, north of Sverdlovsk and Kazan', somewhat north of Zuy, and considerably to the north of Toyohara, South Sakhalin. In another year of minimum (1944), in which the value of  $W$  was almost identical with its 1933 value, the  $S_{DH}$  at Zuy is close to that of 1933, but at

\* There is also a small shift of the center of the eddy during the hours of the day, which is manifested, for example, in the shift of the maximum of  $S_{DZ}$  at Slutsk from 19 hours in 1932 to 1730 hours in 1939. But this shift is very small by comparison with the variation of the latitude of the center of the eddy.

STAT

**POOR ORIGINAL**

Sverdlovsk and Kazan!  $S_D$  is almost of transitional type, which compels us to plot the line of centers in 1944 somewhat to the north of these observatories. In the years of high activity, the line of centers plainly descends to low latitudes, but

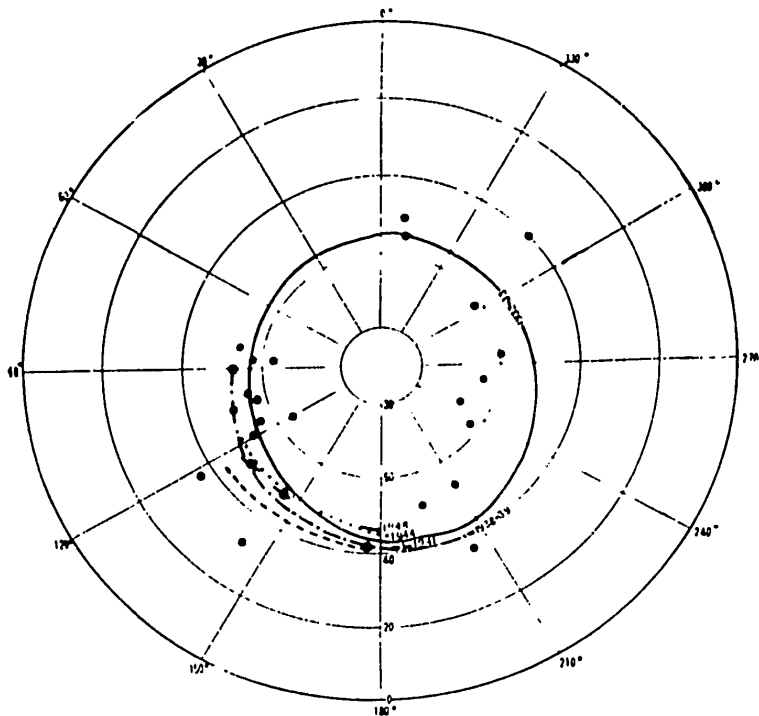


Fig.36 - Position of Line of Centers of Middle-Latitude Eddies in Different Years of the Solar Cycle. The Coordinate Net is Geomagnetic

this movement is not parallel in the entire sector we are discussing. The fact that in a year of exceptionally high activity (1948), the position of the lines according to the data of Kazan!, Sverdlovsk, and Zuy was almost the same as in 1944, appears to be somewhat surprising. Thus the conclusion may be drawn that the fluctuations of the line of the center of the middle-latitude eddy are very complex, and that the data of observatories located at different longitudes must be used for their study.

Considering however, that on the average, with increasing activity, the lines of the centers descend by  $5 - 6^\circ$ , I calculated the intensity of the current of the evening eddy ( $I_{SD}^1$ ) for 1922 - 1944, based on the value of the evening minimum of  $S_D$  at Huancayo and using the above described approximate formula. It was found that

# POOR ORIGINAL

Table 21

 $S_D$  - Variations

Dombas

Universal Time

| Year                         | Hours |      |      |      |      |     |     |      |      |      |      |      |      |     |      |      |      |      |      |      |       |       |       |       |       |
|------------------------------|-------|------|------|------|------|-----|-----|------|------|------|------|------|------|-----|------|------|------|------|------|------|-------|-------|-------|-------|-------|
|                              | 0     | 1    | 2    | 3    | 4    | 5   | 6   | 7    | 8    | 9    | 10   | 11   | 12   | 13  | 14   | 15   | 16   | 17   | 18   | 19   | 20    | 21    | 22    | 23    | 24    |
| D - Declination (in min)     |       |      |      |      |      |     |     |      |      |      |      |      |      |     |      |      |      |      |      |      |       |       |       |       |       |
| 1916                         | -6.5  | -4.0 | -3.8 | 0.0  | 0.5  | 4.4 | 3.3 | 3.7  | 3.0  | 2.8  | 1.5  | 1.5  | 1.1  | 1.1 | -0.1 | 0.0  | -2.5 | -3.5 | -3.5 | -6.0 | -6.0  | -7.5  | -7.5  | -6.7  | -6.7  |
| 1917                         | -5.3  | -6.0 | -5.0 | -3.4 | -1.6 | 0.4 | 2.0 | 2.0  | 1.5  | 0.5  | 0.4  | -0.2 | 0.2  | 1.4 | 1.9  | 3.2  | 2.4  | 1.3  | -0.1 | -0.9 | -3.9  | -7.0  | -7.8  | -6.6  | -6.6  |
| 1918                         | -9.8  | -7.8 | -6.7 | -4.5 | -1.1 | 1.8 | 2.2 | 1.8  | 1.1  | 0.6  | 1.2  | 1.8  | 1.4  | 2.2 | 3.8  | 3.8  | 3.2  | 0.7  | -2.2 | -5.1 | -7.8  | -9.1  | -10.9 | -10.2 | -10.2 |
| 1919                         | -6.6  | -6.8 | -5.3 | -2.4 | 1.4  | 1.5 | 1.2 | -3.0 | -7.3 | -9.0 | -6.8 | -2.2 | 3.3  | 7.4 | 10.5 | 7.7  | 4.5  | 1.4  | -2.3 | -4.4 | -5.7  | -7.7  | -10.7 | -10.7 | -8.6  |
| 1920                         | -6.4  | -6.6 | -6.2 | -4.4 | 1.1  | 1.5 | 1.6 | 1.1  | -0.1 | -1.1 | -1.0 | 0.9  | 2.5  | 3.8 | 4.6  | 3.5  | 2.6  | 1.5  | -0.9 | -5.3 | -6.7  | -9.8  | -10.4 | -8.9  | -8.9  |
| 1921                         | -2.8  | -2.4 | -3.2 | -2.3 | -1.8 | 0.3 | 1.5 | 1.0  | 1.2  | 1.4  | 2.2  | 2.4  | 2.9  | 3.2 | 3.1  | 2.4  | 1.6  | -0.4 | -3.6 | -5.8 | -6.4  | -6.7  | -6.6  | -6.6  | -3.6  |
| 1922                         | -3.3  | -3.6 | -2.6 | -2.2 | -0.2 | 2.2 | 3.4 | 3.9  | 3.4  | 2.7  | 2.4  | 1.7  | 1.5  | 1.6 | 1.2  | 1.8  | -0.6 | -4.1 | -4.1 | -7.7 | -9.7  | -8.8  | -7.2  | -4.7  | -4.7  |
| 1923                         | -3.7  | -2.5 | -1.2 | -1.0 | 0.4  | 1.3 | 1.2 | 1.0  | 0.9  | 0.4  | 0.8  | 1.2  | 1.7  | 1.2 | 1.5  | 1.4  | 0.2  | -0.2 | -2.0 | -4.0 | -5.8  | -7.3  | -6.8  | -5.4  | -5.4  |
| 1924                         | -3.9  | -3.2 | -2.0 | -0.9 | -0.4 | 0.6 | 1.2 | 1.1  | 0.7  | 0.1  | 1.2  | 1.4  | 1.6  | 2.2 | 2.4  | 3.0  | 2.5  | 0.9  | 1.1  | -2.4 | -4.9  | -6.2  | -4.7  | -3.4  | -3.4  |
| 1925                         | -4.2  | -3.3 | -2.0 | -1.1 | 0.5  | 1.4 | 2.1 | 2.9  | 2.6  | 1.4  | 2.1  | 2.4  | 2.5  | 2.5 | 2.5  | 2.5  | 0.9  | 0.9  | 1.0  | -3.5 | -5.6  | -7.3  | -7.8  | -7.5  | -7.5  |
| 1926                         | -7.7  | -7.4 | -4.9 | -2.6 | -0.1 | 2.2 | 4.2 | 3.7  | 2.6  | 1.0  | 1.0  | 1.4  | 1.3  | 0.9 | 1.1  | 1.0  | 1.2  | -0.8 | -1.4 | -4.1 | -6.8  | -7.1  | -6.8  | -7.4  | -5.6  |
| 1927                         | -4.7  | -3.6 | -1.7 | -0.8 | -0.2 | 2.7 | 3.4 | 2.1  | 2.1  | 1.3  | 0.7  | 0.6  | 1.7  | 1.9 | 2.1  | 1.6  | 0.8  | -0.4 | -3.0 | -6.5 | -6.8  | -7.8  | -7.4  | -7.4  | -8.1  |
| 1928                         | -5.5  | -4.6 | -4.2 | -2.0 | -0.2 | 0.7 | 2.3 | 1.4  | 0.2  | -0.8 | 0.0  | 0.4  | 1.3  | 1.8 | 1.8  | 1.7  | 2.5  | 0.4  | -1.4 | -2.9 | -6.3  | -7.6  | -8.9  | -8.1  | -8.1  |
| 1929                         | -6.9  | -5.3 | -4.3 | -3.0 | 0.1  | 2.2 | 2.9 | 2.1  | 0.8  | 0.2  | 0.8  | 0.1  | 0.3  | 0.8 | 1.0  | -0.5 | -1.0 | -1.6 | -3.3 | -6.0 | -7.6  | -10.6 | -10.6 | -9.7  | -8.4  |
| 1930                         | -7.1  | -5.9 | -3.9 | -2.6 | 0.8  | 2.9 | 4.6 | 5.4  | 4.0  | 1.5  | 0.4  | -0.3 | -0.5 | 0.7 | 0.7  | 0.0  | -1.6 | -3.6 | -4.7 | -7.8 | -10.4 | -10.5 | -10.2 | -8.4  | -8.4  |
| 1931                         | -3.3  | -2.8 | -2.2 | -2.1 | 0.0  | 1.6 | 1.5 | 1.4  | 1.5  | 1.2  | 1.6  | 2.1  | 0.6  | 0.8 | 0.1  | 1.0  | -1.4 | -3.3 | -3.3 | -7.3 | -8.1  | -9.6  | -8.5  | -5.0  | -5.0  |
| 1932                         | -3.9  | -3.4 | -2.9 | -2.2 | -0.2 | 1.7 | 3.4 | 3.2  | 1.9  | 0.9  | 0.6  | 0.5  | 0.2  | 1.1 | 1.1  | 0.1  | 0.1  | -1.6 | -4.3 | -7.5 | -8.7  | -9.6  | -11.0 | -7.9  | -5.8  |
| 1933                         | -3.5  | -1.9 | -2.3 | -2.5 | -0.2 | 2.8 | 3.5 | 2.7  | 0.7  | -0.2 | 0.6  | 0.7  | 0.4  | 1.2 | 1.8  | 1.1  | 0.7  | -1.6 | -5.0 | -6.4 | -8.3  | -8.8  | -7.0  | -5.6  | -5.6  |
| 1934                         | -2.1  | -2.3 | -3.2 | -2.1 | -1.3 | 1.6 | 3.2 | 2.9  | 2.3  | 1.4  | 1.0  | 1.3  | 1.8  | 1.8 | 1.5  | 1.7  | 1.3  | -0.8 | -3.1 | -4.9 | -7.5  | -7.5  | -7.6  | -5.8  | -3.0  |
| 1935                         | -6.4  | -4.8 | -4.9 | -3.1 | -1.2 | 1.1 | 2.3 | 2.1  | 2.9  | 2.3  | 2.2  | 2.6  | 2.4  | 1.9 | 1.6  | 0.4  | -0.6 | -1.8 | -4.1 | -4.7 | -7.2  | -8.7  | -9.2  | -7.9  | -7.9  |
| 1936                         | -6.6  | -6.0 | -5.2 | -4.0 | -2.0 | 0.2 | 1.8 | 1.2  | 1.9  | 1.7  | 1.8  | 2.8  | 2.2  | 2.2 | 2.1  | 2.2  | 1.8  | 1.8  | 1.8  | 0.8  | -2.2  | -6.0  | -7.6  | -8.3  | -7.2  |
| H - Component (in $\gamma$ ) |       |      |      |      |      |     |     |      |      |      |      |      |      |     |      |      |      |      |      |      |       |       |       |       |       |
| 1916                         | -20   | -17  | -18  | -11  | -7   | -4  | -3  | -1   | -1   | -1   | 0    | 4    | 6    | 8   | 8    | 8    | 10   | 6    | 4    | 2    | 0     | -6    | -10   | -15   | -12   |
| 1917                         | -14   | -13  | -12  | -8   | -6   | -4  | -3  | -2   | -2   | 0    | 2    | 5    | 8    | 11  | 14   | 16   | 16   | 15   | 12   | 7    | 2     | -4    | -12   | -12   | -13   |
| 1918                         | -29   | -24  | -19  | -15  | -8   | -4  | -4  | -3   | -2   | -1   | 2    | 5    | 9    | 16  | 16   | 16   | 15   | 15   | 14   | 12   | 5     | -6    | -15   | -21   | -23   |

# POOR ORIGINAL

|      |     |     |     |     |     |    |    |    |    |    |    |   |   |    |    |    |    |    |    |    |    |     |     |     |
|------|-----|-----|-----|-----|-----|----|----|----|----|----|----|---|---|----|----|----|----|----|----|----|----|-----|-----|-----|
| 1919 | -29 | -23 | -16 | -10 | -6  | -5 | -4 | -4 | -4 | -4 | 0  | 3 | 9 | 15 | 17 | 19 | 20 | 16 | 12 | 1  | 8  | -17 | -23 | -28 |
| 1920 | -21 | -16 | -8  | -6  | -4  | -4 | -3 | -3 | -3 | -3 | -2 | 0 | 3 | 7  | 7  | 6  | 6  | 7  | 4  | 0  | -5 | -13 | -20 | -20 |
| 1921 | -12 | -10 | -9  | -7  | -6  | -6 | -4 | -4 | -4 | -4 | -1 | 1 | 2 | 5  | 4  | 4  | 4  | 5  | 5  | 4  | -3 | -10 | -12 | -9  |
| 1922 | -12 | -8  | -5  | -4  | -3  | -3 | -2 | -2 | -2 | -2 | 0  | 1 | 2 | 2  | 4  | 7  | 8  | 8  | 0  | 3  | -2 | -6  | -4  | -6  |
| 1923 | -5  | -5  | -3  | -2  | -2  | -1 | -1 | -1 | -1 | -1 | 0  | 0 | 2 | 2  | 3  | 2  | 2  | 1  | 1  | -1 | -2 | -4  | -3  | -3  |
| 1924 | -4  | -5  | -3  | -2  | -2  | -1 | -1 | -1 | -1 | -1 | 0  | 1 | 1 | 4  | 5  | 5  | 5  | 1  | 4  | 1  | -2 | -6  | -9  | -9  |
| 1925 | -10 | -9  | -9  | -5  | -4  | -3 | -3 | -3 | -3 | -3 | 1  | 1 | 8 | 11 | 14 | 16 | 19 | 16 | 10 | 0  | -8 | -15 | -22 | -20 |
| 1926 | -26 | -19 | -14 | -12 | -9  | -7 | -7 | -7 | -7 | -7 | 1  | 5 | 8 | 10 | 12 | 12 | 12 | 8  | 4  | 0  | -8 | -11 | -11 | -15 |
| 1927 | -14 | -13 | -11 | -10 | -7  | -7 | -6 | -6 | -6 | -6 | 0  | 3 | 7 | 10 | 12 | 13 | 12 | 10 | 4  | 0  | -4 | -11 | -16 | -16 |
| 1928 | -14 | -18 | -15 | -10 | -7  | -7 | -6 | -6 | -6 | -6 | 0  | 5 | 8 | 10 | 12 | 13 | 12 | 10 | 4  | 0  | -4 | -11 | -11 | -15 |
| 1929 | -20 | -19 | -15 | -10 | -8  | -7 | -7 | -7 | -7 | -7 | 0  | 2 | 7 | 10 | 12 | 13 | 12 | 10 | 4  | 0  | -4 | -11 | -11 | -16 |
| 1930 | -33 | -32 | -28 | -16 | -11 | -9 | -8 | -8 | -8 | -8 | -3 | 2 | 7 | 10 | 12 | 13 | 12 | 10 | 4  | 0  | -4 | -11 | -11 | -29 |
| 1931 | -11 | -10 | -8  | -6  | -4  | -4 | -4 | -4 | -4 | -4 | -2 | 1 | 4 | 4  | 5  | 4  | 2  | 6  | 11 | 2  | -7 | -16 | -25 | -10 |
| 1932 | -14 | -15 | -12 | -6  | -4  | -4 | -4 | -4 | -4 | -4 | -2 | 1 | 4 | 4  | 5  | 4  | 2  | 6  | 11 | 2  | -7 | -16 | -25 | -10 |
| 1933 | -11 | -11 | -7  | -4  | -4  | -4 | -4 | -4 | -4 | -4 | -2 | 1 | 4 | 4  | 5  | 4  | 2  | 6  | 11 | 2  | -7 | -16 | -25 | -10 |
| 1934 | -6  | -7  | -4  | -3  | -3  | -3 | -3 | -3 | -3 | -3 | 0  | 1 | 3 | 6  | 6  | 6  | 5  | 3  | 2  | 0  | -4 | -6  | -8  | -8  |
| 1935 | -9  | -9  | -7  | -6  | -6  | -6 | -6 | -6 | -6 | -6 | 0  | 1 | 3 | 6  | 6  | 6  | 5  | 3  | 2  | 0  | -4 | -6  | -8  | -8  |
| 1936 | -10 | -11 | -9  | -6  | -5  | -4 | -3 | -2 | -2 | -2 | 1  | 2 | 6 | 9  | 11 | 9  | 10 | 4  | 6  | 2  | -2 | -6  | -9  | -10 |

Z-Component (m)

|      |     |     |     |     |     |     |    |    |    |    |    |    |   |   |   |   |    |    |    |   |   |     |     |     |
|------|-----|-----|-----|-----|-----|-----|----|----|----|----|----|----|---|---|---|---|----|----|----|---|---|-----|-----|-----|
| 1920 | -22 | -17 | -15 | -12 | -10 | -8  | -6 | -5 | -4 | -3 | -2 | -1 | 1 | 3 | 7 | 7 | 9  | 10 | 11 | 7 | 4 | -9  | -18 | -11 |
| 1921 | -16 | -10 | -8  | -7  | -5  | -4  | -3 | -3 | -3 | -3 | 1  | 3  | 6 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1922 | -24 | -21 | -15 | -10 | -9  | -5  | -3 | -3 | -3 | -3 | 1  | 3  | 6 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1923 | -10 | -10 | -8  | -6  | -5  | -4  | -3 | -3 | -3 | -3 | 1  | 3  | 6 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1924 | -9  | -9  | -8  | -7  | -5  | -4  | -3 | -3 | -3 | -3 | 1  | 3  | 6 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1925 | -14 | -13 | -12 | -11 | -10 | -9  | -8 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1926 | -23 | -19 | -16 | -13 | -10 | -9  | -6 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1927 | -16 | -15 | -12 | -10 | -9  | -7  | -6 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1928 | -22 | -19 | -17 | -14 | -12 | -10 | -9 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1929 | -19 | -18 | -17 | -14 | -12 | -10 | -9 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1930 | -25 | -25 | -22 | -18 | -15 | -11 | -9 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1931 | -18 | -18 | -18 | -14 | -11 | -9  | -7 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1932 | -23 | -22 | -22 | -18 | -15 | -11 | -9 | -7 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1933 | -18 | -18 | -18 | -14 | -11 | -9  | -7 | -6 | -6 | -6 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1934 | -14 | -13 | -10 | -8  | -7  | -6  | -4 | -4 | -4 | -4 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1935 | -16 | -16 | -16 | -11 | -9  | -7  | -6 | -4 | -4 | -4 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |
| 1936 | -22 | -22 | -22 | -18 | -14 | -10 | -6 | -5 | -4 | -3 | 1  | 5  | 8 | 8 | 8 | 8 | 10 | 12 | 10 | 9 | 8 | -10 | -14 | -14 |

STAT

POOR ORIGINAL

Table 22

SD - Variations

Slutsk

| Year | Hours |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    | Universal Time |
|------|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----------------|
|      | 0     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24             |

|      | D - Declination (in min) |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |     |     |     |     |     |     |
|------|--------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|-----|-----|-----|-----|-----|-----|
|      | 1.1                      | 0.0  | -0.2 | -1.1 | -1.5 | -1.5 | -1.7 | -1.4 | -1.1 | -1.4 | -1.5 | -1.1 | -1.4 | -1.6 | -1.1 | -0.8 | -0.2 | 0.9  | 1.3  | 2.9 | 2.5 | 3.1 | 2.5 | 2.9 |     |
| 1923 | 1.1                      | 0.8  | -0.2 | -0.7 | -1.5 | -1.4 | -1.9 | -1.7 | -1.4 | -1.1 | -1.4 | -1.5 | -1.1 | -1.4 | -1.6 | -1.1 | -0.8 | -0.2 | 0.9  | 1.3 | 2.9 | 2.5 | 3.1 | 2.5 | 2.9 |
| 1924 | 1.1                      | 0.8  | -0.2 | -0.7 | -1.5 | -1.4 | -1.9 | -1.7 | -1.4 | -1.1 | -1.4 | -1.5 | -1.1 | -1.4 | -1.6 | -1.1 | -0.8 | -0.2 | 0.9  | 1.3 | 2.9 | 2.5 | 3.1 | 2.5 | 2.9 |
| 1925 | 2.1                      | 1.5  | 0.3  | -0.3 | -2.6 | -3.0 | -2.5 | -2.2 | -1.3 | -1.7 | -1.2 | -1.1 | -1.3 | -1.0 | -1.5 | -0.9 | -0.9 | -0.9 | 0.0  | 1.8 | 3.4 | 3.4 | 1.7 | 3.8 | 3.9 |
| 1926 | 4.1                      | 3.4  | 3.1  | 0.7  | -0.5 | -2.1 | -2.8 | -2.5 | -2.2 | -2.2 | -1.5 | -1.1 | -1.5 | -2.0 | -2.6 | -2.4 | -3.2 | -1.6 | 0.4  | 1.4 | 2.6 | 4.8 | 4.8 | 3.6 | 3.5 |
| 1927 | 1.5                      | 1.0  | 0.6  | 0.2  | -2.8 | -1.2 | -1.3 | -2.2 | -1.4 | -1.4 | -1.4 | -1.9 | -1.2 | -0.8 | -2.3 | -1.9 | -1.1 | -1.6 | -0.2 | 1.4 | 3.1 | 4.1 | 3.5 | 3.9 | 3.6 |
| 1928 | 2.2                      | 0.7  | 1.7  | 0.2  | 0.0  | -0.7 | -0.6 | -0.6 | -1.2 | -0.5 | -0.8 | -1.5 | -1.2 | -0.8 | -1.9 | -2.6 | -1.6 | -1.7 | -0.4 | 0.2 | 3.1 | 1.7 | 2.4 | 3.2 | 3.5 |
| 1929 | 1.0                      | -0.1 | -1.7 | -3.4 | -3.8 | -3.8 | -3.5 | -2.9 | -2.1 | -1.8 | -0.9 | -1.3 | -1.6 | -2.2 | -0.8 | 1.5  | 2.1  | 3.6  | 4.0  | 4.1 | 4.3 | 4.3 | 3.1 | 2.5 | 2.5 |
| 1930 | -0.7                     | -1.3 | -1.5 | -2.9 | -3.1 | -2.8 | -2.2 | -2.2 | -1.5 | -0.9 | -1.3 | -1.5 | -0.5 | -0.8 | 0.0  | 1.5  | 2.0  | 2.9  | 2.9  | 4.1 | 4.3 | 4.3 | 3.1 | 2.5 | 2.5 |
| 1931 | -0.1                     | -0.5 | -1.2 | -1.8 | -2.4 | -2.3 | -2.3 | -2.3 | -1.2 | -1.4 | -0.9 | -1.1 | -1.3 | -1.4 | -0.3 | 1.5  | 1.2  | 2.9  | 2.9  | 4.1 | 4.3 | 4.3 | 3.1 | 2.5 | 2.5 |
| 1932 | -0.4                     | -0.5 | -1.1 | -2.3 | -2.5 | -2.8 | -2.4 | -1.7 | -1.3 | -1.1 | -1.2 | -1.6 | -1.5 | -2.2 | -0.7 | 0.7  | 1.5  | 3.8  | 4.0  | 4.1 | 4.3 | 4.3 | 3.1 | 2.5 | 2.5 |
| 1933 | -0.2                     | -0.8 | -0.8 | -1.4 | -2.2 | -2.5 | -2.6 | -1.8 | -1.1 | -1.3 | -0.7 | -0.9 | -1.0 | -0.8 | -0.3 | 0.5  | 1.6  | 3.2  | 3.6  | 3.1 | 2.4 | 1.9 | 1.4 | 0.9 | 0.9 |
| 1934 | 1.0                      | 0.6  | -0.2 | -1.8 | -2.2 | -2.1 | -1.7 | -1.7 | -1.6 | -1.6 | -1.8 | -1.7 | -2.5 | -2.3 | -1.3 | 0.2  | 1.5  | 2.0  | 3.6  | 3.6 | 3.1 | 2.4 | 1.9 | 1.4 | 0.9 |
| 1935 | 1.4                      | 1.1  | 0.0  | -0.9 | -1.5 | -1.3 | -1.3 | -1.4 | -1.4 | -1.5 | -1.3 | -2.1 | -2.4 | -3.0 | -2.3 | -1.0 | -0.2 | 1.4  | 2.3  | 2.5 | 3.2 | 3.0 | 3.0 | 2.9 | 2.9 |
| 1936 | 1.8                      | 0.1  | -1.4 | -2.4 | -2.5 | -2.7 | -2.5 | -2.0 | -1.6 | -1.6 | -1.3 | -1.3 | -1.0 | -3.6 | -2.4 | -2.3 | 0.6  | 1.8  | 2.3  | 3.7 | 4.6 | 5.0 | 5.6 | 5.3 | 5.3 |
| 1937 | 3.1                      | 2.6  | -0.8 | -2.7 | -2.3 | -1.8 | -1.3 | -2.1 | -1.9 | -2.2 | -2.1 | -2.6 | -3.7 | -4.6 | -3.1 | -2.3 | 0.1  | 1.0  | 2.9  | 3.4 | 4.8 | 6.2 | 6.6 | 4.1 | 4.1 |
| 1938 |                          |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |     |     |     |     |     |     |
| 1939 |                          |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |     |     |     |     |     |     |

|      | H - Component (in T) |    |   |    |    |    |    |    |    |    |    |    |    |    |    |   |   |   |   |   |    |    |    |    | STAT |   |
|------|----------------------|----|---|----|----|----|----|----|----|----|----|----|----|----|----|---|---|---|---|---|----|----|----|----|------|---|
| 1923 | 2                    | 1  | 1 | -1 | -1 | 0  | -1 | -1 | -3 | -3 | -2 | -2 | -3 | -1 | 0  | 3 | 5 | 0 | 6 | 3 | -1 | 0  | 1  | 1  | -2   | 0 |
| 1924 | 0                    | 1  | 2 | 1  | -5 | -2 | 0  | -2 | -2 | 0  | -3 | -4 | -1 | 0  | 2  | 2 | 3 | 3 | 2 | 5 | 1  | -3 | 3  | -2 | 0    |   |
| 1925 | 1                    | -2 | 3 | 2  | -3 | 1  | -1 | -4 | -5 | -7 | -2 | -4 | 0  | 1  | -3 | 3 | 6 | 5 | 6 | 3 | 2  | 0  | -5 | -2 | 0    |   |

POOR ORIGINAL

|      |     |     |     |     |     |     |     |     |     |    |    |     |     |     |    |    |    |    |    |     |     |     |     |     |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|-----|-----|-----|----|----|----|----|----|-----|-----|-----|-----|-----|
| 1926 | -18 | -13 | -13 | -10 | -12 | -16 | -16 | -14 | -9  | -6 | -1 | -3  | 10  | 11  | 13 | 21 | 26 | 26 | 29 | 17  | 0   | 1   | -13 | -19 |
| 1927 | -6  | 4   | -1  | -3  | -12 | -3  | -2  | -13 | -10 | -7 | -6 | -1  | -4  | 6   | 8  | 15 | 10 | 12 | 4  | -1  | -5  | -3  | -8  | -8  |
| 1928 | -1  | -4  | -2  | -7  | -9  | -2  | -7  | -12 | -7  | -7 | -8 | -5  | -3  | 4   | 7  | 16 | 12 | 14 | 10 | 8   | 2   | 2   | -6  | -2  |
| 1929 |     |     |     |     |     |     |     |     |     |    |    |     |     |     |    |    |    |    |    |     |     |     |     |     |
| 1930 | -1  | -1  | -6  | -8  | -10 | -9  | -10 | -9  | -11 | -4 | 0  | 2   | 8   | 11  | 12 | 20 | 16 | 14 | 7  | 2   | -4  | -3  | -6  | -9  |
| 1931 | 4   | 4   | 4   | 1   | 0   | 1   | 1   | -2  | -5  | -5 | -6 | -4  | -2  | 2   | 4  | 7  | 2  | 5  | 1  | 2   | -3  | -4  | -6  | -3  |
| 1932 | 1   | 0   | -2  | 0   | 1   | -5  | -4  | -3  | -3  | -7 | -5 | -6  | -2  | 1   | 0  | 4  | 2  | 3  | 3  | 0   | -3  | -3  | -6  | -5  |
| 1933 | 2   | 2   | 0   | -2  | -1  | -3  | -2  | -1  | 2   | 0  | -1 | -2  | -3  | 0   | -1 | 4  | 5  | 1  | 2  | 2   | 1   | 1   | -5  | -4  |
| 1934 | -4  | -3  | -3  | -3  | -3  | -7  | -7  | -6  | -8  | -9 | -8 | -10 | -11 | -11 | -9 | -7 | -6 | -7 | -7 | -13 | -11 | -10 | -11 | -10 |
| 1935 | 5   | 5   | 5   | 4   | 3   | 2   | 0   | -5  | -5  | -3 | 2  | 0   | 2   | 2   | 5  | 2  | 3  | 0  | -2 | -6  | -5  | -7  | -3  | -7  |
| 1936 | 5   | 5   | 4   | 0   | -1  | -4  | -7  | -6  | -4  | -2 | 0  | 3   | 8   | 8   | 8  | 5  | 1  | 0  | -3 | -4  | -5  | -5  | -10 | -8  |
| 1937 |     |     |     |     |     |     |     |     |     |    |    |     |     |     |    |    |    |    |    |     |     |     |     |     |
| 1938 | -14 | -5  | -3  | 1   | 2   | -5  | -5  | -3  | 1   | 5  | 8  | 15  | 24  | 32  | 30 | 32 | 24 | 10 | -5 | -9  | -23 | -33 | -42 |     |
| 1939 | 2   | -4  | -3  | -7  | -3  | -10 | -14 | -10 | -4  | 1  | 6  | 13  | 28  | 37  | 45 | 42 | 27 | 11 | 3  | -15 | -29 | -34 | -43 | -29 |

Z-Component (in)

|      |     |     |     |     |     |     |     |     |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|------|-----|-----|-----|-----|-----|-----|-----|-----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| 1923 | -18 | -16 | -14 | -16 | -13 | -19 | -7  | -3  | 0  | 1  | 3  | 5  | 9  | 13 | 15 | 18 | 20 | 15 | 11 | 3   | -2  | -8  | -17 |
| 1924 | -12 | -14 | -16 | -13 | -13 | -9  | -7  | -4  | -1 | -1 | 2  | 4  | 8  | 13 | 16 | 19 | 18 | 15 | 13 | 7   | 1   | -6  | -12 |
| 1925 | -20 | -25 | -22 | -20 | -18 | -11 | -7  | -3  | 1  | 3  | 7  | 10 | 15 | 17 | 19 | 22 | 24 | 22 | 16 | 9   | 0   | -9  | -17 |
| 1926 | -34 | -34 | -50 | -43 | -34 | -18 | -18 | -11 | -5 | 2  | 9  | 16 | 23 | 33 | 38 | 44 | 41 | 39 | 26 | 15  | -8  | -18 | -30 |
| 1927 | -22 | -23 | -23 | -24 | -26 | -17 | -12 | -11 | -7 | -3 | 2  | 7  | 13 | 21 | 28 | 35 | 34 | 26 | 14 | 0   | -10 | -16 | -20 |
| 1928 | -27 | -28 | -29 | -25 | -25 | -18 | -13 | -9  | -5 | -3 | 2  | 7  | 10 | 15 | 20 | 27 | 27 | 25 | 11 | 0   | -6  | -13 | -18 |
| 1929 |     |     |     |     |     |     |     |     |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| 1930 |     |     |     |     |     |     |     |     |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| 1931 | -27 | -29 | -18 | -26 | -13 | -8  | -3  | 0   | 4  | 8  | 13 | 16 | 21 | 28 | 32 | 34 | 24 | 12 | 1  | -13 | -23 | -31 | -20 |
| 1932 | -19 | -18 | -16 | -16 | -13 | -9  | -5  | -2  | 2  | 4  | 7  | 9  | 15 | 21 | 30 | 30 | 21 | 11 | -6 | -16 | -23 | -24 | -26 |
| 1933 | -16 | -15 | -12 | -10 | -8  | -7  | -5  | -4  | -1 | 1  | 3  | 6  | 11 | 17 | 19 | 22 | 17 | 7  | -2 | -6  | -11 | -16 | -19 |
| 1934 | -21 | -21 | -20 | -17 | 13  | -10 | -6  | -6  | -1 | 3  | 7  | 13 | 18 | 24 | 31 | 28 | 20 | 11 | 3  | -7  | -16 | -26 | -27 |
| 1935 | -26 | -25 | -23 | -20 | -18 | -12 | -7  | -2  | 2  | 6  | 8  | 15 | 22 | 30 | 35 | 26 | 20 | 12 | 1  | -8  | -17 | -22 | -24 |
| 1936 |     |     |     |     |     |     |     |     |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| 1937 |     |     |     |     |     |     |     |     |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
| 1938 | -49 | -49 | -46 | -39 | -28 | -20 | -11 | -7  | 1  | 13 | 17 | 26 | 40 | 59 | 62 | 56 | 48 | 21 | 6  | -24 | -38 | -45 | -30 |
| 1939 | -50 | -50 | -49 | -43 | -32 | -20 | -12 | -3  | 3  | 13 | 22 | 31 | 44 | 59 | 66 | 56 | 40 | 18 | -8 | -28 | -37 | -50 | -17 |

STAT

# POOR ORIGINAL

Table 23  
SD - Variations  
Huancayo

Universal Time

| Year | Hours |   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|------|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|      | 0     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |

D - Declinations (in tenth of a minute)

|      |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1922 | -2 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1923 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1924 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1925 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1926 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1927 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1928 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1929 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1930 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1931 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1932 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1933 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1934 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1935 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1936 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1937 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1938 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1939 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1940 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1941 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1942 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1943 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1944 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |

H - Component (int)

|      |   |   |   |   |   |    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|------|---|---|---|---|---|----|----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1922 | 8 | 0 | 4 | 7 | 9 | 10 | 12 | 9 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 1923 | 0 | 1 | 5 | 3 | 6 | 6  | 6  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 1924 | 1 | 5 | 4 | 4 | 6 | 6  | 6  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 1925 | 4 | 4 | 4 | 4 | 6 | 6  | 6  | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

STAT



# POOR ORIGINAL

|      |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |   |
|------|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|---|
| 1926 | 2 | 9 | 4 | 1 | 3 | 4 | 2 | 4  | 4  | 1  | 0  | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8 |
| 1927 | 9 | 4 | 1 | 3 | 4 | 2 | 4 | 4  | 1  | 0  | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |   |
| 1928 | 4 | 1 | 3 | 4 | 2 | 4 | 4 | 1  | 0  | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |   |
| 1929 | 1 | 3 | 4 | 2 | 4 | 4 | 1 | 0  | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |   |
| 1930 | 3 | 4 | 2 | 4 | 4 | 1 | 0 | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |   |
| 1931 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1932 | 3 | 4 | 2 | 4 | 4 | 1 | 0 | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |   |
| 1933 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1934 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1935 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1936 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1937 | 1 | 3 | 4 | 2 | 4 | 4 | 1 | 0  | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |   |
| 1938 | 3 | 4 | 2 | 4 | 4 | 1 | 0 | 3  | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |   |
| 1939 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1940 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1941 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1942 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1943 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |
| 1944 | 4 | 2 | 4 | 4 | 1 | 0 | 3 | 10 | 5  | 7  | 13 | 13 | 11 | 8  |    |    |    |    |   |

Z-Component (in 1)

|      |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1922 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 3 | 2 | 1 | 2 | 0 |
| 1923 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1924 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1925 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1926 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1927 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1928 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1929 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1930 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1931 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1932 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1933 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1934 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1935 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1936 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1937 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1938 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1939 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1940 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1941 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1942 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1943 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |
| 1944 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 1 | 1 | 0 |

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the march  $I_{SD}^1$  is completely parallel to the march of  $R_H$  at Huancayo. It follows that  $S_D$  depends almost completely on the value of  $X$  and depends little on the extension of the eddy along the meridian. Consequently, the error of  $\pm 4^\circ$  which may possibly have been made in estimating the position of the center of the eddy will distort the value of  $I_{SD}^1$  only slightly.

Table 24

| a)   | b)    | c)    | d)    | e)    | f)    | g)    | h)    | i)    | j)  | W   | $S_H$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-----|-------|
|      | $R_Z$ | $R_Z$ | $R_Z$ | $R_Z$ | $R_Z$ | $R_Z$ | $R_Z$ | $R_H$ |     |     |       |
| 1916 |       |       |       |       |       |       | 28    |       |     | 56  |       |
| 1917 |       |       |       |       |       |       | 28    |       |     | 104 |       |
| 1918 |       |       |       |       |       |       | 43    |       |     | 80  |       |
| 1919 | 20    |       |       |       |       |       | 48    |       |     | 64  | 23,2  |
| 1920 | 18    |       |       |       |       |       | 30    |       | 1,2 | 38  | 15,8  |
| 1921 | 20    |       |       |       |       |       | 18    |       | 1,1 | 26  | 12,4  |
| 1922 | 18    |       |       |       |       |       | 20    | 16    | 1,0 | 14  | 12,4  |
| 1923 | 14    |       | 35    | 75    |       |       | 10    | 15    | 0,7 | 6   | 12,4  |
| 1924 | 12    |       | 30    | 72    |       |       | 7     | 11    | 0,6 | 26  | 15,0  |
| 1925 | 15    |       | 50    | 98    |       |       | 15    | 8     | 0,7 | 44  | 14,4  |
| 1926 | 22    |       | 100   | 125   |       |       | 45    | 24    | 0,8 | 64  | 19,6  |
| 1927 | 24    |       | 60    | 120   |       |       | 30    | 26    | 1,2 | 70  | 22,2  |
| 1928 | 19    |       | 65    | 125   |       |       | 30    | 15    | 1,0 | 78  | 23,0  |
| 1929 | 24    |       |       | 155   |       |       | 35    | 25    | 1,0 | 65  | 19,5  |
| 1930 | 32    |       |       | 240   | 200   |       | 50    | 15    | 1,0 | 36  | 14,8  |
| 1931 | 16    |       |       | 120   | 130   |       | 23    | 18    | 1,0 | 21  | 14,0  |
| 1932 | 17    |       | 60    | 115   | 160   | 15    | 20    | 11    | 0,7 | 11  | 14,4  |
| 1933 | 18    |       | 57    | 110   | 140   | 13    | 18    | 9     | 0,7 | 6   | 13,2  |
| 1934 | 15    |       | 40    | 80    | 100   | 5     | 13    | 12    | 0,7 | 8   | 16,8  |
| 1935 | 21    |       | 58    | 110   | 135   | 20    | 20    | 9     | 0,7 | 36  | 15,0  |
| 1936 | 22    | 22    | 60    | 117   | 130   | 30    | 20    | 9     | 0,8 | 80  | 18,0  |
| 1937 | 23    | 24    |       |       | 112   | 50    | 24    | 19    | 0,9 | 114 | 25,4  |
| 1938 | 31    | 38    | 105   |       |       | 77    | 21    | 14    | 1,4 | 110 | 23,6  |
| 1939 | 32    | 40    | 110   |       |       | 95    | 18    | 1,3   | 1,3 | 89  | 24,2  |
| 1940 | 27    | 47    |       |       |       | 77    | 24    | 1,3   | 1,3 | 68  | 18,4  |
| 1941 | 30    | 49    |       |       |       | 85    | 19    | 1,2   | 1,2 | 46  | 20,0  |
| 1942 | 21    | 26    |       |       |       | 35    | 24    | 0,9   | 0,9 | 27  | 17,0  |
| 1943 | 26    | 28    |       |       |       | 30    | 22    | 0,9   | 0,9 | 15  | 18,4  |
| 1944 | 20    | 22    |       |       |       | 20    |       | 0,8   | 0,8 | 10  | 16,1  |
| 1945 |       | 20    |       |       |       | 15    |       | 0,9   | 1,4 | 35  |       |
| 1946 |       | 52    |       |       |       |       |       | 1,4   | 1,4 | 92  |       |
| 1947 |       |       |       |       |       |       |       |       |     | 152 |       |
| 1948 |       |       |       |       |       |       |       |       |     | 136 |       |

a) Year; b) Watheroo; c) Paris; d) Slutsk; e) Sitka; f) Tromso; g) Lovo;  
h) Dombas; i) Huancayo; j) u-Measure

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The following features must be noted in the march of  $I_{SD}^1$  (cf. Fig. 34):

1. The values of  $I_{SD}^1$  vary by a factor of almost 3 from the year of maximum ac-

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tivity to the year of minimum activity (3.8 in 1925 and 11.0 in 1927 and 1943).

2. After 1938 the values of  $I_{SD}^1$  continue to rise almost monotone, reaching very high values in the years of the minimum (1943 and 1944). Since material for only two cycles is available to us, it is difficult to give an explanation for this phenomenon.

3. In some years (years of high solar activity: 1927, 1928, 1937) the center of the eddy splits into two parts, that is two maxima are found in the  $S_D H$  at Huancayo. The secondary maxima are marked by dots on Fig.34.

4. The agreement between the cyclical variations of  $I_{D_{st}}$  and  $I_{SD}^1$  is small. Thus, for example, the sharp drop in  $I_{SD}^1$  in 1928 corresponds to a smooth march of  $I_{D_{st}}^1$ , and, on the other hand, the maximum of  $I_{D_{st}}^1$  in 1930 corresponds to a minimum in  $I_{SD}^1$ . The fluctuations  $I_{SD}^1$  are less regular than the fluctuations of  $I_{D_{st}}$ ; the latter follow the cyclical variations of  $W$  considerably more closely. The march of  $I_{SD}^1$  displays no tendency to a lag in the time of the maxima with respect to  $W$ , and, on the other hand, a certain lag of the epochs of the minimum does appear. The relatively poor correspondence between  $I_{D_{st}}^1$  and  $I_{SD}^1$  becomes particularly interesting if we bear in mind the fact that the intensity in both systems of current is calculated from one and the same empirical data of the  $X$  component at Huancayo Observatory. This poor correspondence, it seems to us, is one of the indications of the different nature of these two current systems, the  $D_{st}$  currents being more directly correlated with the intensity of solar activity, while the  $S_D$  currents may possibly be affected by other factors as well.

### Section 3. The 11-Year Variation of the Polar Part of the $S_D$ Currents

For studying the fluctuations of the polar eddies, the  $S_D$  variations of the observatories at Dombas, Lovo, Sitka, Godhavn, Sodankyla and other Arctic Observatories were calculated by me. The fluctuations of  $S_D$  at the polar observatories from year to year are considerably greater than at the middle-latitude observatories.

The cyclical variations of the amplitude of  $S_D$  reach their maximum value in the im-

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mediate vicinity of the auroral zone (observatories at Tromso, Sitka, etc). At the observatories with the circumpolar type of variations (Godhavn, and, in part, Tikhaya Bay) the variation of the amplitude is again considerably smaller. Since all the above enumerated observatories are far from the centers of the current eddies, the  $S_D$  variations of the horizontal component are of the same type as to form in all years, and differ only in amplitude. A change in form is observed in the  $S_D$  of the observatories located beneath the zone of the hypothetical linear current: Sodankyla, Matochkin Shar, Dickson. In the Second International Polar Year, a transitional type of Z variations was observed at these observatories: at Sodankyla, it was close to the middle-latitude type, while at Matochkin Shar and Dickson it was close to the polar type. This indicates that the zone of linear current, (or of strong concentration of surface currents) must pass to the north of Sodankyla and to the south of Matochkin Shar and Dickson. In years of high activity, the  $S_D$  variations of Z at all three observatories take on a distinctly polar form, which confirms the well known fact of the descent of the zone to lower latitudes with the growth of activity.

To obtain the numerical data on the location of the zone and the intensity of the current in different years of the 11-year cycle, I calculated the value of  $I$  and  $\Phi_0$ , using the formulas for the linear current (cf. Chapter V), from the data of few observatories, assuming that the height of the current during the entire 11-year cycle did not substantially vary from that of 1933. The replacement of the surface system of currents by linear system as we have seen in Chapter V, allows a rather good estimate to be made of the position of the auroral zone. The assumption of the invariability of  $h$  does not of course correspond to the actual behavior of the heights of the ionospheric layers, and this produces a certain element of the arbitrary in my results. But whatever scanty information on the 11-year fluctuation of the height of the  $F_2$  layer in the polar latitudes is today available indicates that these fluctuations are not large. The calculations made separately for the morning and evening hours (cf. Table 25) show that the latitude of the zone and the intensity

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Table 25

| Year | Time of Day | Sitka          |                   | Dombas         |                   |
|------|-------------|----------------|-------------------|----------------|-------------------|
|      |             | $\Phi_0^\circ$ | $I \times 10^4 A$ | $\Phi_0^\circ$ | $I \times 10^4 A$ |
| 1920 | a)          |                |                   | 66,4           | -9                |
|      | b)          |                |                   | 68,2           | 5                 |
| 1921 | a)          |                |                   | 67,1           | -7                |
|      | b)          |                |                   | 70,3           | 6                 |
| 1922 | a)          |                |                   | 68,5           | -10               |
|      | b)          |                |                   | 67,8           | 6                 |
| 1923 | a)          | 67,4           | -18               | 68,5           | -5                |
|      | b)          | 65,7           | 16                | 69,6           | 3                 |
| 1924 | a)          | 66,8           | -16               | 68,9           | -5                |
|      | b)          | 65,2           | 13                | 71,5           | 11                |
| 1925 | a)          | 66,0           | -19               | 67,1           | -6                |
|      | b)          | 65,2           | 18                | 68,2           | 4                 |
| 1926 | a)          | 62,7           | -19               | 65,3           | -9                |
|      | b)          | 62,2           | 19                | 65,3           | 6                 |
| 1927 | a)          | 62,7           | -19               | 66,4           | -7                |
|      | b)          | 62,2           | 17                | 64,6           | 4                 |
| 1928 | a)          | 63,5           | -23               | 66,7           | -9                |
|      | b)          | 63,0           | 16                | 66,0           | 6                 |
| 1929 | a)          | 64,6           | -28               | 65,6           | -8                |
|      | b)          | 63,8           | 24                | 66,4           | 7                 |
| 1930 | a)          | 63,3           | -42               | 65,3           | -10               |
|      | b)          | 63,2           | 31                | 65,7           | 7                 |
| 1931 | a)          | 67,2           | -32               | 69,7           | -11               |
|      | b)          | 63,2           | 24                | 68,9           | 5                 |
| 1932 | a)          | 64,6           | -23               | 67,8           | -10               |
|      | b)          | 64,6           | 19                | 68,9           | 5                 |
| 1933 | a)          | 65,7           | -23               | 68,2           | -9                |
|      | b)          | 65,4           | 20                | 68,6           | 5                 |
| 1934 | a)          | 69,8           | -28               | 69,3           | -7                |
|      | b)          | 66,2           | 18                | 68,9           | 4                 |
| 1935 | a)          | 68,0           | -65               | 69,0           | -8                |
|      | b)          | 64,6           | 17                | 66,4           | 4                 |
| 1936 | a)          | 62,7           | -20               | 69,0           | -10               |
|      | b)          | 62,7           | 15                | 66,8           | 5                 |

| Year | Time of Day | Chelyuskin     |                   | Dickson        |                   | Matochkin Shar |                   | Uellen         |                   |
|------|-------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|
|      |             | $\Phi_0^\circ$ | $I \times 10^4 A$ | $\Phi_0^\circ$ | $I \times 10^4 A$ | $\Phi_0^\circ$ | $I \times 10^4 A$ | $\Phi_0^\circ$ | $I \times 10^4 A$ |
| 1933 | a)          |                |                   | 61,0           | -30               | 61,9           | -33               |                |                   |
|      | b)          |                |                   | 62,9           | 17                | 64,1           | 18                |                |                   |
| 1934 | a)          |                |                   | 60,2           | -27               | 61,8           | -26               |                |                   |
|      | b)          |                |                   | 62,1           | 17                | 64,3           | 13                |                |                   |
| 1935 | a)          | 45,5           | -100              | 60,8           | -34               |                |                   | 64,6           | -21               |
|      | b)          | 59,5           | 44                | 61,9           | 22                |                |                   | 64,0           | 15                |
| 1936 | a)          | 51,4           | -121              | 60,4           | -32               |                |                   | 65,0           | -19               |
|      | b)          | 59,1           | 55                | 61,8           | 22                |                |                   | 63,4           | 15                |
| 1937 | a)          |                |                   | 60,1           | -39               | 61,5           | -42               | 62,6           | -19               |
|      | b)          |                |                   | 60,9           | 30                | 62,7           | 31                | 62,7           | 16                |
| 1938 | a)          |                |                   | 59,1           | -55               | 59,6           | -72               | 62,3           | -27               |
|      | b)          |                |                   | 59,3           | 48                | 59,6           | 76                | 62,1           | 23                |
| 1939 | a)          | 56,7           | -79               | 59,7           | -46               | 59,5           | -39               | 62,6           | -31               |
|      | b)          |                |                   | 59,3           | 53                | 59,8           | 50                | 62,0           | 19                |
| 1940 | a)          | 49,7           | -123              | 60,7           | -43               | 60,3           | -45               | 63,0           | -26               |
|      | b)          | 47,4           | 150               | 60,6           | 34                | 61,5           | 41                | 62,9           | 19                |
| 1941 | a)          | 54,3           | -106              | 59,3           | -50               | 55,6           | -65               | 62,3           | -17               |
|      | b)          |                |                   | 59,9           | 36                | 61,4           | 29                | 62,6           | 17                |
| 1942 | a)          | 54,6           | -76               | 60,3           | -37               |                |                   | 64,6           | -29               |
|      | b)          | 52,8           | 78                | 60,9           | 26                |                |                   | 63,6           | 18                |
| 1943 | a)          | 56,1           | -96               | 59,0           | -45               |                |                   | 64,2           | -22               |
|      | b)          | 49,6           | -148              | 60,5           | 33                |                |                   | 63,0           | 25                |
| 1944 | a)          | 56,7           | -61               |                |                   |                |                   |                |                   |
|      | b)          | 46,4           | 124               |                |                   |                |                   |                |                   |

a) morning b) evening

of the current vary about equally in the two halves of the day during the course of these years. No systematic difference whatever was found in the cyclical variations of  $I$  or  $\phi_0$  at various times of the day, as should have been the case if the morning and evening disturbance were due to different solar agents (cf. Chapter I).

In view of the fact that the fluctuations of the parameters of the current, from the data of observatories located south of the auroral zone, lead to similar results for all those stations, Fig. 34 gives the mean values for  $I$  and  $\phi_0$  of the morning and evening hours at the Sitka Observatory for 1920 - 1926 and the Uellen Observatory for 1933 - 1943.\* The southward shift of the auroral zone was distinctly manifested during the course of both maxima: this shift amounted to  $4.5^\circ$  in the 1923 - 1933 cycle, and to  $3.5^\circ$  in the 1933 - 1944 cycle. But the return to the high latitudes after the 1938 maximum was not immediate. In 1943, already characterized by low values of the solar activity, the position of the zone was only slightly north of its 1937 position. There is a strikingly good correlation between the position of the zone and the intensity of the zonal current, which is notable not merely in the general tendency of the variations during the 11-year cycle, but also in the oscillations in individual years (for example, the increase of  $\phi_0$  and the decrease of  $I$  in 1940, in 1942, etc). The absence of parallelism between the 11-year fluctuations of the intensity of the current in the polar zone and the middle latitude current eddy indicates that possibly the mechanisms exciting them may be somewhat different. The curve of annual values of the intensity of the polar currents would appear to display two maxima each in the course of an 11-year cycle, of which one is located on the branch of rising activity, and the other on the branch of falling activity (cf. years 1930, 1935, 1938, 1939).

The variations, with the solar cycle, of the position and intensity of the polar current, according to the data of observatories situated in the zone itself or

\* The absolute values of  $I$  and  $\phi_0$ , as already stated in Chapter V, differ somewhat between the different observatories.

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north of it, are less regular. Thus, for example, according to the data of the Dickson Observatory, the southern position of the zone connected with the maximum of 1938 was maintained until 1943 inclusive. The fluctuations of the position of the zone, from the data of the Matochkin Shar and Chelyuskin Observatories, according to the extremely fragmentary information represented by the data of Table 25, appear to be entirely random. The intensity of the polar current, according to the Chelyuskin data had maxima in 1940 and 1943, but according to the Matochkin Shar data, in 1937. It is possible that the results obtained may be interpreted as follows: during the 11-year cycle the width of the auroral zone varies. Its southern boundary is regularly shifted southward in the years of high magnetic activity and northward in the years of low activity. The northern edge of the zone is either little shifted at all during the 11-year cycle or is shifted according to certain peculiar and still imperfectly elucidated laws of its own. These facts are in good agreement with the view of auroral investigators to the effect that the cyclical fluctuations of the auroral frequency in the zone and in the polar cap are different from those at lower latitudes. Thus Vegard denies any existence whatever of a regular cyclical behavior in the auroral frequency. Tromhold indicates a cyclical march inverse to the march of solar activity. On the polar cap, the 11-year oscillations have 2 maxima each (on the branches of falling and rising activity), and in the zone itself the 11-year march has a transitional form. Pushkov and Brunkovskaya (Bibl.28) have found that the southward displacement of the auroral zone on days with elevated magnetic activity does not involve the weakening of the auroral displays to the north of the zone, which once again indicates the possible expansion of the zone with increasing activity.

The question as to the position of the zone of linear current and of its fluctuations in the 11-year cycle is of great importance in calculating the working frequencies of radio waves over routes passing through high latitudes, since this zone is at the same time the zone of maximum absorption. In view of this fact it appears

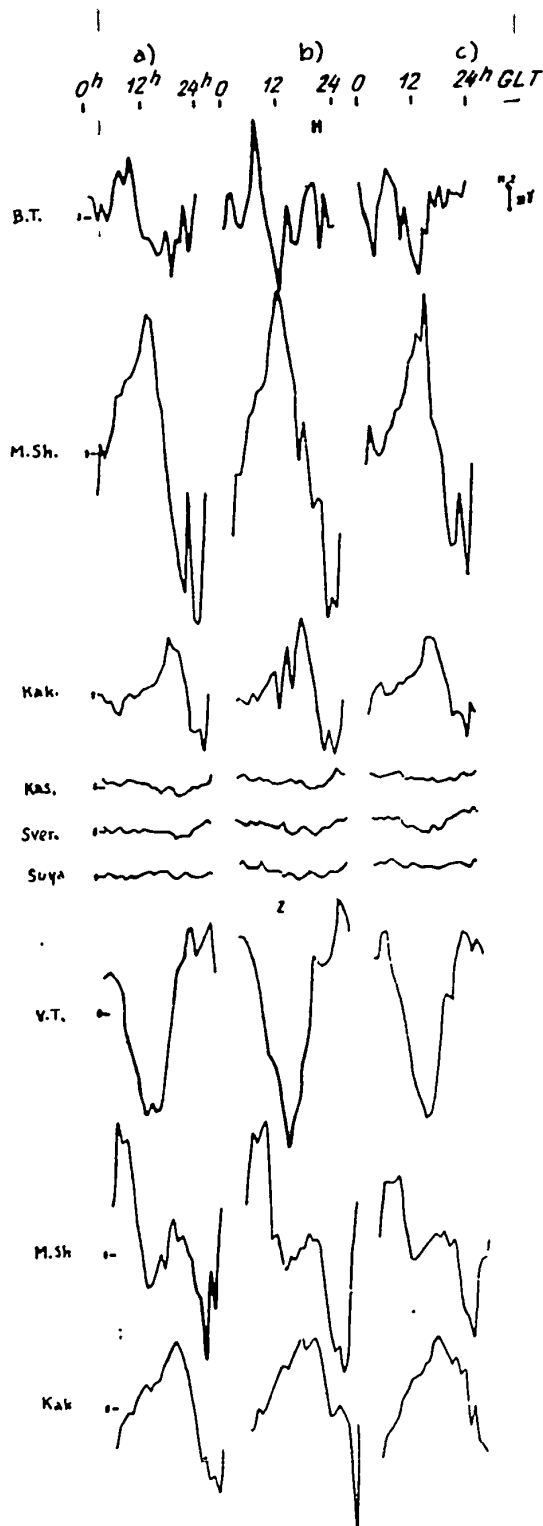


Fig.37 - The  $S_D$  Variation of the H and Z Components of the Magnetic Field in 1932 - 1933 from USSR Observatories  
a) Winter; b) Equinox; c) Summer

to be necessary to continue the accumulation of material on magnetic disturbances and their auroral displays, which will help to pinpoint the position of the zone.

The material on the  $S_D$  variations considered by us allow us to draw two conclusions: first, that the study of the  $S_D$  variations can yield useful information on the diurnal march, on the 11-year fluctuations, and on other peculiarities of the zone, and second, that the presently available data from observatories situated inside the zone are insufficient for the formulation of any reliable picture of the displacement of the northern edge of the zone.

#### Section 4. Seasonal Variations of the $S_D$ Currents

Let us now consider the seasonal variations of the current systems of the  $S_D$  variations. The literature summaries of the  $S_D$  variations (Bibl.4, 6, 61) show that the seasonal variations of the  $S_D$  of all three elements are small, especially in the middle and low latitudes. As in the case of the 11-year fluctuations, the character of the variation of the components with the seasons is completely de-



terminated by the position of the observatory with respect to the current eddies. At observatories far from both the centers of the eddies and the zone of linear current, the seasonal variations reduce, on the whole, to an increase or decrease of the amplitudes. Observatories near the centers of the eddies often note an inversion of the the phase of  $X$ , while observatories located in the neighborhood of the polar current note an inversion of the phase of  $Z$ . Figure 37 gives the  $S_D$  variations of a few observatories for the Second International Polar Year, which give a clear idea of the seasonal variations characteristic for different types of  $S_D$ . The amplitude of the variations is greatest at all latitudes in the epochs of the equinox and is smallest in the winter period. Their position of the centers of the middle-latitude eddies does not remain constant throughout the year, descending southward in the summer months and ascending northward in the winter. From the  $S_D$  variations of the horizontal component at Zuy Observatory, at the latitude of which the centers of the eddies were located during the Second International Polar Year, it is clear that in summer the middle-latitude type of  $S_D$  is observed, while in winter the type observed is low-latitude. In the equinox, when the lines of centers occupy an intermediate position, the  $S_D$  at Zuy are of transitional type with very small and irregular oscillations during the course of the day.

At observatories close to the auroral zone (Dickson, Matochkin Shar) the middle-latitude type of  $S_D$  variations is observed in summer and the high latitude type in the equinox (cf. the  $S_D Z$  components) while in the winter the  $S_D Z$  components have a characteristically transitional form. This indicates that the fluctuations of the auroral zone are similar to the fluctuations of the line of centers, more specifically, that zone occupies an intermediate position in winter, descending to the south in the equinox and ascending to the north in the summer months. The position of the line of centers of the middle-latitude eddies and of the auroral zone, calculated from the data of the whole net of observatories, is shown in Fig.38a. The broken lines in the sector  $180$  to  $270^\circ$  denote the absence of data for these longitudes.

The intensity of the middle-latitude eddies (I) and of the high-latitude eddies (II) (in amperes), calculated by approximate formulas for the surface current, are given in Table 26.

Table 26

|              | a) |    | 1938-1939 |    |
|--------------|----|----|-----------|----|
|              | I  | II | I         | II |
| b) . . . . . | 3  | 12 | 4         | 16 |
| c) . . . . . | 5  | 18 | 7         | 26 |
| d) . . . . . | 4  | 16 | 5         | 21 |
| e) . . . . . | 4  | 15 | 5         | 21 |

Note. The values in Table 26 are given in  $10^4$  amp.

a) Second International Polar Year; b) Winter; c) Equinox;  
d) Summer; e) Year

The seasonal fluctuations differ in years of different solar activity: in the years of high activity they are considerably sharper. Figure 38b gives the position of the line of centers and of the polar zone for 1938 - 1939. The character of the displacement of the line of centers and of the zone remains the same as in the years of the minimum, but the value of the shift is considerably greater. It will be seen from Table 26 that in the year of the maximum the seasonal fluctuations of the intensity of the currents likewise increased.

Summarizing all that has been said in the present Chapter, the following conclusions may be drawn: the seasonal and 11-year fluctuations of the  $D_{st}$  and  $S_D$  variations, which differ in character and amplitude at different observatories and for different elements, find their explanation in the changes undergone by the systems of electric currents responsible for these variations. The 11-year fluctuations of the  $D_{st}$  currents follow the 11-year cycle of solar activity most closely of all,

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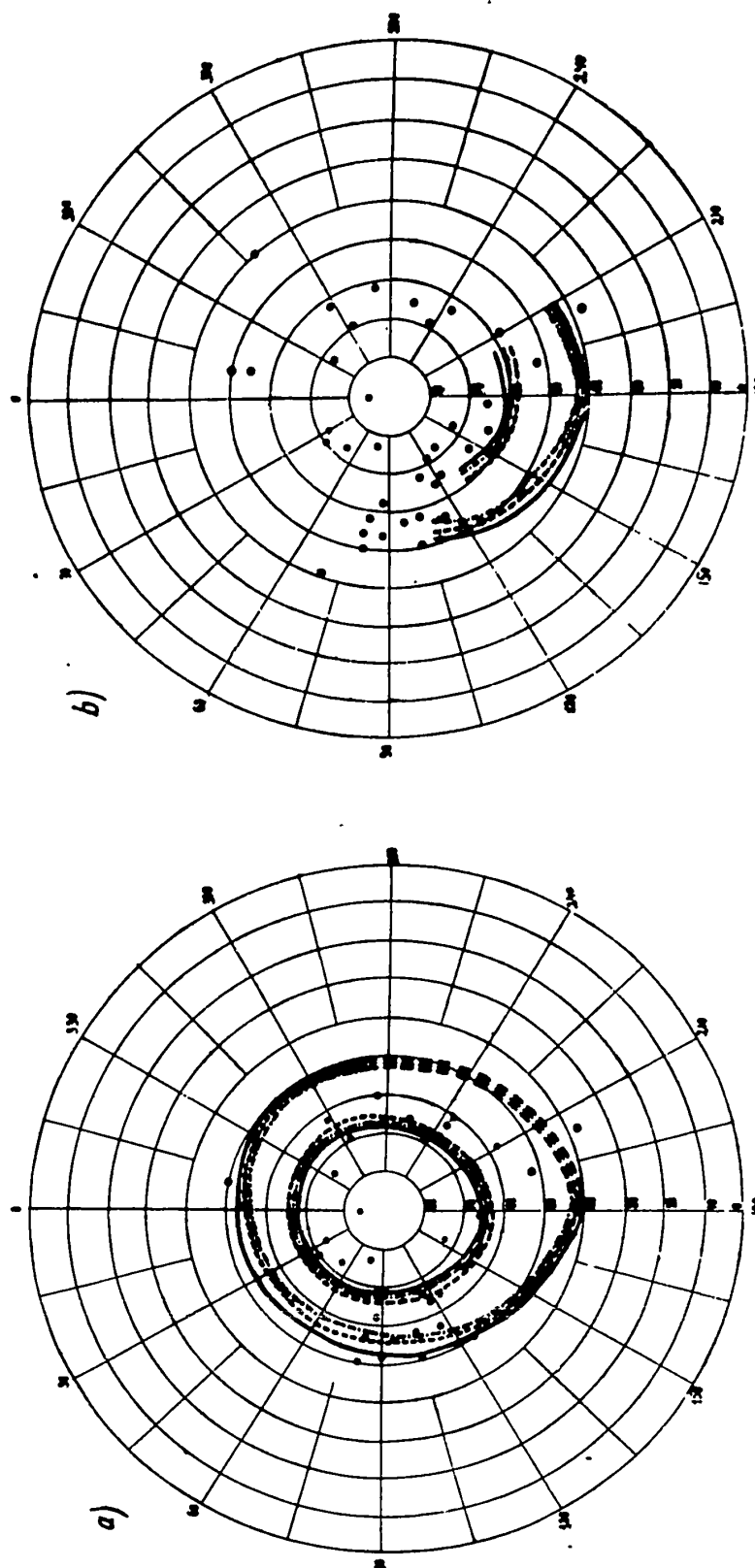


Fig.38 - Position of Auroral Zone and Lines of Centers of Middle-Latitude Eddies  
 (a - 1932 - 1933; b - 1938 - 1939; - - - Equinox, — Summer; -.-.- Winter.  
 The Points Mark the Position of the Magnetic Observatories. The Coordinate Net  
 Used is Geomagnetic)

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with the lag that is characteristic for geophysical phenomena due to corpuscular radiation. The seasonal march of  $D_{st}$  has the pronounced equinoctial maxima which likewise confirm the corpuscular nature of the phenomenon, amplitude and also have an annual march of small amplitude with extreme values at the epoch of the solstices. This second annual wave likewise may be explained within the frame of the corpuscular theory, if we bear in mind that it is not only the heliographic latitude of the earth that varies during the course of the year (Corti effect) but also the angle between the magnetic axis of the earth and the line sun-earth. As stated by Bartels, the variation of this angle during the course of the year changes the direction and magnitude of the field on which the charged particles coming from the sun impinge and, Consequently, also modifies the conditions of the course of the disturbances. The 11-year and seasonal variations of the currents of the  $S_D$  variations are much more complex. The correlation between the 11-year fluctuations of solar activity and the intensity of the  $S_D$  currents is not so close, and is different for the middle-latitude and polar currents. It would seem that the fluctuations of the  $S_D$  currents are not due only to fluctuations in the intensity of the corpuscular radiation, but also to the condition of the upper layers of the atmosphere. This latter differs in different latitudes, depends on the solar radiation of both types (photon and corpuscular), and obeys its own more complex regularities.

## CHAPTER VIII

MORPHOLOGY OF THE DISTURBED IONOSPHERE AND THE CURRENT SYSTEMS  
OF MAGNETIC STORMSSection 1. Ionospheric Disturbances

In the preceding Chapters we have described the calculation of the electric currents corresponding to the external part of the field of magnetic storms, and we have discussed the properties and peculiarities of these currents. But since we used only geomagnetic data in studying these currents, many questions still remained obscure: the distance of these currents from the surface of the earth, the actual physical conditions in the medium in which, as we postulate, the currents are located; whether the current layer can be identified with one ionospheric layer or another; and so on. We have seen in Chapter I that the discussion of these questions in the literature is only beginning. In order to give answers, though only provisional ones, to these questions, it is necessary to formulate an idea as to the variations that take place in the ionosphere during the time of magnetic disturbances. In the present Section we shall briefly set forth certain information of the morphology of ionospheric disturbances, taken from literature sources, and other data obtained as a result of the work up of the data from a number of ionospheric stations.

The first investigators of ionospheric disturbances were Bulatov, Berkner and Wells and Seaton (Bibl.3, 40) whose works give a detailed description of magnetic storms from observations at Tomsk, and in South America and Great Britain. The au-

thors noted the basic features of the behavior of the disturbed ionosphere: the lowering of the critical frequencies of the  $F_2$  layer and the increase in its heights, the appearance of a sporadic layer at the level of the E layer, the fused and scattered reflections, indicating the inhomogeneous, cloudlike structure of the ionosphere, and the increase of absorption. These features were further confirmed by a number of works of Soviet and foreign authors, and a description of them may be found in modern surveys of ionospheric physics (Bibl.1, 2). One of the latest works devoted to the description of the individual disturbances is the paper by Burkhard (Bibl. 39) on the magnetic ionospheric storm of 15 March 1948. The data of about 30 ionospheric observatories were available to Burkhard, who calculated the value for each observatory of  $\Delta = \frac{f^{0z} - f_n^{0z}}{f_n^z}$  where  $f^0$  = critical frequency of  $F_2$  layer on day of storm and  $f_n^0$  = corresponding value for a normal day. The latitudinal distribution of  $\Delta$  discloses obviously decreased values of  $f^0 F_2$  in the high latitudes and increased values in latitudes near the equator. The dispersion of values is relatively small, which forces us to accept, without doubt, the relation found. A work by Yu. D.Kalinin (Bibl.22) is also devoted to the morphology of an ionospheric disturbance. To elucidate the regularities of the behavior of the ionospheric layers, he used statistical methods common to the methods used in geomagnetism. He studied the  $D_{st}$  and  $S_D$  variations of the critical frequencies and the heights of the ionospheric layers for two ionospheric observatories, Leningrad and Tomsk. An analysis of the material showed that the parameters of the E layer remained in fact normal during the time of magnetic disturbances. This conclusion is in full agreement with the well known fact that usually, in the middle latitudes, the disturbance affects only the F region and only in the strongest storms does the disturbance penetrate down to the underlying layers and disturb their structure. In the variations of height of the  $F_2$  layer and particularly of the critical frequencies of that layer, a regular part could be detected. The  $D_{st}$  variations of  $f^0 F_2$  are characterized by an increased in-

index of  $f^oF_2$  in the first hours of a storm, followed by a decrease in the subsequent hours of the storm. The  $S_D$  variations of  $f^oF_2$  differ for the winter and summer months and are not the same at Tomsk and Leningrad. A consideration of the materials for two years for two stations is, under all circumstances, inadequate for any judgment as to the geographic incidence of the disturbed variations, or even as to how much the variations change from year to year. Nevertheless the work has shown that statistical methods are fully applicable to the study of the ionosphere of ionospheric disturbance.

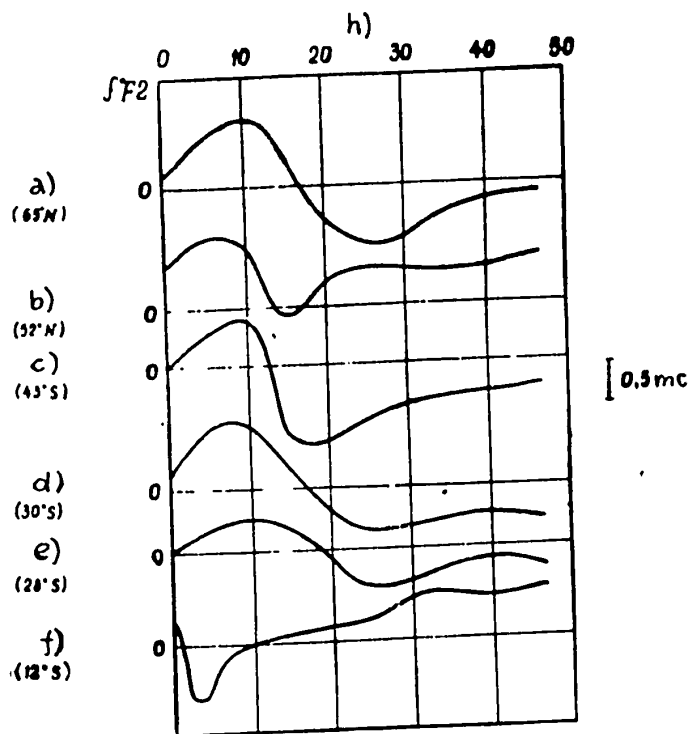


Fig.39 -  $D_{st}$  Variations of the Critical Frequencies of the  $F_2$  Layer

a) Alaska; b) Slough; c) Hobart;  
d) Watheroo; e) Brisbane; f) Huan-cayo; g) 0.5 mc; h) Hours

ment as to the geographic incidence of the disturbed variations, or even as to how much the variations change from year to year. Nevertheless the work has shown that statistical methods are fully applicable to the study of the ionosphere of ionospheric disturbance.

Analogous results have been published by Appleton and Piggott (Bibl. 36) in 1950 on the question of the correlation between magnetic and ionospheric disturbances. After working up the data on the  $F_2$  layer of a number of observatories located at different latitudes, the authors concluded that in the middle latitudes ionospheric disturbance usually

take the following course: during a few first hours of the magnetic storm an increase in the critical frequencies of the  $F_2$  layer is observed. This is the positive phase, which is replaced afterwards by the negative phase, in which the decrease of  $f^oF_2$  in absolute value considerably exceeds its increase during the first phase. The negative phase lasts considerably longer than the positive phase. The return to the normal state of the  $F_2$  layer is slow, dragging out to a few days, as

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occurs with the phase of restoration of the  $D_{st}$  variations of the magnetic field. The negative values of  $f^0F_2$  are observed during the entire magnetic storm. In the high latitudes, on the contrary, the ionospheric disturbances as a rule have only a negative phase, commencing immediately together with the magnetic disturbance. The negative disturbances of the  $f^0F_2$  of the high latitudes differ substantially from the negative phase of the middle-latitude disturbances. But it is precisely the restoration of the normal state of the  $F_2$  layer after the polar disturbance that occurs very rapidly, without a long drawn-out period of after-effect. This fact, it seems to us, is responsible for the negative disturbances in the  $F_2$  layer that accompany polar geomagnetic storms. There are indications in the literature that by now the  $D_{st}$  and  $S_D$  variations of  $f^0F_2$  have been calculated for many ionospheric observatories, but more detailed data on the results of such calculations are not available to us.\*

The papers devoted to the variation of the critical frequencies and the heights of the regular layers during the time of a disturbance have been enumerated above. In addition to works of this kind, there have also been a large number of other investigations with respect to special types of disturbances (for example sudden ionospheric disturbances due to outbursts of ultraviolet radiation), formation of additional layers at various heights during storms, the correlation of E sporadic with the degree of magnetic disturbance, the nonuniformity of the ionosphere, etc. In view of our basic object, to elucidate the ionospheric conditions of a typical magnetic storm, these studies are of less interest for us. Indeed, the existence of an  $E_s$  layer of corpuscular origin, related to and correlated with the degree of magnetic disturbance, is very probable. However, in the middle latitudes, in a large number of cases,  $E_s$  is observed with a completely quiet field, and  $E_s$  is often absent during a storm. There is therefore no reason to consider that its formation leads

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\* This question is also considered in the papers by Martin, Louis Waldo, and Appleton and Martin, published before the completion of the present work (cf. Proc. Roy. Soc. and Journ. Atm. Terr. Phys., 1952 and 1953).



to the formation of the electric currents responsible for the regular parts of the field of magnetic storms.

This applies to an even greater extent to the appearance of additional high lay-

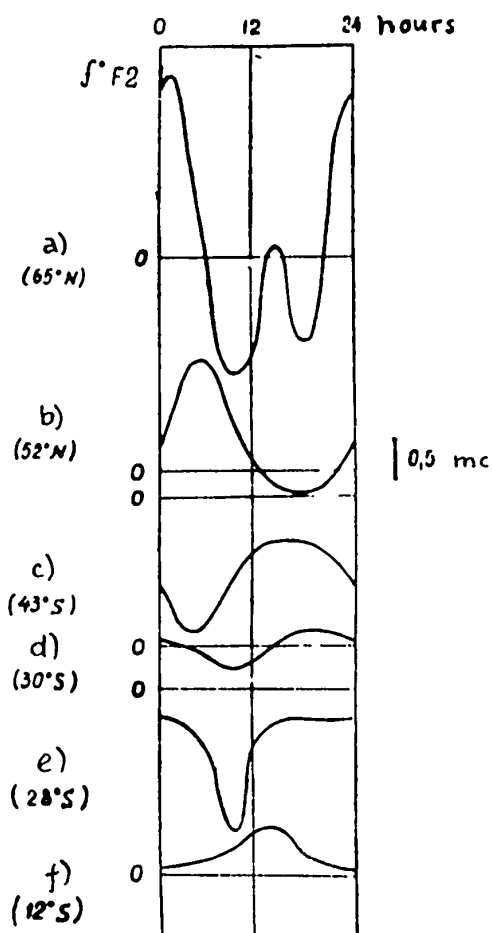


Fig.40 -  $S_D$  Variations of the Critical Frequencies of the  $F_2$  Layer

- a) Alaska; b) Slough; c) Hobart;  
d) Watheroo; e) Brisbane;  
f) Huancayo; g) Hours; h) 0.5 mc

ers during the time of a disturbance. Additional and sporadic layers at the level of the  $F_2$  layer and above it are not invariably observed during the time of magnetic disturbances, and it is not probable that they are connected with the regularly originating currents.

On analyzing similarly the other manifestations of an ionospheric disturbance, it may be concluded that the regular parts of the field of a magnetic storm are most likely to be related to such processes in the ionosphere as variations of density or circulations of large scale. Starting out from these considerations, in the present survey we have touched only on a few investigations devoted to the consideration of precisely these questions.

The additional statistical treatment of the ionospheric data performed by us leads to the following results (Figs.39 and 40):

1. The  $D_{st}$  variations of  $f^oF_2$  have a two-phase character at all latitudes: in the high and middle latitudes, the first phase is positive and the second negative. In the low and equatorial latitudes, on the contrary, the first phase is negative and the second positive. Thus the geographic distribution of the  $D_{st}$  variations of

the magnetic field and  $f^0F_2$  do not resemble each other.

2. The  $S_D$  variations of  $f^0F_2$  vary strongly, depending on the season and on the level of solar activity. Nevertheless certain regularities in the geographic distribution of  $S_D$  can be established: the amplitude of  $S_D f^0F_2$  is smallest in the equatorial regions and greatest in the polar latitudes; the time of the extreme values likewise varies with the latitude: in the low latitudes the minimum is observed in the forenoon hours, and the maximum in the afternoon hours, while in the high latitudes, on the contrary, the minimum occurs in the second half of the day and the maximum in the first half. It follows from this that the geographic distribution of  $S_D f^0F_2$  is analogous to that of the  $S_D$  variations of the magnetic elements.

3. The  $D_{st}$  and  $S_D$  variations of  $f^0F_2$  are considerably less regular than the corresponding variations of the magnetic elements.

No regular disturbed variations of the E layer are detected, either at low latitudes or in the polar regions.

## Section 2. Conductivity of the Ionospheric E and F Layers, and the Dynamo Effects in the $F_2$ Layer

As we have seen in the preceding paragraph, the density of ionization of the  $F_2$  layer undergoes variations during the time of a disturbance, depending on the storm-time ( $D_{st}$  variations) and on the time of day ( $S_D$  variations). Our task is to elucidate the question whether these variations can cause the rise of the electric currents responsible for the  $D_{st}$  and  $S_D$  variations of the geomagnetic field. In order to compare the quantitative characteristics of the ionosphere (for example the density of ionization or the velocity of motion) with the intensity and configuration of the electric currents, it is necessary to have some working hypothesis about the mechanism of excitation of these currents. The hypotheses in the literature as to the causes for the origin of the currents of magnetic disturbances may be divided into two main groups. The first of these groups includes the hypotheses related to the assumption of the deep penetration of solar corpuscles into the earth atmosphere

(to the level of the F region and lower). As we have already shown, this hypothesis has recently been confirmed by auroral spectroscopy, and thus there should be no doubt of the penetration of corpuscles down to the very lowest layers of the ionosphere in the polar latitudes. But the question as to the penetration of corpuscles into the ionosphere of the middle latitudes still remains unsolved. Eckersley (Bibl. 42), Burkhard (Bibl. 39), and a number of other authors consider it possible that the corpuscles penetrate in all latitudes, and explain, by the direct action of the corpuscles, those variations that are observed in the ionosphere and the magnetic field of the earth during the time of a disturbance.

According to Eckersley, the positive ions penetrate somewhat deeper into the atmosphere than the electrons, and the vertical electrostatic field thereby formed is the prime cause of the drift of charged particles and of the excitation of the electric currents responsible for magnetic storms. Without making it my task here to give a complete critical discussion of Eckersley's work, I may say that an electric vertical field should in my opinion prevent the further invasion of the corpuscles into the atmosphere, and thus, the process of a disturbance, as soon as it began, should at once thereafter die out, without leading to the formation of stable current systems.

According to Burkhard, the entrance of corpuscles into the ionosphere leads, in some manner (the author does not specify precisely in what manner) not to the increase of ionization but to its decrease. The corpuscles emitted by the sun during quiet periods penetrate the earth atmosphere at all latitudes and reduce the ionization of the F<sub>2</sub> layer due to ultraviolet radiation. During the time of disturbances, the parameters of the particles vary in such a way that the particles are collected toward the polar regions of the earth, without reaching the low latitudes. Accordingly, there is a particularly strong decrease in the density of ionization in the high latitudes, while in the low latitude there is an increase, connected with the disappearance of the negative corpuscular effect. We have cited Burkhard's reason-

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ing in order to show to what absurd conclusions the speculative idea of the deionizing action of the corpuscular stream, developed without any connection with experimental data, can lead. Not only is the course of the arguments of Eckersley and Burkhard erroneous, in our opinion, but the very penetration of particles into the lower latitudes would appear to be contradicted by a number of facts. First, the geographic distribution of the aurora is such that, at relatively low latitudes, ( $\phi = 30 - 40^\circ$ ), it is observed only during exceptionally strong magnetic storms, while the ordinary moderate and great magnetic storms are accompanied by a shift of the isochasms by only  $5 - 6^\circ$  toward lower latitudes, from their mean position ( $\phi_0 = 67^\circ$ ). The calculation of the paths of the particles in the magnetic field and the determination of the zone of their penetration into the ionosphere that have been made by a number of authors, with various objects in view (Störmer, Bugoslavskiy, Vallarta, Alfven, Martin, and others) are likewise all in agreement that the approach of particles to the earth in the low latitudes is impossible if the velocity of the particles is less than the velocity of light (for instance about 1000 km/sec). It goes without saying that particularly great active formations on the solar surface emit corpuscles at high velocities (about 3000 km/sec and perhaps even higher) which are little deflected by the magnetic field, and produce the aurora in the middle latitudes, while the intensifying the ionization in the high layers of the ionosphere or the (more energetic) lower layers of the ionosphere (cf. work of N.V. Mednikova (Bibl. 24)). But such powerful processes are relatively rare, and consequently, we should not take them as a basis for discussing the possible mechanism of excitation of the electric currents of the regular variations, flowing around the earth during moderate and small magnetic storms.

The following argument against the approach of the corpuscles to the earth surface is provided by the morphology of the magnetic disturbances. The great but smooth deviations from the normal values in the march of the magnetic elements, and the absence of a local character in the course of storms in the equatorial latitudes,

all speak for the view that the fluctuations of the magnetic field are due to stable current systems encompassing the earth as a whole, which are not disturbed by the invasion of streams of charged particles.

It follows from this that it is more advisable to assume that the middle-latitude parts of the currents of the magnetic variations are excited in the ionosphere, if they can be referred to the height of the ionosphere at all, without direct entrance of additional charges into the ionospheric layers.\* The authors of the works placed by us in the second group share this viewpoint. In Chapter I we have already mentioned the investigators (Yu.D.Kalinin, S.M.Matsushita, Khiroiyama) who have attempted to explain the currents of magnetic storms by a dynamo effect in the ionosphere. In addition, the thought has been expressed that with the existence of an external extra-ionospheric primary field varying with time, the currents in the ionosphere would be excited owing to electromagnetic induction, and would make their contribution to the observed disturbance field. These thoughts have been developed in the paper by Ashour and Price (Bibl.37), and in certain papers by Sugiura (Bibl.55). But the dynamo and induction effects are not the only methods for the excitation of currents. It is well known that the excitation of currents in an ionized gas by the combined action of two fields of force on the particles (magnetic and gravitational fields, or magnetic and electric fields) is also possible. The current so excited (drift current) has been used to explain the  $S_q$  variations and the regular field of the sun. A number of considerations, which we shall present below, compels us to consider the drift also as a possible cause of the formation of the currents of magnetic disturbances. It is to the discussion of the dynamo, drift and induction me-

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\* The literature sometimes gives as an argument for the penetration of corpuscles into the ionosphere the so-called "geomagnetic effects" in the  $F_2$  layer (the dependence of ionization density on the geomagnetic latitude, etc). But it would appear to be more plausible to explain these effects by the redistribution of the charges already in the layer under the action of the earth magnetic field.

mechanisms of excitation of ionospheric currents that this and the following Sections of the present will be devoted.

The question of the excitation of electric currents in plasma and of the evaluation of its conductivity has been discussed with great vigor in the literature of recent years. The works of Pedersen, Tamm, Cowling et al (Bibl.23) show that the value of the conductivity of an ionized gas depends substantially on the magnitude and direction of the magnetic and electric fields acting on the particles, on the length of the free path, and on the parameters of the particles; the motion of the particles in the plasma will be completely different from that in the case of a rarefied gas, the interaction between whose particles may be neglected, and which has been considered in their time by Störmer and Chapman. In the works of Tamm and Cowling, the conductivity of an ionized gas is considered specially in its application to the earth atmosphere. Tamm assumes the ionosphere to be completely ionized and gives approximate expressions for the conductivity, one of which expressions is true for regions of short free paths and the other for regions of long paths. Cowling considers the ionosphere as a ternary gas composed of electrons, positive ions, and neutral molecules, and obtains more general expressions for its conductivity. It is not hard however, to show that the conclusions of Tamm and Cowling do not contradict each other. If the charged particles of the plasma are under the action of a magnetic field ( $\vec{H}$ ), an electric field ( $\vec{E}$ ) and a gravitational field (acceleration of gravity  $\vec{g}$ ) and, is also undergoing motion of translation under the action of certain other forces, at the velocity  $\vec{w}$ , then according to Tamm, the density of the current formed by the translation of particles of one kind will be:

$$j = eN \left\{ \vec{w} + \frac{4\lambda V_m}{3\sqrt{2\pi kT}} \left( \vec{g} + \frac{e}{m} \{ \vec{E} + [\vec{w}\vec{H}] \} - \frac{kT}{m} \frac{\nabla N}{N} - \frac{k}{2m} \nabla T \right) \right\} \quad r \gg \lambda, \quad (1)$$

$$j_{\perp} = eN\vec{w} + \frac{1}{H^2} [Nm\vec{g} + Ne(\vec{E} + [\vec{w}\vec{H}]) - \text{grad}(kTN)] \vec{H} \quad r \ll \lambda. \quad (2)$$

Here the density of the given gas ( $N$ ) and the temperature ( $T$ ) are not assumed to be uniform,  $r$  is the radius of vortex motion of the particles about the lines of

force of the magnetic field, and  $\lambda$  = free path of the particles. The component of density of the current parallel to  $\vec{H}$  in the field of the long free paths ( $r \ll \lambda$ ) is likewise described by eq.(1). If we have a binary gas ( $n_+ \simeq n_- = N$ ), then, from eq.(1), neglecting the temperature gradient and density gradient, we get

$$j_{\perp} = \sigma_0 (\vec{E} + [\vec{w}\vec{H}]) \quad r \gg \lambda, \quad (3)$$

where

$$\sigma_0 = \frac{2e^2 N}{3 \sqrt{2} \pi k T} \left( \frac{\lambda_+}{v_{m_+}} + \frac{\lambda_-}{v_{m_-}} \right) = \frac{Ne^2 \lambda_+}{m_+ v_+} + \frac{Ne^2 \lambda_-}{m_- v_-} = \frac{Ne^2}{m_+ v_+} + \frac{Ne^2}{m_- v_-} \quad (4)$$

is the conductivity of the gas in the absence of a magnetic field. Here  $v$  = kinetic velocity of molecules, and  $\nu$  = number of collisions per second ( $\lambda \nu = v$ ). The current described by eqs.(3) and (4) is the dynamo current used by Schuster and Chapman to explain the  $S_q$  variations.

According to Cowling, in an ionized gas under the action of the crossed, mutually perpendicular electric and magnetic fields  $\vec{E}'$  and  $\vec{H}$ , an electric current of density

$$j = \sigma' \vec{E}' + \sigma'' \frac{[\vec{H}\vec{E}']}{H}, \quad (5)$$

is excited, where  $\vec{E}'$  must be understood as meaning not only the proper electrostatic field  $\vec{E}$  of some external origin, but also the electric field arising as a result of the motion of the mass of gas in the field  $\vec{H}$  at velocity  $\vec{w}$ , that is

$$\vec{E}' = \vec{E} + [\vec{w}\vec{H}], \quad (6)$$

and, consequently,

$$j = \sigma' (\vec{E} + [\vec{w}\vec{H}]) + \sigma'' \left( \frac{[\vec{H}\vec{E}]}{H} + \vec{H}\vec{w} \right). \quad (7)$$

The first term in the expression for  $j$  denotes the Schuster-Chapman dynamo effect, while the second indicates the formation of a current in the direction of the velocity of motion of the gas  $\vec{w}$ , or in a direction perpendicular to the crossed mag-

netic and electric fields. The expression  $\sigma^I$ , according to Cowling, is equal to

$$\sigma^I = \frac{\sigma_0}{1 + \omega^2 \tau^2}, \quad (8)$$

where  $\tau$  = time of free path ( $\tau = v^{-1}$ ), and  $\omega$  = angular velocity of procession of the particle ( $\omega = \frac{eH}{m}$ ,  $v_{\perp} = r\omega$ ). It is identical with the expression for the conductivity of a gas in a direction perpendicular to the magnetic field, introduced into the literature by Pedersen:

$$\sigma_{\perp} = \frac{\sigma_0 v^2}{v^2 + \omega^2}. \quad (8')$$

For a region of short free paths ( $\frac{r}{\lambda} = \frac{v_{\perp}}{\omega} \ll 1$ ) neglecting the value of  $\omega^2$ , we have  $\sigma_{\perp} \approx \sigma_0$ , as is put in the Tamm equations. The conductivity  $\sigma^{II}$  is determined by the expression

$$\sigma^{II} = \frac{\sigma_0 \omega \tau}{1 + \omega^2 \tau^2}. \quad (9)$$

In the field of short free paths,

$$\sigma^{II} \approx \sigma_0 \frac{\omega}{v} \text{ and } \sigma^{II} \ll \sigma^I. \quad (10)$$

Consequently, Tamm did not make a large error by neglecting the current in the direction of  $\vec{w}$  (or perpendicular to  $\vec{H}$  and  $\vec{E}$ ) for this region. In the region of long free paths ( $v/\omega \ll 1$ )

$$\left. \begin{aligned} \sigma^I &\approx \sigma_0 \frac{v^2}{\omega^2}, \quad \sigma^{II} \approx \sigma_0 \frac{v}{\omega} \\ \sigma^I &\ll \sigma^{II} \ll \sigma_0 \end{aligned} \right\} \quad (11)$$

and, consequently, the current in the direction of  $\vec{w}$  is considerably greater than the current in the direction of  $\vec{E}$ . It is therefore entirely natural that in the approximate equation of Tamm, out of the terms describing the dependence of the current on  $\vec{H}$  and  $\vec{E}$ , only the term  $\frac{Ne}{H^2} [(\vec{E} + [\vec{w}|\vec{H}]) \cdot \vec{H}]$  should be retained.

The scalar Coefficient  $\frac{Ne}{H^2}$  is the same as the coefficient  $\frac{\sigma^{II}}{H}$  in eq.(5).

In fact,

$$\frac{\sigma^{II}}{H} \approx \frac{Ne^2}{m v} \frac{v}{H \omega} = \frac{Ne^2}{m H} \frac{m}{e H} = \frac{Ne}{H^2}.$$



Thus eqs.(1) and (2) of Tamm and eq.(5) of Cowling do not contradict each other.

Let us see now to what extent the identification of this or that ionospheric layer with the region of long or short free paths is correct. The angular velocity of motion of a particle ( $\omega = \frac{eH}{m}$ ) depends only on the parameters of this particle and the magnitude of the magnetic field. Neglecting the variation of the earth magnetic field with height for the region of the ionosphere, we find that for all ionospheric layers the velocity of an electron  $\omega_e = 5 \times 10^6$  and the velocity of an ionized oxygen molecule  $\omega_i = 10^2$ . The number of collisions in the ionosphere has been repeatedly determined from the experimental data on the absorption (Bibl.1), and has also been calculated by the formulas (Bibl.11):

$$\left. \begin{aligned} v_m^e &= \frac{4\pi a^2}{3} N_m \bar{v} \\ v_i^e &= \frac{\pi e^4}{(kT)^2} N_i \bar{v} \ln \left( 0,37 \frac{kT}{e^2 N_+^{1/2}} \right) \\ v_m^i &= \frac{16\pi}{3} a^2 N_m \bar{v} \\ v_i^i &= \frac{\pi}{(kT)^2} \bar{v}^2 N_i \ln \left( 0,37 \frac{kT}{e^2 N_+^{1/2}} \right) \end{aligned} \right\} \quad (12)$$

Here  $v_m^e$  denotes the frequency of collision of electrons with neutral molecules,  $v_i^e$  with positive ions,  $v_m^i$  the number of collisions of ions with molecules, and  $v_i^i$  of ions with ions;  $a$  = effective diameter of a particle (for air the value  $\pi a^2 = 7 \times 10^{-16}$  is usually taken);  $\bar{v}$  = kinetic velocity of the particles. The collisions of electron with electron and ion with ion with the same sign may be neglected in evaluating the total number of collisions, and therefore the term  $v_i^i$  will have a substantial value only in those regions where there is a sufficient number of both positive and negative ions. Radio methods enable us to determine directly only the effective ionization density  $N_{ef}$ , while the actual number of charged particles  $N = N_{ef} \frac{m_i}{m_e}$  remains unknown. But a number of supplementary considerations (the magneto-ionic splitting of a deflected radio signal, etc) allow us to judge the ratio between electrons and negative ions in the layer  $1 = \frac{n_-}{n_e}$ . There is no doubt today

that the conductivity of the  $F_2$  and  $F_1$  layers is due primarily to electrons ( $n_- < n_e$ ). It is also probable that, in the E layer as well, the conductivity is determined mainly by the electrons, since in the D layer the number of free electrons is in all probability small. The first columns of Table 27 give the values of  $N_{ef}$  adopted in the modern literature for all layers, together with the possible values of  $l$ ; the following columns give the values of  $n_e$ ,  $n_-$ ,  $n_+$  and the most probable values of  $n_m$ , all calculated on the basis of  $N_{ef}$  and  $l$ . Columns 7 - 12 give the values of  $v_e \dots v_i^i$  calculated by eq.(12), as well as the total number of collisions for particles of a given kind,  $v^e$  or  $v^i$ . Column 13 gives the value of  $v$  determined from experimental data. It will be seen from the tables that in the D layer, the total number of collisions is determined by the collision of charged particles with neutral particles, and none of the three assumptions as to the value of  $l$  contradicts the order of the observed  $v_{ef}$ . For the E and  $F_1$  layers, as will be seen from the table, the data on  $v_{ef}$  agree only with the assumption  $l = 0$ , that is, with absence of any substantial number of negative ions. The value of  $n_m$  for the  $F_2$  layer is determined only indirectly, namely on the basis of the number of collisions. For an ionization density of the order of  $10^6$  ions/cm<sup>3</sup> and the assumption that ionization in the layer is due to electrons and positive ions ( $l = 0$ ), this number of collisions corresponds to the effective number of collisions between electrons and ions (cf. Table 4, p.97, of the Ginzburg monograph Bibl.11). About the same number of collisions takes place for electrons and neutral molecules, if the molecular density  $n_m \approx 10^{11}$ . From this it is concluded that the number of neutral molecules in the  $F_2$  layer does not exceed  $10^{11}$  molecules/cm<sup>3</sup>. It is true that the literature also contains hypotheses of the complete ionization of the  $F_2$  layer, particularly in the daytime.

The data of Table 27 show that the D layer is a region of short paths of both ions and electrons, that the E layer is a region of short paths for the ions and long ones for the electrons, and that the  $F_1$  and  $F_2$  layers are a region of long free paths for particles of both kinds. The conductivity  $\sigma^I$  which determines the dynamo

effect has the smallest value in the D layer and rises for the higher layers just as the conductivity  $\sigma^{II}$  does. In the D layer  $\sigma^{II} \ll \sigma^I$ , while in the overlying layers  $\sigma^I$  and  $\sigma^{II}$  are of comparable value, and  $\sigma^{II}$  is even somewhat greater than  $\sigma^I$ . Evaluating the integral conductivity of the R region, Cowling shows that it is possible that the conductivity of this region is considerably less, since the presence of current in the magnetic field leads to the excitation of ponderomotive forces [wH] which retard the further motion of the charged particles, that is, it is as though they decreased the value of the conductivity. Thus the current that arises should be damped after the time  $-\frac{\rho}{\sigma^{II}}$  (where  $\rho$  is the density of the mass), which amounts to 45 days for the E layer,  $3\frac{1}{2}$  hours for the  $F_1$  layer and 20 min for the  $F_2$  layer. The damping of the currents does not occur if the particles are under the constant action of a force, that is, if the motion of the particles is accelerated, or if under the action of the magnetic field a polarization of the gas occurs, neutralizing the retarding force [wH], or if currents screening the internal parts of the volume from the action of the magnetic field are induced on the surface of the moving mass of gas.

In accordance with the above, Cowling considers that the conductivity\* of the  $F_2$  layer in reality does not exceed  $\sigma^I = e \times 10^{-9}$ , and that the conductivity of the E layer is practically constant (for instance,  $\sigma^I = 10^{-7}$  for  $l = 25$ ). Cowling concludes from this that the total conductivity of the entire ionosphere must be within the range from  $10^{-7}$  to  $10^{-8}$  and must be due primarily to the charged particles of the E layer.

The values of  $\int \sigma^I dh$  given in Table 27 force us to apply the following corrections. Since the more probable value of the conductivity for the E layer would seem to be  $\int \sigma^I dh = 10^{-9}$ , then the integral conductivity of the entire ionosphere, causing the dynamo effect, is probably not more than  $10^{-9}$ , while both layers of E and F

\* Under the condition that the motion takes place under the action of tidal forces. From what has been said it follows that the conductivity differs for different kinds of motion of the gas.

possibly yield equal contributions to the value of the conductivity. The conclusions drawn as to the conductivity of the E and F layers are based on the values of the ionization density for a normal day. On a disturbed day, however, (cf. Chapter II), the order of magnitude of the ionization density remains the same and, consequently, the order of magnitude of the conductivity should likewise not differ markedly from that of a normal day.\*

It follows from Chapter III and VII of the present work that the  $D_{st}$  variations of the geomagnetic field may cause ionospheric currents flowing westward along the parallels of latitude and having a density of about  $3 \times 10^{-9}$  CGS in a year of moderate solar activity. If these currents are attributed to the action of the dynamo effect, then it would be necessary to assume the presence in the ionosphere of a stable wind of meridional direction with a speed of the order of

$$w = \frac{3 \times 10^{-5}}{0.3 \times 10^{-9}} = 10^5 \text{ cm/sec.} = 1 \text{ km/sec.}$$

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\* After the present work had been completed, I learned of the paper by J.K. Csada, Acta Phys. Acad. Sc. Hungaricae I (3) 235 - 246, 1952, which considers the variation of the electromagnetic parameters of a gas under the influence of turbulent processes. It is shown that the local magnetic field formed in presence of turbulence lead to an increase of magnetic permeability and to a decrease of the electric conductivity of the gas. The turbulent processes occurring in stellar atmospheres may, according to Csada's calculations, reduce the conductivity of the atmospheric gas by several orders of magnitude. During magneto-ionospheric disturbances, it is generally known that turbulent processes also develop in the ionosphere. However, as shown by rough preliminary calculations, owing to the low temperature and the low degree of ionization of the ionosphere of the earth, the turbulent processes in it cannot lead to such great changes of the electromagnetic parameters as occur in stellar atmospheres.

A number of experiments in recent years (cf. Bibl. 2, 3, and 27) indicate the existence of horizontal movements of the clouds in the ionosphere in both its lower layers and the  $F_2$  layers. In most cases, however, the authors give lower values for the velocities. Thus, according to the data of Australian stations, a systematic displacement of clouds in the  $F_2$  layer, having a meridional direction and a velocity of 80 - 400 m/sec, has been found. Observations at Slough have shown displacement from time to time, of the  $F_2$  layer as a whole (or of parts of it) at velocities of 120 m/sec in east-west direction. Velocities of the order of one kilometer a second are noted considerably less often. Thus, for example, from the observations in Australia the usual rates of motion of the clouds of the  $F_2$  layer (of the order of 400 - 500 m/sec) increase, sometimes to 1800 m/sec, during magnetic storms. It would thus appear that the dynamo-excitation of the  $D_{st}$  currents requires somewhat higher rates of motion in the ionosphere than those usually observed. A still more weighty argument against the dynamo hypothesis of the  $D_{st}$  variations is the configuration of the current system, which is a latitudinal distribution of the current lines from eastward during the first phase of the storm and westward during the second stage. To explain such a form it would be necessary for the  $D_{st}$  variations of conductivity (and, consequently, of the critical frequency of the  $F_2$  layer) to be of a very regular character, which would be the same over the entire earth, with an increase of  $f^0_{F_2}$  in the first phase of a storm and a decrease in the second phase. However, as will be seen from Chapter II of the present work, the  $D_{st}$  variations of  $f^0_{F_2}$  only have such a form in the middle latitudes, while in the low latitudes, their form, on the contrary, is negative in the first phase of the storm and positive in the second phase. The irregularity and instability of the  $D_{st}$  variations of  $f^0_{F_2}$ , which is particularly striking on a comparison with the  $D_{st}$  variations of the magnetic field, compels the definitive recognition of the impossibility of explaining the latter by the dynamo currents flowing in the  $F_2$  layer of the ionosphere. These same considerations as to the dissimilarity of the  $D_{st}$  variations of  $f^0_{F_2}$  and of the magnetic

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Table 27

| Layer | $\mu$           | $l$ | $n_e$             | $n_{-}$              | $n_{+}$              | $n_{m-}$            | $\nu$             | $y_m$         |
|-------|-----------------|-----|-------------------|----------------------|----------------------|---------------------|-------------------|---------------|
| D     | $10^6$          | 1   | $0.5 \times 10^8$ | $2.5 \times 10^7$    | $2 \times 10^7$      | —                   | $1.5 \times 10^6$ | $10^7 - 10^8$ |
|       | —               | 20  | 20                | $5 \times 10^7$      | $5 \times 10^7$      | $10^{10} - 10^{11}$ | —                 | $10^7 - 10^8$ |
|       | —               | ∞   | —                 | $5 \times 10^7$      | $5 \times 10^7$      | —                   | —                 | $10^7 - 10^8$ |
| E     | $3 \times 10^6$ | 0   | $3 \times 10^8$   | —                    | $3 \times 10^8$      | —                   | $2 \times 10^3$   | $10^8$        |
|       | —               | 1   | $1.5 \times 10^8$ | $7.5 \times 10^9$    | $7.5 \times 10^9$    | $10^{11}$           | $2 \times 10^7$   | $10^8$        |
|       | —               | 50  | $6 \times 10^8$   | $1.5 \times 10^{10}$ | $1.5 \times 10^{10}$ | —                   | —                 | $10^8$        |
| F1    | $5 \times 10^6$ | 0   | $5 \times 10^8$   | —                    | $5 \times 10^8$      | $10^{11}$           | $2 \times 10^8$   | $10^8$        |
|       | —               | 1   | $2.5 \times 10^8$ | $1.2 \times 10^{10}$ | $1.2 \times 10^{10}$ | —                   | $4 \times 10^7$   | $10^8$        |
| F2    | $2 \times 10^6$ | 0   | $2 \times 10^8$   | —                    | $2 \times 10^8$      | $10^{10} - 10^{11}$ | —                 | $10^8 - 10^9$ |

| Layer | $\sigma_{\alpha}$     | $\sigma_e^I$          | $\sigma_e^{II}$       | $\sigma_i^I$          | $\sigma_i^{II}$       | $\frac{3}{2} y_m$ |
|-------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-------------------|
| D     | $10^{-17}$            | $1.2 \times 10^{-18}$ | $6 \times 10^{-20}$   | $10^{-17}$            | $10^{-22}$            | $2 \times 10^8$   |
|       | $2.5 \times 10^{-16}$ | $5 \times 10^{-20}$   | $2.5 \times 10^{-21}$ | $2.5 \times 10^{-16}$ | $2.5 \times 10^{-21}$ | $2 \times 10^8$   |
|       | $2.5 \times 10^{-16}$ | —                     | —                     | $2.5 \times 10^{-16}$ | $2.5 \times 10^{-21}$ | $2 \times 10^8$   |
| E     | $2.5 \times 10^{-16}$ | $3 \times 10^{-15}$   | $1.5 \times 10^{-14}$ | $2.5 \times 10^{-16}$ | $2.5 \times 10^{-18}$ | $2 \times 10^8$   |
|       | —                     | —                     | —                     | —                     | —                     | —                 |
|       | —                     | —                     | —                     | —                     | —                     | —                 |
| F1    | $4 \times 10^{-14}$   | $10^{-17}$            | $2.4 \times 10^{-14}$ | $1.5 \times 10^{-14}$ | $2 \times 10^{-14}$   | $7.5 \times 10^8$ |
|       | —                     | —                     | —                     | —                     | —                     | $7.5 \times 10^8$ |
| F2    | $3 \times 10^{-13}$   | $2 \times 10^{-16}$   | $1 \times 10^{-13}$   | $3 \times 10^{-14}$   | $10^{-13}$            | $1.5 \times 10^7$ |

Note. In this Table the values of  $N_{ef}$ ,  $n$ , and  $\nu$  are given in  $1/\text{cm}^3$ ,  $\sigma$  in the CGS-system, and  $y_m$  in cm, while  $l$  and  $\frac{\omega}{v}$  are dimensionless quantities.

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Table 27 (cont.)

| $v_e$           | $v_i$           | $v_m$           | $v_l$  | $v_d$       | $\frac{v_e}{v_i}$  | $\frac{v_e}{v_l}$ | $\sigma_e$            |
|-----------------|-----------------|-----------------|--------|-------------|--------------------|-------------------|-----------------------|
| $10^8$          | $2 \times 10^8$ | $6 \times 10^8$ | $10^7$ | —           | $5 \times 10^{-2}$ | $10^{-5}$         | $1.3 \times 10^{-12}$ |
| $10^8$          | $5 \times 10^8$ | $6 \times 10^8$ | $10^7$ | $10^8-10^7$ | $5 \times 10^{-2}$ | $10^{-5}$         | $3 \times 10^{-10}$   |
| $10^8$          | $5 \times 10^8$ | $6 \times 10^8$ | $10^7$ | —           | $5 \times 10^{-2}$ | $10^{-5}$         | —                     |
| $10^8$          | —               | $6 \times 10^8$ | $10^4$ | —           | 50                 | $10^{-2}$         | $7.5 \times 10^{-13}$ |
| $10^7$          | $5 \times 10^8$ | $6 \times 10^8$ | —      | $10^8$      | —                  | —                 | —                     |
| $5 \times 10^7$ | $1 \times 10^8$ | $6 \times 10^8$ | —      | —           | —                  | —                 | —                     |
| $2 \times 10^3$ | —               | 60              | 60     | $10^4$      | $4 \times 10^4$    | 1.7               | $6.2 \times 10^{-11}$ |
| $4 \times 10^7$ | $7 \times 10^8$ | 60              | —      | —           | —                  | —                 | —                     |
| $1 \times 10^4$ | —               | 6-60            | 30     | $10^3-10^4$ | $5 \times 10^8$    | 3.3               | $5 \times 10^{-11}$   |

| $\int \sigma_e^I dh$  | $\int \sigma_e^{II} dh$ | $\int \sigma_l^I dh$  | $\int \sigma_l^{II} dh$ | $\int \sigma^I dh$  | $\int \sigma^{II} dh$ |
|-----------------------|-------------------------|-----------------------|-------------------------|---------------------|-----------------------|
| $2.4 \times 10^{-12}$ | $1.2 \times 10^{-13}$   | $2.0 \times 10^{-11}$ | $2 \times 10^{-16}$     | $2 \times 10^{-11}$ | $1 \times 10^{-13}$   |
| $1.0 \times 10^{-13}$ | $5 \times 10^{-15}$     | $5 \times 10^{-10}$   | $5 \times 10^{-15}$     | $5 \times 10^{-10}$ | $5 \times 10^{-15}$   |
| —                     | —                       | $5 \times 10^{-10}$   | $5 \times 10^{-15}$     | $5 \times 10^{-10}$ | $5 \times 10^{-15}$   |
| $6.0 \times 10^{-9}$  | $3 \times 10^{-8}$      | $5 \times 10^{-10}$   | $5 \times 10^{-12}$     | $6 \times 10^{-9}$  | $3 \times 10^{-8}$    |
| —                     | —                       | —                     | —                       | —                   | —                     |
| —                     | —                       | —                     | —                       | —                   | —                     |
| $7.5 \times 10^{-11}$ | $2 \times 10^{-7}$      | $1 \times 10^{-7}$    | $1.5 \times 10^{-7}$    | $10^{-7}$           | $10^{-7}$             |
| —                     | —                       | —                     | —                       | —                   | —                     |
| $3 \times 10^{-9}$    | $1.5 \times 10^{-6}$    | $4.5 \times 10^{-7}$  | $1.5 \times 10^{-6}$    | $4 \times 10^{-7}$  | $10^{-6}$             |

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field also force us to abandon other possible mechanisms of excitation of the ionospheric currents, although these, too, may not lead to any quantitative contradictions with respect to conductivity or motion in the ionosphere.

Let us consider in greater detail the possibility of current originating in the ionosphere in the direction of motion of the gaseous masses. It follows from Table 27 that the conductivity  $\int \sigma^{II} dh$  of the  $F_2$  layer in the direction of motion is one order of magnitude greater than the conductivity  $\int \sigma^I dh$ . Moreover, in the case of the formation of a current in the direction of motion of the retarding mechanical force  $\sigma^{II} H^2 w$ , which arises as a result of the motion of charged particles in a direction transverse to the magnetic field, would cause not a decrease in conductivity as with Cowling's examination of the dynamo effect, but the excitation of a Hall current of perpendicular direction. Thus the value of the conductivity  $\int \sigma^{II} dh$  in the  $F_2$  layer would hardly be much less than  $10^{-9}$ , and, consequently, if there are any displacements of ionized masses or winds in the layer, they would lead to the excitation of currents of relatively high intensity in the direction of these motions. It follows from eq.(7) that to explain the  $D_{st}$  variations, very low velocities would be sufficient:

$$w = \frac{3 \times 10^{-5}}{0.3 \times 10^{-6}} \cong 1 \text{ m/cek.}$$

The presence of such small motions in a latitudinal direction would appear not to be in contradiction with the empirical data. Nevertheless, as we have already pointed out, it would hardly be possible to explain the origin of the  $D_{st}$  currents in this way, since the fluctuations in the ionization density of the  $F_2$  layer during storms does not satisfy the necessary requirements.

The density of the drift current produced by the combined action of a magnetic field and, for instance, of the gravitational field, on the charges, in exactly the same way should be proportional to the ionization density (for further details see below), and, consequently, the drift currents which have been repeatedly used by in-

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investigators to explain the disturbances (Hulburt, Eckersley) are likewise unable to explain the regular character of the  $D_{st}$  variations of the magnetic field. Thus a consideration of the  $D_{st}$  variations of the ionospheric parameters and a survey of the possible mechanisms of excitation of currents forces us to consider that the most plausible explanation of the  $D_{st}$  variations of the magnetic field would be an extra-ionospheric ring current. The great radius of the ring by comparison with the ionosphere ( $3 - 4 R$  according to our calculations) well explains the regularity and the absence of local anomalies in the  $D_{st}$  variations, which are very difficult to explain if the distance between the earth and the current-carrying layer is assumed to be short.

### Section 3. Explanation of the $S_D$ Variations of the Magnetic Field by Drift Currents

In the brief survey of the literature presented in Chapter I we stated that the  $S_D$  variations of the magnetic field might be explained either by means of an extra-ionospheric ring, assuming it to be elliptic, or by the aid of ionospheric current systems. Most of the arguments, however, are in favor of the ionospheric system. The  $S_D$  variations of the parameters of the  $F_2$  layer which we have just described likewise do not contradict the attribution of the  $S_D$  currents to the height of the  $F_2$  layer. The fundamental facts supporting this point of view, it seems to us, may be considered to be the similarity of the geographic distribution and of the 11-year fluctuations of the  $S_D$  variations of the magnetic field and of the ionization density of the  $F_2$  layer. From the example of the  $D_{st}$  variations that we have discussed we have seen that the quantitative relations between the current density necessary to explain the magnetic disturbances and the possible values of the conductivity, and these relations between that current density and the possible motions in the ionosphere, lead to promising results. The current density of the  $S_D$  variations in the temperate latitudes (between  $\phi = \pm 50^\circ$ ) amounts, in years of moderate magnetic activity, to a few units of  $10^{-4}$  amp or of  $10^{-5}$  CGS. At latitudes  $\phi = 50 - 67^\circ$ , the

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current density is 3 to 4 times as great, that is, it reaches  $10^{-4}$  CGS. Thus, to excite currents of the necessary density in the  $F_2$  layer, the existence of systematic displacements of masses at a velocity of several meters a second would be sufficient, the direction of the current coinciding, on the whole, with the direction of the wind. In the light of experiments disclosing the motion of ionized gases at velocities of tens and hundreds of meters a second, the existence of such small velocities is very possible. Moreover it seems to us that arguments may be adduced according to which the formation of such storms during magnetic storms would be very plausible from the theoretical point of view as well. These arguments are as follows. Since we have recognized that the formation of an equatorial ring current is the most probable explanation of the  $D_{st}$  variations, the influence of the field of this ring current on the electromagnetic processes in the ionosphere must be examined. We have already mentioned one possible effect of the ring current, the electromagnetic induction of currents in the ionosphere under the action of the alternating field of the ring. As we shall show below (Section 3), this effect could hardly be of major importance for the formation of the  $S_D$  currents. In this Section we shall turn to a different type of the action of the ring on the ionosphere. Since the ionosphere participates in the diurnal rotation of the earth about its axis, while the field of the equatorial ring may be considered in sun-bound coordinates, as constant or slowly varying, it follows that it is necessary to consider the problem of the rotation of a conducting spherical layer in a quasi-constant field  $H$ . Let us simplify our problem by considering, at first, the rotation in the magnetic field  $H$  of an individual charged particle of mass and charge  $e$ . If this particle is bound to the earth (of mass  $M$ ) by the forces of gravitational attraction, then its motion, in fixed coordinates, will be described by the equation

$$\frac{kmM}{R^2} + eR\omega H = mw_n, \quad (13)$$

where  $\frac{kmM}{R^2}$  = force of gravitation;  $mw_n$  = centripetal force;  $eR\omega H$  = Lorentz force

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acting on a charge moving at velocity  $\omega R$  in the field  $H$ ; and  $\omega$  = initial velocity of rotation. On introducing a new system of coordinates rotating at velocity  $\Delta\omega$ , it is easy to show (Bibl.33) that the motion of the charge in the new system will be as follows:

$$m\omega_n = \frac{mkM}{R^2} + eR\omega H + 2mR\omega\Delta\omega + mR(\Delta\omega)^2. \quad (14)$$

If  $\Delta\omega$  is selected such that

$$eR\omega H + 2mR\omega\Delta\omega + mR(\Delta\omega)^2 = 0, \quad (15)$$

then it will be found that the charge will continue to move along its orbit, but with a changed velocity equal to  $\omega + \Delta\omega$ . If the motion of our charge were governed only by eq.(13), then the quantity  $\Delta\omega$  would be found to be so great for an electron and an ion that they would practically not participate at all in the diurnal rotation of the earth, but would obey only electromagnetic forces. However, as soon as the charge begins to move with respect to earthbound coordinates, it will be under the action of the geomagnetic field  $H_0$ , which considerably exceeds the field of the ring  $H$ . The resultant motion under the action of the fields  $H$  and  $H_0$  is described by the equation:

$$\frac{kmM}{R^2} + eR[(\omega + \Delta\omega)H + \Delta\omega H_0] + 2mR\omega\Delta\omega + mR(\Delta\omega)^2 = m\omega_n. \quad (16)$$

By an appropriate choice of  $\Delta\omega$ , we get

$$eH\omega + eH\Delta\omega + eH_0\Delta\omega + 2m\omega\Delta\omega + m(\Delta\omega)^2 = 0, \quad (17)$$

whence, neglecting the small terms,

$$\Delta\omega = -\frac{H}{H+H_0}\omega. \quad (18)$$

Taking  $H = 10^{-3}$  CGS,  $H_0 = 0.3$  CGS,  $\omega = 7 \times 10^{-5}$ , we have  $\Delta\omega \approx -3 \times 10^{-3}$  and the linear velocity  $\omega_n \approx 1 - 2$  m/sec, which is the same for charges of both signs. It

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goes without saying, of course, that these arguments are not entirely correct, since the collisions between particles will disturb the regular drift of particles in the westward direction. Nevertheless, owing to the low molecular density of the atmosphere at the  $F_2$  level, it may be considered that the interaction between the field of the ring and the main field of the earth will lead to a certain mean displacement of the charges in a latitudinal direction. If this displacement, all the same, should amount to centimeters or meters a second, it would still be sufficient to produce a current of density  $10^{-5}$  CGS in latitudinal direction. This current would have a maximum density at the equator and a minimum density at the poles. Its role in the deformation of the  $S_D$  variations could therefore be substantial only in the middle latitudes.

The inductive influence of the extra-ionospheric current ring, however, is not the only cause leading to the formation of stable currents. A consideration of the drift of particles in the  $F_2$  layer likewise leads us to the formation of an analogous current of latitudinal direction. It follows from eq.(2) that, in the region of long free paths, a drift of particles under the action of the gravitational and magnetic fields will occur, provided that the force of gravity per unit volume  $\vec{g}Nm$  is not balanced completely by the partial pressure grad  $kTN$ . Since, in Tamm's opinion, there is a rule a disturbance of the equilibrium distribution by a barometric law in the ionosphere, the term  $[(\vec{g}Nm - \text{grad } kTN) \vec{H}]$  may play a substantial role in the formation of the  $S_q$  variations. Cowling gives the following expression for the density of the drift current:

$$j = \frac{1}{H^2} [\vec{H} (2 \frac{\partial p_e}{\partial r} - \frac{p_e}{p} \frac{\partial p}{\partial r})] \quad (19)$$

Here  $p_e$  is the partial pressure of electrons,  $p$  = pressure of gas as a whole, and  $r$  = radius vector.

For a layer whose molecular density obeys the barometric law, and whose ionization density obeys the Kryuchkov-Chapman law,

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$$p_e = P_e \left( \frac{p}{P} \right)^{\frac{1}{2}} e^{\frac{1}{2} \left( 1 - \frac{p}{P} \right)} \quad (20)$$

and

$$j = \frac{1}{H} H \frac{P_e}{P} \left( \frac{p}{P} \right)^{\frac{1}{2}} e^{\frac{1}{2} \left( 1 - \frac{p}{P} \right)} \frac{\partial p}{\partial r} \quad (21)$$

Here  $P_e$  = value of  $P_e$  at level of maximum electron density, while  $P$  = corresponding value of  $p$ .

Integrating eq.(20) over the entire thickness of the ionosphere, and putting  $P_e = 2.5 \times 10^{-7}$  CGS, Cowling obtains the result for the equator  $j = 3 \times 10^{-6}$  CGSM, that is, a quantity smaller by one order of magnitude than what we need to explain the disturbance field. But if we bear in mind that the actual distribution of molecular or electron density may differ strongly from both the Chapman law and the barometric law, then a calculation by eq.(20) leads to other results. At the present time experiments do not yield so great a material on vertical motions in the ionosphere as we have on horizontal winds, but still it is possible to find certain opinions on this subject in the literature. First of all, investigators have several times succeeded in noting a variation in the height of the level reflecting a radio signal, deducing such variation from the Doppler shift of the frequency. The velocity of the displacement so revealed was found to be of the order of meters per second. A consideration of the behavior of the active heights of the  $F_2$  layer during disturbances shows that  $H$  often varies by 100 - 150 km in one or two hours, which makes 10 - 20 m/sec.\* This can also be noted both in the analysis of individual

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\* We shall not dwell here on the tidal motions of the  $F_2$  layer experimentally found and theoretically considered in a number of works by Martin and other authors. The existence of horizontal vertical motions connected with tidal effects is today beyond all doubt, but it would still seem to be more advisable to correlate them with the geomagnetic variations in discussing the normal diurnal march  $S_q$  instead of the <sup>STAT</sup> disturbances.

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cases and in calculating the mean parameters. At the present time we have no opportunity to establish whether these changes in the height of the reflecting layer constitute actual displacements of air masses or wavelike fluctuations of density. But, under either of these assumptions, we should have a deviation of density from the barometric equilibrium and, consequently, a drift term that does not vanish in eqs. (2) and (19). It is very probable that the disequilibrium is intensified on days of magnetic disturbances, when the scattered and diffused reflections and the appearance of additional layers speak for the cloudlike structure and motions in the layer. If we assume that the fluctuations with density with height may be by a factor of several times (2, 5, 10), then the order of the term  $\vec{g}Nm - \text{grad } kTN$  is the same as the order of  $\vec{g}Nm$ . Then, as follows from eq.(2), the density of the drift current:

$$j = \frac{10^3 \times 3 \times 10^6 \times 5 \times 10^{-23} \times 10^7}{0.3} = 5 \times 10^{-6} \text{ CGS}$$

is half an order of magnitude smaller than the density of the  $S_D$  currents. The vector of densities of the drift current must be perpendicular to the magnetic and gravitational field, that is, directed according to latitude. Thus both the influence of the equatorial ring current discussed by us, and that of the drift charges under the combined action of the gravitational and magnetic fields, should lead to the formation of currents of latitudinal direction and density  $10^{-5}$  to  $10^{-6}$  CGS. Let us now consider whether these currents could lead to the formation of the current system of the  $S_D$  variations. The mathematical formulation of the theory of drift currents of the  $S_q$  variations proposed in 1920 by Chapman has not been carried to completion and gives only a qualitative scheme of the formation of the  $S_q$  currents, instead of a quantitative calculation, which the dynamo theory does give. Nevertheless, this aspect of the work, the possibility of the formation of currents of the necessary configuration, has not evoked objections either from Tamm or from critics of this theory. I therefore deemed it possible to transfer this qualitative scheme of the formation of the  $S_q$  currents to the formation of the  $S_D$  currents as well.

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The fundamental proposition of Chapman's theory of the  $S_q$  variations is as follows: the interaction of the gravitational and magnetic fields leads to the formation of a drift current in an easterly direction. The mean diurnal value of this drift,  $I_0$ , corresponding to the mean diurnal density of ionization of  $N_0$ , makes a contribution to the main field of the earth, somewhat increasing the H component. In the noon hours there is an excess of ionization  $\Delta N$ , to which there likewise corresponds an excess current of easterly direction  $\Delta I$ . To the deficit of ionization in the night hours there corresponds the negative current  $-\Delta I$ . As a consequence of these additional currents, there is an accumulation of positive charges on the evening side of the earth and of negative charges on the morning side. Since the conductivity of the ionosphere along the lines of force of the magnetic field is very great, these charges will tend to be displaced toward the higher latitudes along the lines of force, and will form two current eddies: a more intense one on the daylight side of the earth and a less intense one, negative in sign, on the night side of the earth. In its application to the  $S_D$  variations, this scheme must be modified as follows.

If we assume that: 1) the  $S_D$  variations of the ionization density of the  $F_2$  layer are responsible for the  $S_D$  currents; and 2) the vector  $\vec{g}Nm - \text{grad } kTN$  is directed vertically upward,\* then an additional current will begin to flow in westerly direction on the evening side of the low latitudes ( $0$  to  $45^\circ$ ), while an easterly current

\* Since only the term  $\vec{g}Nm$ , directed vertically downward, entered into the Chapman drift theory, this predetermined the formation of the current  $I_0$  of easterly direction. Following Tamm, we assume that the drift is determined by the difference vector  $\vec{g}Nm - \text{grad } kTN$ , which may be either positive or negative. The assumption of a  $\vec{g}Nm - \text{grad } kTN$  directed downward leads to signs of currents opposite those obtained from geomagnetic data. The assumption  $\vec{g}Nm - \text{grad } kTN$  directed upward leads to the formation of a westerly mean current  $I_0$ , which, being superimposed with the western current, due to the effect of the equatorial ring, gives the necessary signs of  $\Delta I$  on the morning and evening sides in the low and middle latitudes. STAT

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will begin to flow on the morning side. In the middle latitudes ( $45$  to  $60^\circ$ ) the direction of the currents on the evening and morning sides will be opposite, and the closure of the currents along the lines of force of the magnetic field leads to the formation of a negative eddy on the evening side and positive eddy on the morning side, the centers of the eddies being located at the latitudes  $40 - 50^\circ$ , where we have an inversion of the phase of  $S_D f^0 F_2$ . This scheme of formation is a rough one, intended merely to show that the explanation of the  $S_D$  variations by ionospheric current is possible in principle with respect to both the order of magnitude and configuration of the current lines. Which of these two mechanisms we have discussed, the influence of the equatorial ring or of the drift current, yields the greater contribution to the formation of the  $S_D$  currents at one latitude or another, remains obscure without performing exact mathematical calculations. The question as to whether these effects are capable of explaining all features of the  $S_D$  variations likewise remains unanswered. To elucidate these and other questions that may arise in connection with the explanation of the  $S_D$  variations, a detailed development of the theory would be necessary. The above presented reasoning is but an attempt to determine the direction in which this theory can be developed.

#### Section 4. Currents in the Ionosphere Induced by the External Field

In this Section we shall discuss the role of the currents induced in the ionosphere by the alternating magnetic field of the equatorial ring current. Let us denote the field external with respect to the ionosphere (that is, the field of the equatorial ring) by the letter  $E$ , and the field of internal origin (with respect to the outer edge of the ionosphere) by the letter  $I$ . The field  $I$  is made up of fields induced by the external field in the conducting layer of the earth and the ionosphere. Let us assume for simplicity, an ionosphere beyond the homogeneous conducting spherical layer of conductivity  $\sigma_d$  ( $\sigma$  = specific conductivity,  $d$  = thickness of layer). Then, denoting the external and internal fields observed directly under the

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current layer by  $E'$  and  $I'$ , we have, by the Whitehead formulas:

$$\left. \begin{aligned} E' &= E - \frac{C_0}{n} \frac{d}{dt} [nE - (n+1)I] \\ I' &= I - \frac{C_0}{n+1} \frac{d}{dt} [nE - (n+1)I] \end{aligned} \right\} \quad (22)$$

where  $C_0 = \frac{4\pi a_0}{2n+1}$  and  $a_0$  = radius of the conducting spherical layer. Let us estimate the influence exerted on the  $E$  and  $I$  fields by the currents induced in the ionosphere, that is, in other words, let us estimate the differences  $E' - E$  and  $I' - I$ . Considering only the first harmonic of the series representing the fields  $E$  and  $I$ , and assuming the terms  $E$  and  $I$  to be expressed by exponential term of the form

$$E = A_e e^{-\alpha_e(t-t_0e)}, \quad I = A_i e^{-\alpha_i(t-t_0i)}, \quad (23)$$

we have

$$\left. \begin{aligned} \frac{dE}{dt} &= -\alpha_e E, \quad \frac{dI}{dt} = -\alpha_i I, \\ E' &= E - C_0 \frac{d}{dt} (E - 2I), \quad I' = I - \frac{C_0}{2} \frac{d}{dt} (E - 2I), \\ E' &= E(1 + C_0 \alpha_e) - 2C_0 \alpha_i I \\ I' &= I(1 - C_0 \alpha_i) + \frac{C_0 \alpha_e}{2} E \end{aligned} \right\} \quad (24)$$

The quantities  $E$  and  $I$  are completely unknown to us, while the values of  $E'$  and  $I'$  may be judged on the basis of the field of  $D_{st}$  variations observed on the earth surface. Thus, eq.(24) enables us to determine the values of  $E$  and  $I$  if we only make definite assumptions as to the conductivity of the ionosphere. After eliminating  $I$  from eq.(24), we have

$$E = \frac{E'(1 - C_0 \alpha_i) + 2C_0 \alpha_i I'}{1 + C_0 \alpha_e - C_0 \alpha_i}, \quad (25)$$

whence

$$E - E' = \frac{2C_0 \alpha_i I' - C_0 \alpha_e E'}{1 + C_0 \alpha_e - C_0 \alpha_i}. \quad (26)$$

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It follows from a consideration of table 27 that if  $\alpha_d$  is understood to mean

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the conductivity even of the entire first half of the ionosphere, it would even be fairly be reasonable to evaluate  $\beta_p$  as a quantity, greater than  $5 \times 10^{-7}$  CGS. Hence the approximate quantity

$$C_0 \approx \frac{12.5 \times 6.4 \times 10^8}{3} 5 \times 10^{-7} \cong 1.5 \times 10^3.$$

The numerical values of  $\alpha_e$  and  $\alpha_i$  are unknown to us, but an idea of their order of magnitude can be formulated in the following way. Let us assume that the field of the equatorial ring during two days (the mean duration of a moderate storm) falls to 0.1 of its greatest value observed at the instant  $t^0$ . In that case it follows, from eq.(23), that

$$0.1 A_e = A_e e^{-\alpha_e 48 \times 3600} \text{ and } \alpha_e \cong 10^{-5}.$$

If the storm dies away still more slowly, then the value of  $\alpha_e$  is even less. The coefficient  $\alpha_i$  is of an analogous order of magnitude. Thus,

$$C_0 \alpha \cong 1.5 \times 10^{-2}.$$

and

$$E - E' \cong 1.5 \times 10^{-2} (2I' - E'). \quad (27)$$

The values of  $E'$  (that is, the field of the equatorial ring, allowing for the influence of the currents induced in the ionosphere) and of  $I'$  (the field of the currents induced in the earth) are known to us, not directly "under" the current layer, but on the earth surface. It follows from eq.(3 III), however, that the value of the field at the earth surface and at the level of the lower edge of the ionosphere ( $h = 100 - 300$  km) are little different from each other, and, thus, the value of the difference eq.(27) can be estimated from the values of  $I$  and  $E$  in Tables 7. Thus, for instance: for  $\tau = 20$  hours,  $E' - E = 1.5 \times 10^{-2} (54\gamma - 28\gamma \times 2) \cong 0$ ; for  $\gamma = 30$  hours,  $E' - E = 1.5 \times 10^{-2} (51\gamma - 17\gamma \times 2) \cong 0.3$ .

From these results the difference  $I - I'$  was calculated.

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For more rapid fluctuations (for example weakening of the storm to 0.1 of its initial value in 12 hours or, on the other hand its development at the beginning of

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the main part), the value of  $\epsilon$  is generally greater (about  $5 \times 10^{-2}$ ), in which case with which the differences  $\epsilon_1 - \epsilon_2$  are also increased, but for no values whatever of  $\epsilon_1$  and  $\epsilon_2$  (cf. Table 7), will they exceed 1 - 2%. It follows from the examples given that the currents induced in the ionosphere by variations as slow as  $D_{31}$  cannot exert any substantial influence on the magnetic field on the earth surface and, in any case cannot be called upon for an explanation of the  $S_D$  variations. They can likewise in our opinion not be used to explain the seasonal fluctuations of  $D_m$ , either (cf. Chapter VII, Section 1). On the other hand, in studying the rapid-course fluctuations (pulsations or aperiodic fluctuations of the type of sudden commencements), the current induced in the ionosphere may furnish a substantial contribution to the observed variations.\*

The discussion of the dynamo, drift, and induction mechanisms of current excitation given in Sections 4 and 5 had the object of explaining the origin of the middle-latitude part of the  $S_D$  variations. The formation of the polar part of the  $S_D$  currents and of the currents of the P-storms may possibly originate in entirely different ways. First of all, in the polar regions, the direct penetration of charged particles occurs, and it goes without saying that this cannot but affect the conditions of the electromagnetic field. Then, as we have pointed out above, the disturbances of the polar ionosphere as a rule are accompanied by a sharp increase of ionization in the lowest layers of all (the D and E layers). Thus it may be assumed that the current systems disturbing the normal geomagnetic field are formed in these layers as well, and not only in the  $F_2$  layer, as is the case in the middle latitudes. A discussion of the electromagnetic processes in the polar regions would go beyond the scope of the present work, since any opinions on this question would have to be based on a special study of magnetic and ionospheric material that are unavailable

\* It is easy to show by analogous calculations that the induction produced in the E layer by the magnetic fields of currents flowing in the  $F_2$  layer, is likewise very slight.

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to me. It would, however, appear advisable to say the following. It is very probable that the  $S_D$  currents in the polar latitudes flow, as in the middle latitudes, at a great height (the height of the  $F_2$  region). This is indicated both by the geomagnetic data (determination of the height of the linear current, cf. Chapter V) and the great influence of the disturbances on the  $F_2$  layer (the decrease in the ionization of the  $F_2$  layer in the high latitudes, as we have seen, is far more substantial than in the middle latitudes). The penetration of corpuscles down to the lower layers of the ionosphere, resulting in an elevated ionization at the level of the D and E layers, may possibly be responsible for the origin of polar storms. This proposition is based on the statistics given by Wells and a number of other authors, disclosing the correlation between the appearance of the  $E_s$ , the aurora, the bay-shaped disturbances, and the disruption of radio communication, as well as the highly local nature of the course of P-storms, which does not allow us to refer the  $S_D$  current to great heights. All determinations of the height of the current of the P-storms, under the assumption of the linearity of the current, may lead, as we have seen in Chapter I, to heights of the order of 100 - 120 km, or less. As for the mechanisms of excitation of the P-currents in the low layers of the ionosphere, it would appear not impossible that the dynamo effect plays a great role in their formation. If, as follows from Nagata's work (Bibl.52), the ionization density of the lower levels increases tenfold during a disturbance, then the conductivity  $\sigma^I$  of the D and E layers may reach such a value (about  $10^{-6}$ ), that the displacement of ionized masses at relatively low velocities is able to produce currents of the necessary intensity.

But conduction currents are not the only possible cause of the polar storms. In the light of the modern theories of the aurora, which have been developed with particular success by the Soviet scientist A.I. Lebedinskiy, it would appear more probable that the storm field is induced by currents of the discharge type.

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## CHAPTER IX

## CURRENT SYSTEMS OF INDIVIDUAL STORMS

Section 1. Polar Storms

The preceding Chapters of this work have been devoted to the discussion of the average features of the field of magnetic disturbance and to the description of the average current systems. The few attempts to construct the system of currents corresponding to individual storms (cf. Chapter I) have shown that the individual systems are of the same character as the average systems. The task of the present Chapter is to check this proposition on a large amount of empirical material.

A calculation of the external current systems of the  $S_D$  and  $D_{St}$  variations and of the P-storms, performed by rigorous analytic methods, showed that the approximate method of constructing currents gives good results both with respect to the configuration of the currents and to their intensity. This conclusion must be understood to the effect that with a given relatively sparse distribution over the earth surface of points with observed values of the magnetic elements, the analytic and approximate methods give a similar and rather coarse picture of the currents. With a more complete starting material, the analytic methods, of course, would yield more accurate results, while the accuracy of the approximate methods would not be increased. In the present case, the consideration of the individual disturbances <sup>STATE</sup> number of observatories whose materials can be used proves to be still smaller than the number used by us in our study of the average features of the field. For this

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reason it seems advisable to construct the individual current systems by the approximate method, estimating the current density by the Biot-Savara law (cf. eq.(1.V)). The ratio between the external part of the field  $E$  and the observed field was assumed, in accordance with the results of an analysis of the  $S_D$  variations, to be 0.8 for the middle latitudes and 0.9 for the high latitudes.

The current systems are calculated for 26 separate instants of polar and worldwide storms. The current systems of two polar and one worldwide storm are given as an example in Figs.41 - 43.

All the figures were constructed of exactly the same type. The legend under the drawings indicates the Universal Time, and the local time for the various meridians corresponding to that Universal Time is indicated at the edge of the coordinate net. The coordinate net composed of solid lines is formed by the geomagnetic parallels and meridians, while the net composed of dashed lines is formed by the geographic parallels and meridians. The negative values of the current function (the current flows clockwise about the extremum) are indicated by dashed lines, while the positive values (the current flows counter clockwise around the extremum) by solid lines. The horizontal component of the vector of the disturbance field is shown by an arrow. The current lines are so drawn that a current of 10,000 amp flows between adjacent lines. The legend under each drawing gives the intensity of the largest current eddy.

Let us turn at first to a consideration of the current systems of polar storms. In order best to bring out the characteristic features of the P-storms, all the examples were selected on quiet days, when one disturbance is not piled on top of the other. Figures 41a and 41b represent the maps of the currents for two consecutive instants of the polar storm of 9 October 1932.\* The vectors plotted on the maps correspond to the mean hourly values of the magnetic elements. On both figures the

\* The starting data for the disturbances of 9 October 1932 and 23 February 1933 have been taken from Vestine (Bibl.62).

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current system consists of two pairs of eddies. The eddy with the current of positive direction is located on the evening hours of the polar latitude, while the eddy with the negative current is located on the morning hours. In the middle-latitude pair of eddies, the signs are opposite. On Fig.41a, corresponding to the greatest development of the storm, the polar eddies considerably exceed the middle-latitude eddies in intensity. The intensity of the morning and evening eddies is not the same. Of the polar eddies, the most intense is the morning eddy, of the middle latitude eddies, the most intense is the evening eddy. Figure 41b shows the end of the polar disturbance, when the values of the magnetic elements have almost returned to the normal state. The form of the current lines and their location with respect to the local time persists in both figures. A comparison of Figs.41a and 41b with Fig.32 shows that the distribution of currents during the disturbance of 9 October 1932 is very much like the currents of the idealized P-storm, both with respect to the sign and position of the current eddies, and to the configuration of the current lines. It is true, of course, that it is necessary to note that the centers of the eddies on the idealized picture are located at earlier morning hours than in all instances of the disturbance of 9 October 1932. The second example (Fig.42) yields a picture of a more intense polar storm. The intensity of the current in the polar cap reached one million amperes at 1400 hours, 23 February 1933. The current strength in the middle latitudes in this case remains very small. The general form of the current lines and the distribution of the signs of the current function are the same as in the first example. Our attention is attracted by the strong asymmetry in the intensity of the morning and evening polar eddies: the morning eddy is 5 times as intense as the evening eddy. Such asymmetry in the distribution of currents gives the impression that the storm is observed only in a narrow longitudinal sector of the Arctic over North America. Since in many cases the distribution of the degree of disturbance in a polar storm is also characterized by such asymmetry, an uncritical consideration of the material has led many investigators to the conclusion that

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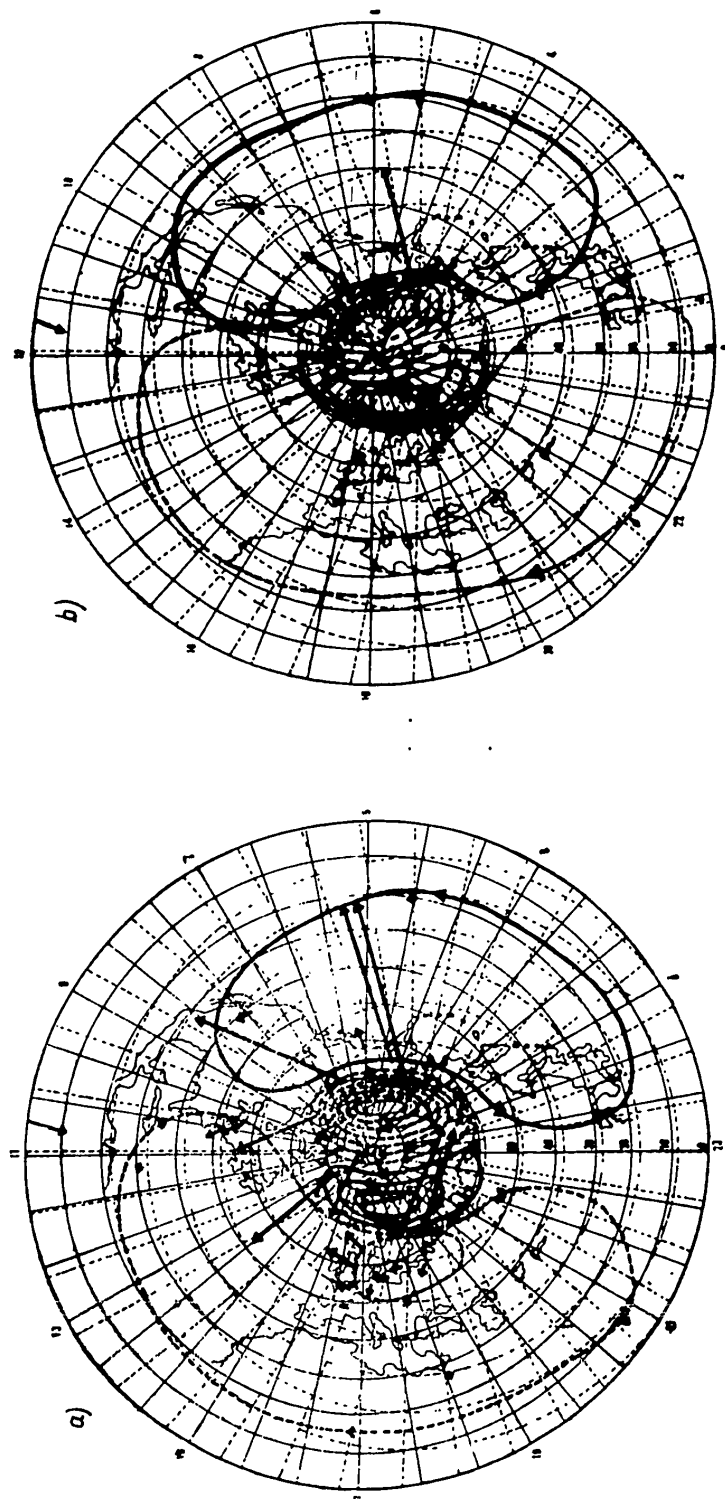
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Fig.41 - Polar Storm of 9 October 1932

(a - 1500 Hours,  $I = 70 \times 10^4$  amp; b - 1600 Hours,  $I = 40 \times 10^4$  amp)

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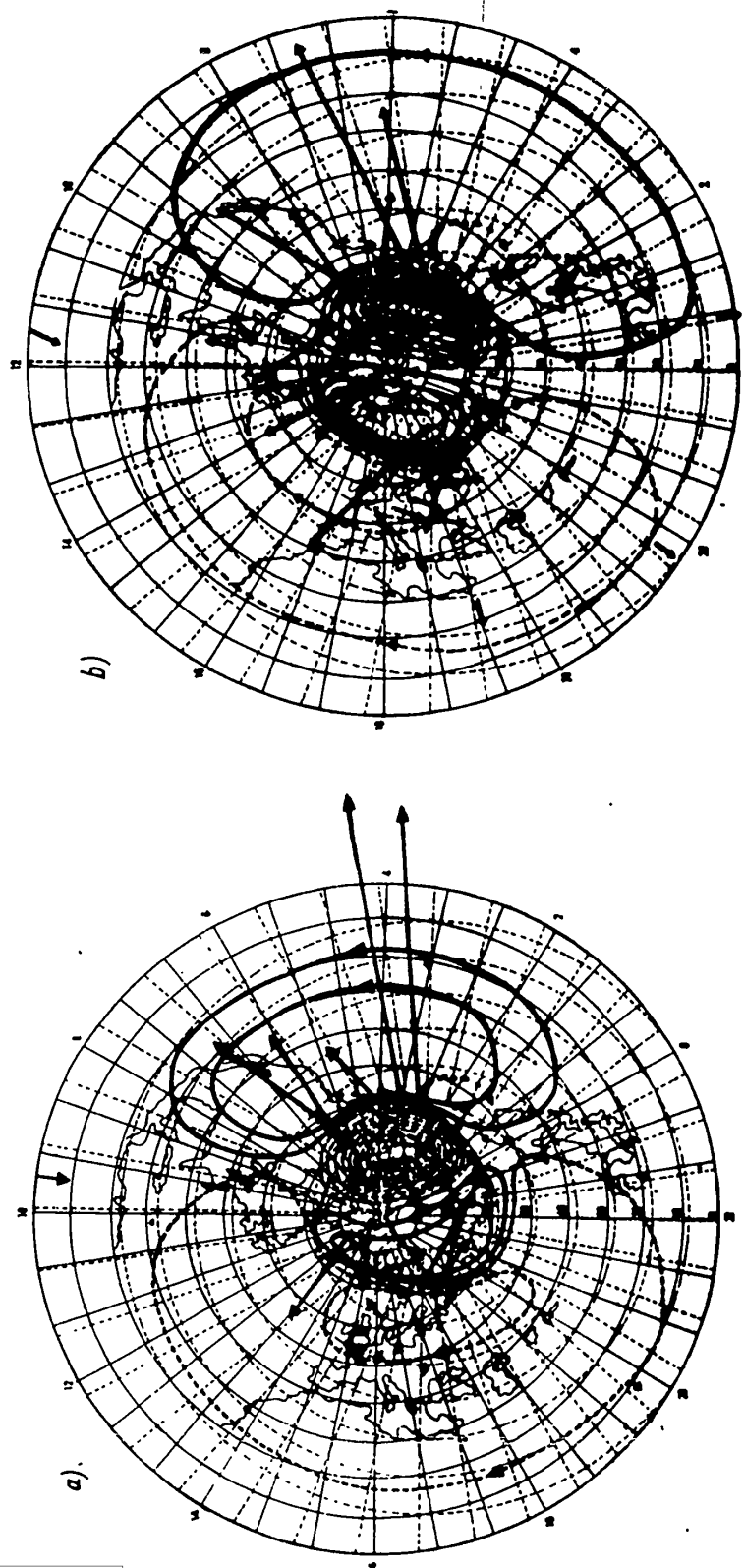
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Fig.42 - Polar Storm of 23 February 1933

(a - 1400 Hours,  $I = 100 \times 10^4$  amp; b - 1600 Hours,  $I = 60 \times 10^4$  amp)

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polar storms are local in character, that is, that a polar storm usually covers only a small longitudinal sector. The examples of polar storms we have considered show, on the contrary, that an intense negative disturbance on one side of the earth is always accompanied by a small positive disturbance on the other side. Thus the current system of a polar storm always consists of a pair of current eddies of different sign and intensity. The disturbance of 23 February 1933 vividly illustrates still another property of the current system of a P-storm: the current system is fixed with respect to the sun but not with respect to the earth. During the two hours that elapsed between 1400 and 1600 hours, 23 February, the centers of the current eddies were so displaced with respect to the earth that at both 14 and 16 hours the center of the main polar eddy remained at 4 hours local time.

The vectors represented on Figs. 41 and 42, as already mentioned, represent the mean hourly values of the elements. They show that the smoothed course of the individual polar storms, as represented by the mean hourly values, is in good agreement with the typical picture of the polar storm described in Chapter VI. For other polar storms, the diagrams of instantaneous values of the vectors have been constructed and the current systems corresponding to them have been drawn.

A typical polar disturbance was observed on 7 March 1946. The magnetograms of the Sitka and Tucson Observatories revealed a barely perceptible curvature of the quiet march of the magnetic elements at the Tucson Observatory, while at Sitka the amplitude of the fluctuations reaches  $200\gamma$ . The first diagram of the currents corresponding to maximum deviation of the H and Z components from normal (that is, to the moment of maximum development of the disturbance) shows a negative current eddy of considerable strength on the morning side of the polar cap ( $I = 170 \times 10^4$  amp) and a weak eddy on the evening side. The middle-latitude eddies, positive on the morning side and negative on the evening side, are developed relatively well, but there are no currents flowing along the parallels of latitude and responsible for the  $D_{st}$  variations. At 14 hours the current strength in the polar and middle

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latitude eddies weakened, and the polar eddy shifted to the night hours. The signs of the current eddies remained the same as at 123 hours. Thus the instantaneous distribution of the field of the polar storm is likewise in good agreement with the field of the averaged storm. The same picture of currents typical for polar storms is shown by the disturbances of 14 March and 4 July 1946. In all cases (except one), the polar part of the system consists of two eddies, a positive on the evening side and a negative on the morning side. The exception is the current system for 14 March 1946, 20 hours, on which the negative eddy moved over to the evening side. The morning eddy, as a rule is more intense than the evening eddy and is usually extended to the central part of the polar cap. Only in two cases (12 hours and 1530 hours, 4 July) did the evening eddy prove to be more intense. The middle latitude eddies have the opposite sign to the polar eddies: the morning is positive and the evening is negative. In intensity they are either both the same, or the evening eddy is stronger. In almost all cases the intensity of the middle latitude eddy is considerably less than the intensity of the polar eddies, and only at the beginning, or, on the contrary on the extinction of the disturbance, do the middle latitude and polar eddy become comparable in power. It was not possible in even a single case to draw even a single current line along the parallel, which would be due to a vector of disturbance equal for all meridians ( $D_{gt}$  variations).

The form of the current lines, the intensity and locations of the centers of the eddies, fluctuate from instant to instant, and from storm to storm, within wide limits. Thus the center of the morning polar eddy is sometimes observed at 7 hours and is sometimes shifted to 2 - 3 hours. The position of the center of the evening eddy fluctuates within just as wide limits (from 15 to 19 hours). The line of maximum crowding together of the current lines is in all cases located between  $\phi = 60^\circ$  and  $\phi = 70^\circ$ , regularly shifting to the lower latitudes with increasing disturbance. The greatest current intensity registered,  $I = 170 \times 10^4$  amp (1230 hours, 7 March 1946) is about three times as intense as the current of the average bay (Chapter VI).

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But the storm of 7 March 1946 is not particularly great; stronger disturbances are often encountered with a current strength probably much exceeding this amount. It follows from all that has been said that the penetration of corpuscles in the high latitudes always lead to the formation of a current system of a definite type. The most characteristic features of this system are: the formation of a powerful negative eddy on the morning side and in the central part of the polar zone, and the formation of a weaker positive eddy on the evening side. The fluctuations in the intensity, form, and position of these current eddies results in an infinite diversity of disturbances. But with all the multiplicity of the currents of the P-storms, the fundamental features of the system always persist, which is evidence of the definite regularities which the formation of these currents obeys.

## Section 2. Worldwide Storms

The storm of 8 April 1947 is a moderate storm. The currents calculated for 22 and 23 hours, 8 April, characterized the first phase of the storm in elevated values of  $H$ . The current encircling the earth along the circles of latitude is directed eastward in both cases. At 22 hours, both polar eddies of the  $S_D$  variations show up rather distinctly on the diagrams, while, of the pair of middle latitude eddies, only the positive morning eddy still persists. During the course of an hour, the position of the eddies was considerably modified, and the polar evening eddy spread out to the middle latitudes as far as  $40^\circ$ , while the polar negative eddy occupied the central part of the polar cap. Three consecutive instants of the disturbance (7, 11, and 17 hours, 9 April, Fig.43) were selected in the negative phase of the storm, during which the  $D_{st}$  current was directed westward. From the current system of the  $S_D$  variations, two polar and the negative evening middle latitude eddy persisted for all these three instants.

The storm of 8 August 1946 is a small magnetic storm (with amplitudes  $R_H = 70$  gammas at Sitka and  $R_H = 25$  gammas at Honolulu) without great irregular fluctuations.

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All the three instants of time for which the current systems were constructed are in the second phase of the storm. The system of currents of 22 hours, 8 August, consists of three current eddies in the polar cap and a current flowing along the circles of latitude in the middle and low latitudes. The negative eddy in the morning hours of the latitude zone  $60 - 70^\circ$  is the most intense; the weaker eddy of positive

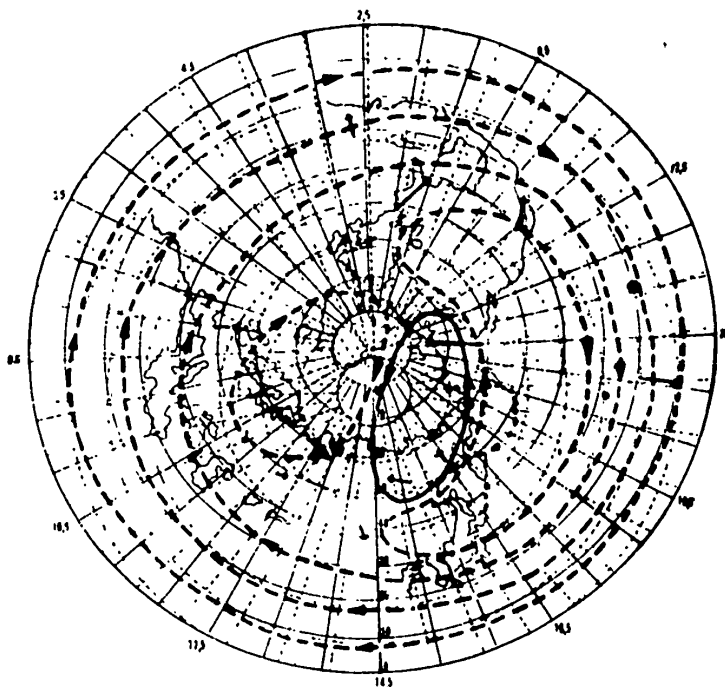


Fig.43a - The Worldwide Storm of 9 April 1947  
0700 Hours

current is located at the center of the polar zone. In these eddies the pair of polar eddies of the  $S_D$  variations may be recognized. The third negative eddy on the afternoon side of the earth is in all probability the evening eddy of the middle latitude part of the  $S_D$  variation. The positive eddy on the morning hours paired with it is absent, neutralized by the negative current girdling the entire earth. This last current (presented on the basis of two stations, Honolulu and San Juan) gives an indication of the existence of  $D_{st}$  currents symmetrical with respect to the earth axis. For three instants of the storm of 8 August 1946, the  $D_{st}$  current is weak

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( $I \sim 10^4$  amp) and is noted only in the low latitudes. The configuration of the current lines varies considerably from hour to hour, but the general character of the distribution persists. From this hasty description it follows that the current lines of the storms of 8 April 1947 and 8 August 1946 may be considered as the result of the composition of the  $S_D$  and  $D_{st}$  systems.\* The direction of the current, the ratio

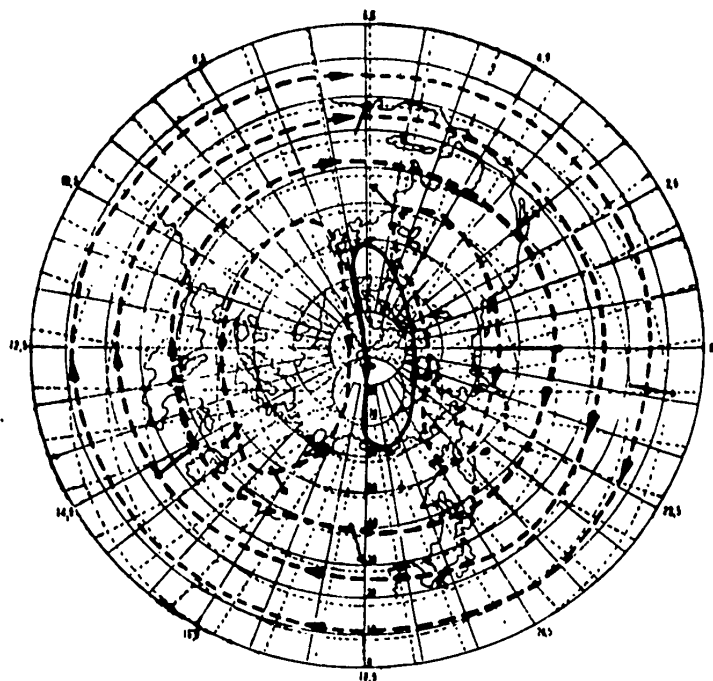


Fig.43b - Worldwide Storm of 9 April 1947

1100 Hours

of the corresponding intensities of the eddies, and the location of the centers of the eddies - all these features of the current system of a given disturbance find

\* The current systems of Fig.43 have been calculated, for convenience, under the assumption that all the current flows in a spherical layer at the height of the ionosphere. However, as follows from the preceding, it is more probable that the  $D_{st}$  currents form an equatorial ring several earth-radii in size. For this reason the current maps that we have described are arbitrary, and they must not be considered as a proof that all the current actually does flow at one level.

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explanation in the fluctuations of the current systems of the average  $D_{st}$  and  $S_D$  variations.

The principal features of the  $D_{st}$  and  $S_D$  current systems are likewise manifested in two other examples of disturbances discussed, 17 April and 5 June 1947. These storms are great storms and the intensity of the currents during them reaches 60 and  $70 \times 10^4$  amp. The variation in the configuration of the current lines and the variation of the current intensity from hour to hour in both cases are very great, but the general character of the system remains unchanged.

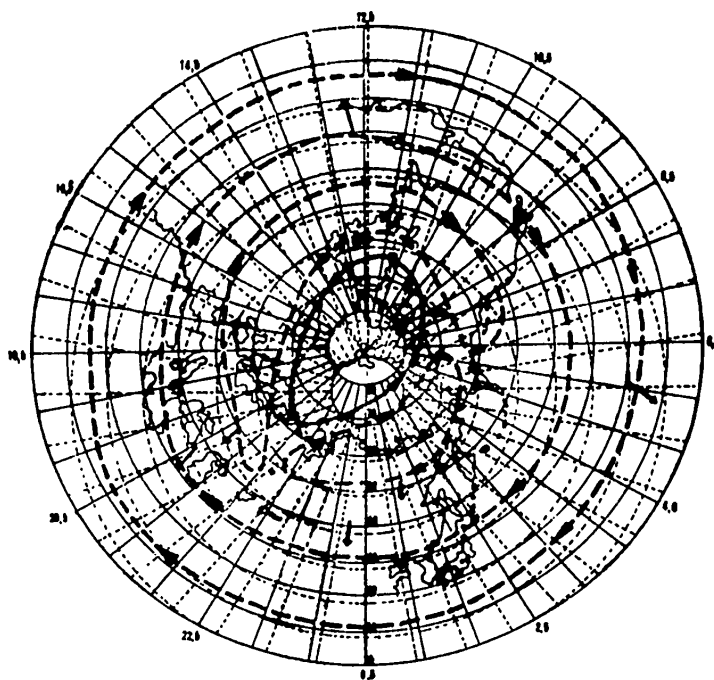


Fig.43c - The Worldwide Storm of 9 April 1947

1700 Hours

It follows from the four examples we have discussed that, in spite of the very complex and outwardly random course of worldwide magnetic storms, the field of disturbance, even at individual instants of time, obeys definite regularities which are well described by the average  $D_{st}$  and  $S_D$  variations. The multiplicity of the fluctuations of the magnetic elements during a storm is explained by the fluctuations of

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the parameters of these current systems and the superimposition on them of polar disturbances, the piling up of which gives an irregular and grotesque form to the variations of the magnetic elements. A comparison of diagrams for successive instants of one and the same storm clearly shows how the current systems are deformed (their configuration and intensity both vary), thereby producing the variations of the geomagnetic field.

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## CHAPTER X

## THE INTERNAL PART OF THE DISTURBANCE FIELD

Section 1. The Inductive Origin of the Inner Part of the Field. Survey of the Results Obtained

It appears beyond question today that the part of the field of magnetic variations whose sources are located inside the earth is not independent; its existence is due to currents induced by the alternating field in the conducting regions of the earth. The study of this internal part is of great interest, since it is still the only source of our knowledge of the electromagnetic properties of the deep parts of the earth. The idea on which the induction theory of the internal part of the magnetic variations is based is very simple. Every external alternating magnetic field  $E(t)$  induces, in a conductor of conductivity  $\kappa$  and magnetic permeability  $\mu$ , magnetization (magnetic induction) and a certain distribution of currents (electromagnetic induction). The field of these currents ( $I'$ ) and of inductive magnetization ( $I''$ ) can be calculated theoretically if the function  $E(t)$  and the parameters  $\kappa$  and  $\mu$  of the medium are known, that is,

$$I' + I'' = I(E, \kappa, \mu). \quad (1)$$

On the other hand, by substituting in eq.(1) the values of the external and internal fields known from the experimental data, the parameters  $\kappa$  and  $\mu$  can be expressed in terms of  $E$  and  $I$ . In practice, however, the finding of  $I(E, \kappa, \mu)$ , and all the more the solution of the inverse problem, to find  $\kappa$  and  $\mu$  from observations of

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E and I on the earth surface, encounters great mathematical difficulties. The problem has been solved only for a few very special cases. Thus, Lamb (Bibl.40), developed a theory of electromagnetic induction in a sphere of uniform conductivity ( $\kappa = \text{const}$ ) for the case when the field of  $E(t)$  is a periodic function. Lamb's formulas, used by a number of authors (see (Bibl.9) for more details) in the analysis of the solar-diurnal variations ( $S_q$ ), show that the earth could be represented as consisting of a conducting core ( $\frac{\kappa}{\mu} = 10^{-12} - 10^{-13}$  CGSM), surrounded by a nonconducting shell 200 - 400 km thick.\* A consideration of the diurnal variations does not enable us to determine the values of  $\kappa$  and  $\mu$  separately, but it appears unlikely that  $\mu$  in the deep parts of the earth should differ much from unity. To verify this proposition, Chapman and Whitehead undertook a qualitative investigation of the slow  $D_{st}$  variations. The potential of the field of induced magnetization ( $I''$ ) should be expressed by the same harmonic as the induced field  $E(T)$ , and the ratio of the harmonic coefficients of the internal field to the external not depending on the rate of change of  $E(t)$  (for more details, see Section 6). Therefore on the 5th to 10th day of a storm, when the variations of  $D_{st}$  are extremely slow, and, consequently, the influence of electromagnetic induction is very slight,  $I''$  should exert most of the influence on the field observed at the earth surface. This influence decreases H and increases the Z component of the E field. For example, for  $\mu = 10$ , the expected effect should be equal to  $\frac{3}{4} E$  in H and  $\frac{3}{2} E$  in Z. During the first days of a storm, with rapid variation of  $D_{st}$ , the field of electromagnetic induction  $I'$  should have a great weight. This field, as is commonly known, is expressed by the same harmonics, but with reversed sign, that is, increasing H and decreasing Z. But Chapman and Whitehead (Bibl.40) found no substantial difference in the ratio of the observed H and Z of the  $D_{st}$  field on the first and subsequent days of a storm, which is evidence that magnetic induction plays a small part in the establishment of the inter-

\* The term "core" here employed is not identical with the notion, generally adopted in geophysics, of the earth core, with a radius of 0.6R.

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nal field.

The rigorous solution of the inductive problem for an aperiodic field (for example the  $D_{st}$  field) involves major computational difficulties. For a sphere  $\kappa = \text{const}$  this problem has been solved by Price, who found that to explain the ratio of  $E$  and  $I$  of the first harmonic of the  $D_{st}$  field it is necessary to assume different parameters for the earth, namely a thickness of 400 km for the nonconducting layer, and  $\kappa = 4 \times 10^{-12}$ . The most probable explanation of this disagreement would appear to be the phenomenon of the skin effect. Let us assume for simplicity that the field  $E$  is proportional to  $\sin pt$ . Then the magnetic flux  $f$  through any contour at a certain level in the earth will be proportional to  $\sin pt$ , that is,  $f = f_0 \sin pt$ , and the electromotive force due to it,  $\varepsilon = -\frac{df}{dt} = -pf_0 \cos pt$ , whence it follows that the current induced in this layer is proportional to  $p$  and  $\kappa$ . The field  $I'$  created by this current will neutralize the inducing field  $E$  and will prevent it from penetrating into deeper layers. The depth at which the full screening of the internal region from the field  $E$  takes place will be inversely proportional to the intensity of  $I'$  or  $\varepsilon$ , that is, proportional to  $\frac{1}{\kappa p}$ . Consequently the more rapid fluctuations of  $E(t)$  will induce currents concentrated in a thinner surface layer of the earth, while to the slow fluctuations will correspond currents extending to great depths. Thus the increased value of  $\kappa$  and the increased depth of the nonconducting shell for  $D_{st}$  indicates that the currents of  $D_{st}$  penetrate deeper within the earth, and that increases toward the center of the earth. Accordingly, Lahiri and Price (Bibl.47) have made calculations for a model of the earth with a core of nonuniform conductivity,  $\kappa = \kappa_0 \sigma^{-s}$  ( $\sigma = \frac{r}{a}$ , where  $a$  = radius of core), which explains the ratio of the fields  $\frac{I}{E}$  for both the  $S_q$  and the  $D_{st}$  variations. Calculations showed that for a depth of the order of 600 km, the  $\kappa$  differs little from the  $\kappa$  of dry rocks, i.e., it is  $10^{-14}$  to  $10^{-15}$  CGSM, and that below that level it strongly increases with the depth.

A number of authors have attempted to evaluate the field of currents induced in

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the oceans and upper conducting layers of the earth crust. They have succeeded in finding that a uniform layer of seawater, 1.5 km thick, markedly changes the ratio  $\frac{I}{E}$  observed on the earth surface (for instance, for  $P_2^2$ , from 0.4 to 0.5). But the continents, alternating with the mainlands, reduce the effectiveness of the currents in the oceans, and as a result it may be considered that the internal parts of the  $S_q$  and  $D_{st}$  fields are almost entirely due to currents flowing in the deep parts of the earth. It goes without saying that for the more rapid variations of the magnetic field the influence of the surface currents is greater, and, in the case, for instance, of the pulsations, which are periodic fluctuations with a period of a few seconds, the upper conducting layers may possibly completely screen the conducting core.

S.Sh.Dolginov (Bibl.15) has obtained interesting results by using the spherical analysis of the noncyclical variations to evaluate the earth conductivity. Considering the noncyclical variations as the derivative of  $D_{st}$  and using the approximate Chapman-Whitehead formulas, Dolginov confirmed the value  $\kappa = 4 \times 10^{-13}$  for a radius of the conducting core from 0.88R to 1.00R which had been obtained by Chapman from  $S_q$  data.

In 1949, A.N.Tikhonov (Bibl.31) found a new possibility of estimating the conductivity of the earth by using simultaneous observations of the geomagnetic variations and the earth currents. The equations relating the variations of the magnetic and electric vectors at a given point permit the determination of the conductivity and the thickness of the earth crust for different regions of the earth, and thereby yield material that is valuable for geotectonics and geology.

All the authors mentioned by us, except S.Sh.Dolginov, started out from one and the same experimental material, the spherical analysis of the  $D_{st}$  field performed by Chapman and Whitehead. Only the first term of the spherical series, the harmonic  $P_1$ , was examined in this case. In view of this fact, the literature has repeatedly pointed out the necessity of repeating the evaluation of the parameters, using new

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experimental materials. We performed such work on the basis of the separation of the disturbance field into E and I parts, as described in the preceding chapters, and the results of this work are given below.

Since the theory of induction in the form developed in the works of Lamb and Price can be successfully used only in cases when the E and I parts of the field are represented by series of special functions, the data on the  $S_D$  variations remained unused. It proved possible to employ only the  $D_{st}$  variations and the polar storms for the evaluation of  $\kappa$ . In the calculations based on  $D_{st}$  we used the formulas of Price for a homogeneous sphere and of Lahiri and Price for a sphere of nonuniform conductivity. Since the field of a polar storm was represented by a series of Bessel functions, the induction problem was solved in cylindrical coordinates in order to make the use of these data possible. For convenience, the exposition of the solution of this problem has been placed in a separate section (Section 2), while Sections 3 - 6 are devoted to the calculation of the conductivity from the data of  $D_{st}$ , and the results are discussed in Section 7.

## Section 2. Solution of the Induction Problem in Cylindrical Coordinates

Assume that the earth, beginning at a certain depth  $z_1$ , has the constant conductivity  $\kappa$ . The upper half-space (the upper layers of the earth and the atmosphere) constitute an ideal dielectric. Then the magnetic field may be described in the dielectric by the scale of potential  $v$ , and in the conductor by the vector potential  $\vec{A}$ , which, as shown by Lamb, satisfies the equations:

$$\Delta \vec{A} - a^2 \frac{\partial \vec{A}}{\partial T} = 0 \quad (2)$$

and

$$\text{div } \vec{A} = 0. \quad (3)$$

Here  $a^2 = 4\pi\mu$ , and  $T$ , as before, denotes Universal Time. The condition of continuity of the tangential components of the field  $H$  and of the radial component of

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induction  $B$  at the surface of separation of the two media ( $z = z_1$ ) enables us to find the constant  $a$  entering into the expression for vector potential, and thus to evaluate  $\kappa$  and  $\mu$ . It is therefore necessary first of all to solve the system of equations (2) and (3), and, from the vector potential so found, to calculate the components of the vectors of induction and field intensity.

Putting  $\vec{A} = R(r)\Phi(\varphi)(z)T(T)$ , we may, without prejudice to the generality of the solution assume that  $T(T) = e^{-inT}$ , where in the case of aperiodic variations  $n$  is a real number, for aperiodic variations an imaginary number, and in the general case, a complex number. Denoting  $R(r)\Phi(\varphi)Z(z)$  by  $\vec{A}_0$ , we have

$$\left. \begin{aligned} \vec{A} &= \vec{A}_0 e^{-inT} \\ \frac{\partial \vec{A}}{\partial T} &= -\vec{A}_0 in e^{-inT} \end{aligned} \right\} \quad (4)$$

and

$$\Delta \vec{A}_0 + k^2 \vec{A}_0 = 0, \quad (5)$$

where

$$k^2 \cong in4\pi\kappa\mu = ina^2. \quad (6)$$

In curvilinear coordinates, the meaning of the symbol  $\Delta$  of the vector is defined by the identity  $\text{rot rot } \vec{F} = \text{grad div } \vec{F} = \Delta \vec{F}$ , from which it follows that

$$\text{rot rot } \vec{A}_0 - k^2 \vec{A}_0 = 0. \quad (7)$$

In cylindrical coordinates, the  $r, \varphi, z$  components of eq.(7) are written as follows:

$$-\frac{1}{r^2} \left( \frac{\partial^2 A_\varphi}{\partial \varphi \partial r} - \frac{\partial^2 A_r}{\partial \varphi^2} \right) + \frac{\partial^2 A_z}{\partial z^2} + k^2 A_r = 0, \quad (8)$$

$$\frac{\partial^2 A_\varphi}{\partial z^2} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial r A_\varphi}{\partial r} \right) - \frac{\partial^2}{\partial r \partial \varphi} \frac{A_r}{r} + k^2 A_\varphi = 0, \quad (9)$$

$$\frac{1}{r} \frac{\partial^2 r A_r}{\partial r \partial z} + \frac{1}{r} \frac{\partial^2 A_\varphi}{\partial \varphi \partial z} = 0. \quad (10)$$

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Here  $A_\varphi$  and  $A_r$  denote the respective components  $\vec{A}_0$  and  $A_z = 0$ , which corresponds

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to the fact that the induced currents are assumed to be parallel to the plane  $z = 0$ .

Setting  $R(r)\psi(\varphi) = Y(r, \varphi)$ ,  $A_r = ZY_r$  and  $A_\varphi = ZY_\varphi$ , we have, from eq.(8):

$$-\frac{Z}{r^2} \left( \frac{\partial^2 Y_\varphi}{\partial \varphi \partial r} - \frac{\partial^2 Y_r}{\partial \varphi^2} \right) + Y_r Z'' + k^2 Y_r Z = 0$$

and

$$\frac{Z'}{Z} + k^2 - \frac{1}{r^2 Y_r} \left[ \frac{\partial^2 Y_\varphi}{\partial \varphi \partial r} - \frac{\partial^2 Y_r}{\partial \varphi^2} \right] = 0,$$

whence

$$\frac{Z''}{Z} + k^2 = \lambda^2, \quad (11)$$

$$-\frac{1}{r^2 Y_r} \left[ \frac{\partial^2 Y_\varphi}{\partial \varphi \partial r} - \frac{\partial^2 Y_r}{\partial \varphi^2} \right] = -\lambda^2. \quad (12)$$

Introducing the notation  $\lambda^2 - k^2 = f^2$ , we have

$$Z'' = f^2 Z \text{ and } Z = e^{-fz}. \quad (11')$$

Equation (10) is equivalent to the identity

$$\frac{\partial r Y_r}{\partial r} + \frac{\partial Y_\varphi}{\partial \varphi} = 0. \quad (10')$$

On replacing  $\frac{\partial Y_\varphi}{\partial \varphi}$  in eq.(12) by  $-\frac{\partial r Y_r}{\partial r}$ , we have

$$\frac{r}{R} \frac{\partial^2 R_r}{\partial r^2} + \frac{1}{R} \frac{\partial r R_r}{\partial r} + \lambda^2 r^2 = -\frac{1}{\Phi_r} \frac{\partial^2 \Phi_r}{\partial \varphi^2} = \text{const} = h^2. \quad (13)$$

It follows from the right side of eq.(13) that

$$\Phi_r = \sin(h\varphi + \varepsilon). \quad (14)$$

Putting  $rR_r = \mathcal{U}$  and  $\lambda_r = \rho$  in the left side, we obtain after several transformations,

$$\rho^2 \mathcal{U}'' + \rho \mathcal{U}' + (\rho^2 - h^2) \mathcal{U} = 0. \quad (15)$$

Equation (15) is the Bessel equation of order  $h$ , whose integral

$$\mathcal{U} = c J_h(\rho) \text{ and } R_r = \frac{c}{r} J_h(\lambda r), \quad (16)$$

where  $c$  is a constant coefficient. It follows from eqs.(11', 14, and 16) that

$$A_r = c e^{-fz} \sin(h\varphi + \varepsilon) \frac{1}{r} J_h(\lambda r). \quad (17)$$

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Substituting eq.(17) and eq.(10"), we get

$$A_{\varphi} = \frac{c}{h} e^{-fz} \cos(h\varphi + \varepsilon) \frac{\partial J_h(\lambda r)}{\partial r}. \quad (18)$$

From eq.(17) and eq.(18), which completely determine the vector  $\vec{A}_0$ , we find the components of the magnetic induction  $\vec{B} = \text{rot } \vec{A}$ . For  $T = 0$

$$B_r = -\frac{\partial A_{\varphi}}{\partial z} = \frac{c}{h} f e^{-fz} \cos(h\varphi + \varepsilon) \frac{\partial J_h(\lambda r)}{\partial r}, \quad (19)$$

$$B_{\varphi} = \frac{\partial A_r}{\partial z} = -c f e^{-fz} \sin(h\varphi + \varepsilon) \frac{1}{r} J_h(\lambda r), \quad (20)$$

$$B_z = -\frac{c\lambda^2}{h} e^{-fz} \cos(h\varphi + \varepsilon) J_h(\lambda r). \quad (21)$$

It follows from eqs.(19) - (21) that the components of the vector  $B(T)$  are the real parts of the expressions

$$B_r = \text{Re} \frac{cf}{h} e^{-fz - i(h\varphi + \varepsilon) - inT} \frac{dJ_h(\lambda r)}{dr}, \quad (22)$$

$$B_{\varphi} = \text{Re} - i h \frac{cf}{h} e^{-fz - i(h\varphi + \varepsilon) - inT} \frac{1}{r} J_h(\lambda r), \quad (23)$$

$$B_z = \text{Re} - \frac{c\lambda^2}{h} e^{-fz - i(h\varphi + \varepsilon) - inT} J_h(\lambda r). \quad (24)$$

The field of the polar storm discussed in Chapter VI is a function of the local time  $t$ . Putting  $h = n$  and  $T + \varphi = t$  in eqs.(22) - (24), we have, for  $Z = Z_1$ :

$$H_r = \mu B_r = \frac{cf}{n} \mu e^{-fz_1 - i(nt + \varepsilon)} \frac{dJ_n(\lambda r)}{dr}, \quad (25)$$

$$H_{\varphi} = \mu B_{\varphi} = -in \frac{cf}{n} \mu e^{-fz_1 - i(nt + \varepsilon)} \frac{1}{r} J_n(\lambda r), \quad (26)$$

$$B_z = -\frac{c\lambda^2}{n} e^{-fz_1 - i(nt + \varepsilon)} J_n(\lambda r). \quad (27)$$

In the nonconducting half-space, the potential of the field satisfies the Laplace equation and consequently may be represented, under the condition  $h = n$ , by

the series

$$V = \text{Re} [E e^{-z_1 - i(nt + \varepsilon)} + i e^{z_1 - i(nt + \varepsilon)}] J_n(\lambda r), \quad (28)$$

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where  $E$ ,  $I$ ,  $\gamma$  and  $\zeta$  are known constants (cf. Table 17, Chapter VI). From eq.(28), for  $Z = Z_1$ :

$$H_r = \mu [Ee^{-z_1\lambda - i(n\tau + \gamma)} + Ie^{z_1\lambda - i(n\tau + \zeta)}] \frac{dJ_n(\lambda r)}{dr}, \quad (29)$$

$$H_\varphi = -in\mu [Ee^{-z_1\lambda - i(n\tau + \gamma)} + Ie^{z_1\lambda - i(n\tau + \zeta)}] \frac{1}{r} J_n(\lambda r), \quad (30)$$

$$B_z = \lambda [-Ee^{-z_1\lambda - i(n\tau + \gamma)} + Ie^{z_1\lambda - i(n\tau + \zeta)}] J_n(\lambda r). \quad (31)$$

From the condition of the continuity of  $H_r$  or  $H_\varphi$  for  $Z = Z_1$ , we get, by equating eq.(22) and eq.(17), or eq.(22) and eq.(23), and putting  $\mu = 1$ :

$$\frac{c}{n} \sqrt{\lambda^2 - ina^2} e^{-\sqrt{\lambda^2 - ina^2} z_1 - i\epsilon} = Ee^{-z_1\lambda - i\gamma} + Ie^{z_1\lambda - i\zeta}. \quad (32)$$

From the condition of continuity of  $B_z$  on  $Z = Z_1$ , we have

$$-\frac{c}{n} \lambda e^{\sqrt{\lambda^2 - ina^2} z_1 - i\epsilon} = -Ee^{-z_1\lambda - i\gamma} + Ie^{z_1\lambda - i\zeta}. \quad (33)$$

On dividing eq.(32) by eq.(33) to eliminate the unknown constants  $c$  and  $\epsilon$ , we have

$$-\frac{\sqrt{\lambda^2 - ina^2}}{\lambda} = \frac{Ee^{-z_1\lambda - i\gamma} + Ie^{z_1\lambda - i\zeta}}{-Ee^{-z_1\lambda - i\gamma} + Ie^{z_1\lambda - i\zeta}}. \quad (34)$$

Equation (34), connecting these complex quantities with each other, is entirely sufficient for the determination of the constants  $a^2$  and  $Z_1$  in which we are interested. Introducing the notation

$$\frac{I}{E} e^{2z_1\lambda} = x \text{ and } \frac{na^2}{\lambda^2} = \alpha,$$

we get

$$1 - i\alpha = \frac{(1 + xe^{i\delta})^2}{(1 - xe^{i\delta})^2}. \quad (35)$$

Separating the real and imaginary parts in eq.(35), we get

$$1 = \frac{(1 - x^2)^2 - 4x^2 \sin^2 \delta}{(1 - 2x \cos \delta + x^2)^2}, \quad (36)$$

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$$-\alpha = \frac{4x(1-x^2)\sin\delta}{(1-2x\cos\delta+x^2)^2} \quad (37)$$

It is easy to obtain a computational expression for  $x$  from eq.(36):

$$x = \frac{1 + \sin\delta}{\cos\delta} \quad (38)$$

From eq.(37) we have

$$x = \frac{x(x^2-1)\sin\delta\lambda^2}{\rho^2\pi\epsilon(1+x^2-2x\cos\delta)^2} \quad (37')$$

The coefficient  $\epsilon = \frac{2}{24 \times 60 \times 60} = \frac{1}{1.5} \cdot 10^{-4}$  is introduced in connection with the fact that the time  $t$  is indicated in eqs.(25) - (31) not in angular measure but in seconds. The numerical value of the coefficient  $\rho$  (cf. Chapter VI) is  $4.5 \times 10^3$  km, or  $4.5 \times 10^8$  cm.

From eq.(37') and eq.(38) the quantities  $\kappa$  and  $z_1$  may be found, if the field observed on the earth surface is separated into an external and an internal part. Thus the solution of the induction problem in cylindrical coordinates leads to very simple formulas which are entirely convenient for practical calculations. \*

The values of  $\kappa$  and  $z_1$  calculated from the data of Table 18 are presented in Table 28. For five terms I did not succeed in getting a reasonable value of the conductivity, owing to negative values of the numerator in eq.(37), but in evaluating the results it must be borne in mind that the values of the coefficients  $E$  and  $I$  for a few terms would hardly exceed the margin of accuracy of the analysis.

The results obtained are discussed in more detail in Section 7.

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\* After these calculations had been completed, I learned of the work by Yu.D. Kalinin (Bibl.17) in which the plane problem is solved in rectangular coordinates, and still simpler formulas for determining  $\kappa$  and  $z_1$  are obtained. Kalinin's formulas, however, are applicable only in the case where the field depends on a  $\sin_{\text{STAT}}$  argument.

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Table 28

|                       | $n \backslash m$ | 1                  | 2                   | 3                  | 4                  | 5                  |
|-----------------------|------------------|--------------------|---------------------|--------------------|--------------------|--------------------|
| $\chi$ , CGSM . . . . | 1                | $8 \cdot 10^{-13}$ | $14 \cdot 10^{-13}$ | —                  | $1 \cdot 18^{-14}$ | $2 \cdot 10^{-12}$ |
| $z_1$ , KM . . . . .  | 1                | 144                | 256                 | —                  | 198                | 45                 |
| $\chi$ , CGSM . . . . | 2                | —                  | —                   | $7 \cdot 10^{-15}$ | —                  | —                  |
| $z_1$ , KM . . . . .  | 2                | —                  | 12!                 | 103                | —                  | 81                 |

### Section 3. Determination of the Earth Conductivity from the Data of the First Harmonic of $D_{st}$ (the Lamb Model)

If the alternating magnetic field of origin external to the earth surface is represented by the sum of terms of the form

$$RE_n^{mh} \left(1 - e^{-\alpha_n^{mh} t}\right) \left(\frac{r}{R}\right)^n S_n^m, \quad (39)$$

then, under the condition that the conductivity of the earth is uniform, to each term  $E_n^{mh}$  will correspond a term in the inductive field, represented by the same harmonic:

$$I_n^{mh}(t) = \left(\frac{r}{R}\right)^{-n-1} E_n^{mh} \varphi_n^{mh}(t) S_n^m. \quad (40)$$

Here  $\varphi_n^{mh}(t)$  is a function expressing the time dependence of  $I_n^{mh}$  and represented by the infinite series

$$\varphi_n^{mh}(t) = -\frac{nq^{2n+1}}{n+1} 2(2n+1) (k_n^{mh})^2 \times \sum_{s=1}^{\infty} \frac{e^{-\alpha_{ns}^{mh} t} - e^{-\alpha_n^{mh} t}}{(k_{ns}^{mh})^2 [(k_n^{mh})^2 - (k_{ns}^{mh})^2]}. \quad (41)$$

The constants  $k^2$  and  $\alpha$  are connected with each other by the relation

$$k^2 = -4\pi\alpha \quad (42)$$

and  $q = \frac{a}{R}$ , where  $a$  is the radius of the conducting core. The constants  $k_{ns}^{mh}$  and  $\alpha_{ns}^{mh}$

are not arbitrary, but satisfy the following conditions:

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$$-(k_{ns}^{mh})^2 q^2 a^2 = (\alpha_{ns}^{mh})^2 = 4\pi\alpha_{ns}^{mh} q^2 a^2 \quad (43)$$

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and

$$J_{n-\frac{1}{2}}(x_{ns}^{mh}) = 0. \quad (44)$$

The terms containing  $e^{-\alpha_{nst}^{mh}}$  are introduced in the expression for  $\varphi(t)$  by Price in accordance with the fact that the equations describing the boundary conditions (analogous to eq.(30) and eq.(31) of Section 2) have solutions even in the absence of the induced field ( $E_n^{mh} = 0$ ). These solutions indicate the existence of free electric systems which are damped by an exponential law.

In considering the periodic fluctuations, which do not require the satisfaction of initial conditions, the free systems may be left out of consideration, but in solving the problem of induction by aperiodic variations, these systems must be included in the equations of the initial conditions. The failure to allow for the free systems led Chapman and Whitehead to erroneous conclusions.

The imposition of initial conditions (vanishing of the total field for  $t = 0$ ) causes the terms containing  $e^{-\alpha_s t}$  to obey eq.(43) and eq.(44).

For convenience in calculating series eq.(41), Price proposed selecting, for the representation of the induced field, the constants  $\alpha_n^{mh}$ , equal to a certain  $\alpha_s$  satisfying the conditions eq.(43) and eq.(44). In that case, putting  $\alpha_n^{mh} = \alpha_p$ , we must omit the term  $s = p$  from the sum eq.(41) and add the term

$$\frac{nq^{2n+1}}{n+1} \frac{2(2n+1)te^{-\alpha_p t}}{q^2 a^2 4\pi x}.$$

Fixing our attention on one term of eq.(40), and omitting the indexes  $n, m$ , and  $h$  in eq.(41), we have

$$\begin{aligned} \varphi_p(t) &= \frac{2n(2n+1)}{n+1} q^{2n+1} \left[ \frac{te^{-\alpha_p t}}{4\pi x q^2 a^2} - \sum_{s=1}^{\infty} \frac{\alpha_p (e^{-\alpha_s t} - e^{-\alpha_p t})}{\alpha_s q^2 a^2 (k_p^2 - k_s^2)} \right] = \\ &= \frac{2n(2n+1)}{n+1} q^{2n+1} \left[ \frac{At}{\pi^2} e^{-\alpha_p t} - x_p^2 \sum_{s=1}^{\infty} \frac{e^{-\alpha_s t} - e^{-\alpha_p t}}{x_s^2 (x_s^2 - x_p^2)} \right], \end{aligned} \quad (41')$$

where  $A = \frac{\pi}{4\pi q^2 a^2}$

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Chapman and Price performed their calculations for the case  $n = 1$ , for which eq.(44) is transformed into

$$I_{\frac{1}{2}}(x) = \frac{\sin x}{\sqrt{\pi x}} = 0 \text{ and } x = s\pi, \quad (45)$$

where  $s$  takes on the series of successive values 0, 1, 2...

Under this condition, on the basis of eq.(43), we have

$$k_s^2 = -\frac{s^2\pi^2}{q^2a^2}; \quad \alpha_s = \frac{s^2\pi}{4\kappa q^2a^2} = s^2A \quad (46)$$

and

$$\varphi_p(t) = \frac{3q^2}{\pi^2} \left[ \left( At + \frac{9-2p^2\pi^2}{12p^2} \right) e^{-p^2At} - p^2 \sum_{s=1}^{\infty} \frac{e^{s^2At}}{s^2(s^2-p^2)} \right]. \quad (47)$$

To solve our problem, which is to find  $\kappa$  and  $q$ , we must calculate the value of  $\varphi(t) = \frac{I(t)}{E}$  from values of  $I$  and  $E$  known on the earth surface, and must then by

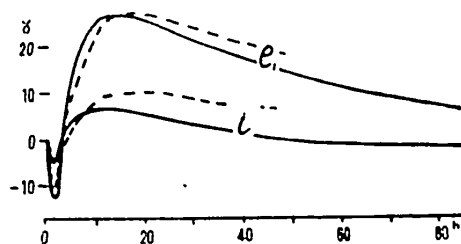


Fig.44 - Coefficients of Spherical

Analysis of the  $D_{st}$  Variations

(after Chapman)

——— observed value;

----- calculated values

some numerical or graphical method determine

$q$  and  $\kappa$  from eq.(47). In view of the ex-

treme complexity of this method, Chapman and

Price attempted to satisfy the observed val-

ues of  $\frac{I(t)}{E}$  calculating  $\varphi(t)$  from the val-

ues of  $q$  and  $\kappa$  found on the basis of the  $S_q$

variations ( $q = 0.96$ ,  $\kappa = 4 \times 10^{-13}$ ,  $A = 5.7$

$\times 10^{-6}$ ). The poor agreement between the cal-

culated and observed values of  $I(t)$  (cf.

Fig.44) forced them to assume that to the

$D_{st}$  variations correspond greater depths of

penetration of the currents ( $q = 0.94$ ) and greater values of the conductivity

( $\kappa = 44 \times 10^{-13}$ ). Evaluating  $\alpha$  and  $q$  from the data of the spherical analysis of  $D_{st}$

performed by us, we considered it more advisable to begin the tests of the calcula-

tion of  $I(t)$  with the values of  $\kappa$  and  $q$  proposed by Chapman. For  $\kappa = 4 \times 10^{-12}$  CGSM,

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we found the value  $\Lambda = 5 \times 10^{-17}$ , or, if  $t$  is expressed in hours, then  $\Lambda \approx 2 \times 10^{-3}$ . The time dependence of  $E_1$  so obtained (cf. Table 6, Chapter III and Fig. 45) was approximated by four terms of the form  $a_s(1 - e^{-s^2 t})$ , whose coefficients  $a_s$  are pairwise equal, i.e.,

$$E_1(t) = -a_{1,2}(e^{-s_1^2 \Lambda t} - e^{-s_2^2 \Lambda t}) + a_{3,4}(e^{-s_3^2 \Lambda t} - e^{-s_4^2 \Lambda t}), \quad (48)$$

where  $s_1 < s_2$ ,  $s_3 < s_4$ , and  $s_4 < s_1$ . Such a selection of constants assures the representation by eq. (48) of the first and second phases of the magnetic storm (the terms  $s_1$  and  $s_2$  being mainly responsible for the first phase and  $s_3$  and  $s_4$  for the second).

The most favorable numerical values of  $s$  and  $a$  were found in the following way. Denoting the value of  $E(t)$  for  $t > 5$  hours by  $E_{II}$ , we have

$$E_{II} \approx a_{3,4}(e^{-s_3^2 \Lambda t} - e^{-s_4^2 \Lambda t}), \quad (49)$$

$$\frac{dE_{II}}{dt} = a_{3,4}(-x A e^{-x \Lambda t} + y A e^{-y \Lambda t}), \quad (50)$$

where  $x = s_3^2$  and  $y = s_4^2$ .  $E_{II}$  reaches its maximum value  $E_{II \max} = 55$  at  $t = 20$  hours, whence we have the two equations:

$$x A e^{-20 A x} = y A e^{-20 A y}, \quad (51)$$

$$55 = a_{3,4}(e^{-20 A x} - e^{-20 A y}). \quad (52)$$

As the third equation necessary for the unique determination of  $x$ ,  $y$ , and  $a_{3,4}$ , let us take the value of  $E_1$  at  $t = 40$  hours:

$$35 = a_{3,4}(e^{-40 A x} - e^{-40 A y}). \quad (53)$$

It follows from eqs. (51) - (53) that the pair  $x = 16$  and  $y = 36$  will be most favorable, whence  $s_3 = 4$  and  $s_4 = 6$ ; the  $a_{3,4}$  corresponding to them is 190. The coefficients  $a_{1,2} = 200$ ,  $s_1 = 13$  and  $s_2 = 18$  were found in a completely analogous way.

The formula so obtained

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$$E_1 = 200(e^{-0.34t} - e^{-0.05t}) + 190(e^{-0.037t} + e^{-0.072t}) \quad (54)$$

approximates the observed relation  $E_1(t)$  rather well. The observed values of  $E_1(t)$  are shown on Fig.45 by the solid lines, while those calculated by eq.(54) are shown

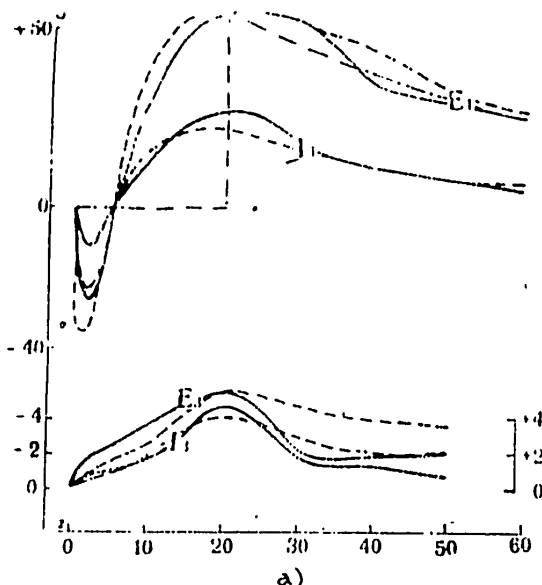


Fig.45 - Coefficients of Spherical

Analysis of  $D_{st}$  Variations

(Analysis II)

a) Hours

by the dashed lines.

The calculations of the induced field of  $I_1$ , corresponding to the field  $E_1$ , thus led to the calculation of four functions by eq.(47):  $\varphi_4(t)$ ,  $\varphi_6(t)$ ,  $\varphi_{13}(t)$ , and  $\varphi_{18}(t)$ .

The calculations were made with accuracy to the eight term of the series, which provided the fifth place in the value of  $\varphi(t)$ . The resulting function

$$I_1(t) = -a_{18,13}(\varphi_{18} - \varphi_{13}) + a_{6,4}(\varphi_6 - \varphi_4) = 200(\varphi_{13} - \varphi_{18}) + 190(\varphi_6 - \varphi_4) \quad (55)$$

is shown on Fig.45 by the dashed curve. It corresponds to the observed  $I_1$  (the solid curve) considerably better than the analogous

curve calculated by Chapman and Whitehead (cf.Fig.44).

Thus the calculations above presented confirm Chapman's hypothesis that the values  $q = 0.94$  and  $\kappa = 4 \times 10^{-12}$  well correspond to the first harmonic  $D_{st}$ .

#### Section 4. Determination of the Constants $q, s, \kappa_0$ (Lahiri Model)

The values found for  $q$  and  $\kappa$  are in good agreement with those previously calculated by us on the basis of the  $S_q$  variations (Bibl.9)  $q \cong 0.935$  and  $\kappa = 5 \times 10^{-12}$ . STAT But this agreement can hardly be regarded as an argument in favor of the correctness of the assumption of a uniform conductivity for a core of radius  $qR$ . An explanation

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of this agreement might rather be that in both cases variations of about the same velocities were investigated ( in the case of  $S_q$ , the gradients were of the order of  $2 - 3 \gamma$  an hour, in the case of  $D_{st}$ ,  $3 \gamma$  an hour on the first day of the storm and  $1 \gamma$  an hour on the second day), and consequently gave information about one and the same layers of the earth. In view of this it appeared advisable to evaluate the earth conductivity in accordance with the Lahiri model ( $q, \kappa = \kappa_0 \rho^{-s}$ ). Lahiri and Price (Bibl.47) selected the constant coefficients  $q, \kappa_0, s$  such that they satisfied the ratio of the amplitudes and phase difference of the harmonic  $P_3^2 S_q$ , and then separated, from the series so obtained, those values best corresponding to the ratio  $\frac{1}{E}$  for  $P_1 D_{st}$ . In contrast to this method of calculation, we made simultaneous use of the data of several harmonics of  $S_q$  and  $P_1 D_{st}$ , which enabled us to set up the number of equations sufficient for the determination of all the coefficients.

It follows from the Lahiri formulas that if the inducing part of the periodic field is represented by the expression

$$E(t) = E_n^m e^{i(\omega_m t + \phi_n^m)}, \quad (56)$$

and the induced part by

$$I(t) = I_n^m e^{i(\omega_m t + \phi_n^m)}, \quad (57)$$

then (omitting the indexes  $n$  and  $m$ ), we have

$$\frac{I}{E} = \text{mod } N(ix), \quad (58)$$

$$j - \epsilon = \arg N(ix), \quad (59)$$

where

$$N(ix) = \frac{nq^{2n+1}}{n+1} \frac{\Gamma(1-\nu)}{\Gamma(1+\nu)} \left(\frac{x}{2}\right)^{2\nu} \left(\cos \frac{\nu\pi}{2} + i \sin \frac{\nu\pi}{2}\right), \quad (60)$$

$$\nu = \frac{2n+1}{s-2}, \quad (61)$$

$$x = \frac{4Rq \sqrt{\pi \kappa_0 a_m}}{s-2}, \quad (62) \text{ STAT}$$

$$a_m = m^2$$



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and

$$\xi = \frac{2\pi}{24 \cdot 60 \cdot 60} \quad (63)$$

Equation (60) is approximate, and is true for  $v < 1$  and small enough values of  $\kappa$ . Denoting  $\frac{I_n^m}{E_n^m}$  by  $f_n^m$ , we have

$$\begin{aligned} f_n^m &= \frac{nq^{2n+1}}{n+1} \frac{\Gamma(1-v)}{\Gamma(1+v)} \left(\frac{x}{2}\right)^{2v} \\ &= \frac{nq^{2n+1}}{n+1} \frac{\Gamma(1-v)}{\Gamma(1+v)} \left(\frac{2Rq \sqrt{\pi \kappa_0 m \xi}}{s-2}\right)^{\frac{2(2n+1)}{s-2}} \end{aligned} \quad (64)$$

Selecting from among the coefficients of the expansion of  $S_q$  to a series in spherical functions, two terms with the same indexes  $n$  and different orders  $m_1$  and  $m_2$ , we obtain, on the basis of eq.(64):

$$\frac{f_n^{m_1}}{f_n^{m_2}} = \left(\frac{m_1}{m_2}\right)^{\frac{2n+1}{s-2}} \quad (65)$$

Equation (65) allows us to evaluate  $s$  from known  $f_n^{m_1}$  and  $f_n^{m_2}$ . From Table (15) of the above cited work (Bibl.9), it follows that  $f_2^1 = 0.43$ ,  $f_3^1 = 0.38$ ,  $f_2^2 = 0.53$ ,  $f_3^2 = 0.43$ .

Thus,

$$\frac{f_2^1}{f_2^2} = \frac{1}{2} \frac{s}{s-2} \text{ and } \frac{f_3^1}{f_3^2} = \frac{1}{2} \frac{7}{s-2} \quad (66)$$

The first of the eq.(66) leads to the value  $s \simeq 23$ , the second to the value  $s \simeq 33$ , which allows us to take the mean value  $s \simeq 26$ .

Putting in eq.(64),  $n = 2$ ,  $m = 1$ ,  $f_2^1 = 0.43$  and  $v = 0.21$ , we obtain the following equation connecting the unknown  $q$  and  $\kappa_0$  with each other:

$$\lg \kappa_0 = -13.27 - 25.70 \lg q. \quad (67)$$

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To obtain the second equation for the determination of  $q$  and  $\kappa_0$ , let us turn to  $P_{1D_{st}}$ . The formulas obtained by Lahiri are sufficiently convenient only in the case where the external, inducing field is approximated by an exponential function and

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the unique Heaviside function\*

$$E(t) = Ae^{-at}H(t), \quad (68)$$

where  $H(t) = 0$  for  $t < 0$ , and  $H(t) = 1$  for  $t \geq 0$ .

In this case the function  $\varphi(t)$  (for its definition, see Section 3) is represented by the infinite series

$$\begin{aligned} \varphi(t) = & \frac{nq^{2n+1}}{(n+1)\Gamma(1+\nu)} \left\{ \left(\frac{\tau}{4t}\right)^\nu M_{-\nu}(at) + 2\left(\frac{\tau}{4t}\right)^{1+\nu} M_{-1-\nu}(at) + \dots + \right. \\ & \left. + \frac{2[\Gamma(1-\nu)]^2}{\Gamma(2+\nu)\Gamma(1-2\nu)} \left(\frac{\tau}{4t}\right)^{1+2\nu} M_{-1-2\nu}(at) + \dots \right\}, \end{aligned} \quad (69)$$

where  $\tau = \frac{16\pi n^2 q^2 \kappa_0}{(s-2)^2}$  and  $M_{-\lambda}$  is the confluent hypergeometric function

$$M_{-\lambda}(at) = 1 - \frac{at}{1-\lambda} + \frac{(at)^2}{(1-\lambda)(2-\lambda)} - \dots \quad (70)$$

The series eq.(69) and eq.(70) are convergent for small values of  $\frac{\tau}{4t}$  and  $at$ . For the calculation of  $\varphi(t)$  by eq.(69), the observed curve of  $E_1(t)$  was replaced by the curve

$$E_1(t) = Ae^{-\alpha(t-20)}H(t),$$

repressed in Fig.45 by the dashed (dash-dot) line.

Here  $H(t) = 0$  for  $t < 20$ ;  $H(t) = 1$  for  $t \geq 20$  hours.

Starting out from the values  $E_1(20 \text{ hours}) = 55$  and  $E_1(60 \text{ hours}) = 25$ , the numerical values of the parameters were found:  $A = 55$  and  $\alpha = 0.02$  (if  $t$  is expressed in hours).

The approximate formula so obtained

$$E_1(t) = 55e^{-0.02(t-20)}H(t) \quad (71)$$

(the broken line in Fig.45) satisfactorily represents the observed curve for  $t > 20$

\* For convenience of exposition, the notation in the Lahiri formulas has been modified, and in addition, the misprints have been corrected.

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hours. The replacement of the smooth variation of the values of  $E_1$  during the first phase of the storm by a sudden increase at the moment of the maximum should not, in Lahiri's opinion, strongly distort the subsequent results. For assigned values  $n = 1$ ,  $s = 26$  and  $\alpha = 0.02$ , we get, for the instant  $t = 30$  hours (that is,  $t - t_0 = 10$  hours) the following numerical values of the quantities entering into eq.(69):

$$\tau = 3.5 \cdot 10^{16} q^2 x_0, \nu = 0.12, M_{-1,12} = 2.33, M_{-0,12} = 0.795, M_{-1,24} = 1.828, \frac{\tau}{4(t-t_0)} = 0.25 \cdot 10^{12}.$$

Confining ourselves in eq.(69) to three terms of the series, and bearing in mind that  $\phi(30 \text{ hours}) = \frac{18}{55} = 0.33$ , we have

$$0.62 = 9.8 x_0^{0.12} q^{3.24} + 2.72 \cdot 10^3 x_0^{1.12} q^{5.24} + 4.58 \cdot 10^{14} x_0^{1.24} q^{5.48}. \quad (72)$$

By graphic solution of eq.(67) and eq.(72) we get  $q = 0.925$  and  $x_0 = 4.0 \times 10^{-13}$ .

If the data for the term  $P_3 S_q$  are used in eq.(64), then  $q = 0.909$  and  $x_0 = 4.0 \times 10^{-13}$ .

Thus an investigation of the  $D_{st}$  and  $S_q$  variations has shown that if we start out from the Lahiri model of the earth, then the conductivity of the core varies by the law

$$x = 4.0 \cdot 10^{-13} \rho^{-26}, \quad (73)$$

Its radius is  $0.91R - 0.92R$ , and the thickness of the nonconducting upper shell  $d = 500 - 600 \text{ km}$ .

#### Section 5. Allowance for the Upper Conducting Layer

As stated in Section 1, Whitehead and Lahiri considered the question how much the parameters of the core would be modified if we allow for the effect of the currents induced in the upper conducting layers of the earth (oceans and wet soil).

They found the following formulas. If the conductivity of the upper spherical shell is  $\kappa_1$ , the radius of its lower surface is  $q_1$  and its thickness  $\partial = (1 - q_1)R$ , then the field in the nonconducting layer under the shell ( $q_1 > \frac{r}{R} > q$ ) is connected with

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the field of the earth surface by the equations:

$$\left. \begin{aligned} E_i &= E - \frac{C_0}{n} \frac{d}{dt} \{nE - (n+1)I\} \\ I_i &= I - \frac{C_0}{n+1} \frac{d}{dt} \{nE - (n-1)I\} \end{aligned} \right\} \quad (74)$$

Here  $E_i$  denotes the field of origin external with respect to the surface  $q_1 R$ ,  $I_i$  the internal field,  $E$  and  $I$ , as before, respectively the external and internal fields observed on the surface  $r = R$ , and

$$C_0 = \frac{4\pi R}{2n+1} \partial x_1. \quad (75)$$

The parameters of the core may be calculated in this case by the formulas of Section 4, if the values of  $E_i$  and  $I_i$  are used instead of the quantities  $E$  and  $I$ . We give below the calculations of the errors introduced in allowing for the upper conducting layers in the evaluations of  $s$ ,  $q$ , and  $\kappa_0$ , obtained by us.

If the field observed on the earth surface is a periodic function of time (cf. eq.(56) and eq.(57),

$$E(t) = E_n^m e^{i(\alpha_m t + \epsilon_n^m)}; \quad I(t) = I_n^m e^{i(\alpha_m t + j_n^m)},$$

then eqs.(74) are reduced to the form:

$$E_i = E e^{i(\alpha_m t + \epsilon)} - \frac{C_0 i \alpha_m}{n} \{n E e^{i(\alpha_m t + \epsilon)} - (n-1) I e^{i(\alpha_m t + j)}\}, \quad (76)$$

$$I_i = I e^{i(\alpha_m t + j)} - \frac{C_0 i \alpha_m}{n+1} \{n E e^{i(\alpha_m t + \epsilon)} - (n+1) I e^{i(\alpha_m t + j)}\}. \quad (77)$$

Here, to save space, the indices  $n$  and  $m$  after the quantities  $\epsilon$  and  $j$  are omitted.

Equation (76) allows us, after several transformations, to calculate, from assigned values of  $f = \frac{I}{E}$  and  $\epsilon - j$ , the values of  $f_i = \frac{I_i}{E_i}$  and  $\epsilon_i - j_i$ . We give below the values of  $f_i$  for four harmonics obtained under the assumption that the conducting shell consists of a continuous ocean, 1 km deep, of conductivity  $\kappa_1 = 5 \times$

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$$\times 10^{-11} \text{ CGS } (\kappa_1 \partial = 5 \times 10^{-6} \text{ and } C_0 \alpha_m = 4\pi R \kappa_1 \partial \varepsilon \frac{m}{2n+1} = 1.3 \frac{m}{2n+1}):$$

$$\begin{array}{cccc} m, n & \dots & 1,2 & 2,3 & 2,2 & 1,3 \\ f_1 & \dots & 0,42 & 0,40 & 0,43 & 0,34 \end{array}$$

A comparison of the values  $f_i$  so obtained with the initial values of  $f$  shows that allowing for the conductivity of the oceans in all cases decreases the ratio  $\frac{1}{E}$ .

The evaluation of the parameter  $s$  from corrected data, performed as described in Section 4, yielded the following results: from a comparison of  $f_3^2$  and  $f_3^1$ ,  $s = 30$ ; from  $f_2^1$  and  $f_2^2$ ,  $s > 100$ .

The latter value was rejected owing to the unreliable value  $j_2^2 - \varepsilon_2^2 = -15^\circ$ , and for convenience of calculations,  $s = 32$  was taken. A recalculation of eq.(64) with the new values of the constants:

$$v = 0,17, \frac{x}{2} = 0,62 \cdot 10^6 q \sqrt{x_0}, f_2^1 = 0,42,$$

$$v = 0,23, \frac{x}{2} = 0,89 \cdot 10^6 q \sqrt{x_0}, f_3^2 = 0,40$$

gave the following relations:

$$\left. \begin{array}{l} \text{for } P_2^1 \lg x_0 = -13,31 - 31,41 \lg q \\ \text{, } P_3^2 \lg x_0 = -13,61 - 32,44 \lg q \end{array} \right\} \quad (78)$$

Allowance was made for the influence of the surface currents on the  $D_{st}$  variations in the following manner. The internal part of the first harmonic was represented by the approximate formula

$$I_1 = 25e^{-0,04(t-20)} \quad (79)$$

(time in hours).

For  $n = 1$ ,

$$\begin{aligned} D &= \frac{d}{dt} [nE - (n+1)I] = \frac{dE_1}{dt} - 2 \frac{dI_1}{dt} = \\ &= 3,1 \cdot 10^{-4} e^{-0,02(t-20)} + 5,6 \cdot 10^{-4} e^{-0,04(t-20)}, \\ C_0 &= 1,34 \cdot 10^6, \end{aligned}$$

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$$C_0 D = 4,2e^{-0,02(t-20)} + 7,5e^{-0,04(t-20)}.$$

For  $t = 20$  hrs  $C_0 D = +3,3$   $E_i = 55 - 3,3 = 52$ ,  
 "  $t = 30$  hrs  $C_0 D = +1,6$   $E_i = 45 - 1,6 = 43$ ,  
 "  $t = 60$  hrs  $C_0 D = -0,4$   $E_i = 25 + 0,4 = 25$ .

Whence  $E_i$  may be approximated by the expression

$$E_i = 52e^{-0,018(t-20)}. \quad (80)$$

The revised value of  $\varphi(t)$  for  $t = 30$  hours is completely identical with the previous value. Indeed, for  $t = 30$  hours,  $I_i = 18 - 0.8 = 17$ , and  $\varphi(30 \text{ hours}) = \frac{17}{52} = 0.33$ .

For the new values of the constants ( $v = 0.1$ ,  $\alpha = 0.018$ ,  $\tau = 2.3 \times 10^{-16} \kappa_0 q^2$ ), the numerical quantities entering into eq.(69) are somewhat modified, and  $q$  and  $\kappa_0$  are connected with each other by the following equation:

$$0,67 = 10,8 \kappa_0^{0,1} q^{3,2} + 1,04 \cdot 10^{13} \kappa_0^{1,1} q^{5,2} + 9,14 \cdot 10^{13} \kappa_0^{1,2} q^{5,4}. \quad (81)$$

On solving eq.(81) in turn by the first and second equation of eqs.(78), we get the following results:

|                              | $P_2^1$              | $P_3^2$              |
|------------------------------|----------------------|----------------------|
| $q \dots \dots \dots$        | 0,949                | 0,921                |
| $\kappa_0 \dots \dots \dots$ | $2,5 \cdot 10^{-13}$ | $3,7 \cdot 10^{-13}$ |

Since the mean value of  $\kappa_0$  for the upper layers of the earth is obviously less than the value  $5 \times 10^{-6}$  CGS taken by us, the most probable values of the parameters  $q$  and  $\kappa_0$  may be expected to lie between the above values and those calculated in the preceding Section.

#### Section 6. External and Internal Parts of the Harmonic $P_3$ of the $D_{st}$ Field

In the present Section we shall discuss the application of the Lamb-Price induction theory to the third order harmonic of the  $D_{st}$  field. From eq.(42), which

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holds for the case  $\mu = 1$ , it follows that  $\varphi(t) > 0$ , since all terms of the series

$$\sum_{s=1}^{\infty} \frac{e^{-\alpha_s t} - e^{-\alpha_p t}}{x_s^2 (x_s^2 - x_p^2)}$$

are always  $< 0$ .

Indeed: for  $s < p$ ,  $\kappa_s^2 < \kappa_p^2$ , and  $\alpha_s < \alpha_p$ , consequently  $e^{-\alpha_s t} - e^{-\alpha_p t} > 0$ , and  $\kappa_s^2 - \kappa_p^2 < 0$ ; for  $s > p$ ,  $\kappa_s^2 > \kappa_p^2$  and  $\alpha_s > \alpha_p$ , consequently  $e^{-\alpha_s t} - e^{-\alpha_p t} < 0$ , and  $\kappa_s^2 - \kappa_p^2 > 0$ .

From this it follows that each term of the potential of the induced fields  $I_n^{mh}(t)$  will be of the same sign as the term of the inducing field  $E_n^{mh} (1 - e^{-\alpha_n^m t})$  corresponding to it, and the ratio  $\frac{I_n^{mh}}{E_n^{mh}}$  for any instant  $t$  must lie within the limits\*

$$0 < \frac{I_n^{mh}}{E_n^{mh}} < 1.$$

Accordingly, the negative ratio  $\frac{I}{E}$  for the harmonic  $P_3$  (cf. Table 8) is very surprising. The data obtained earlier by other authors (cf. Table 8), however, do not contradict our results. It will be seen from the Table that the negative values of  $\frac{I}{E}$  for the harmonics  $P_3$  and  $P_7$  are also obtained by Mc.Nish in the spherical analysis of  $D_m$ . The values of the coefficients  $E$  and  $I$  for the third and fifth harmonics in the Chapman-Whitehead analysis are at the limit of accuracy of the analysis. But all the same it does seem possible to assume that with these authors  $\frac{I_3}{E_3} < 0$ , while  $\frac{I_5}{E_5} > 0$ . A positive sign for  $\frac{I_3}{E_3}$  was obtained only once by S.Sh.Dolginov, in the analysis of the noncyclical variations. Thus the four spherical analyses of the aperiodic part of the storm field, made by different authors and from different starting ma-

\* The erroneous assertion of Chapman and Whitehead that  $\frac{I}{E}$  can vary within any limits from  $-\infty$  to  $+\infty$  is connected, as was shown later by Chapman and Lahiri, with the failure to allow for the free damping currents, which has already been mentioned in Section 1 of the present Chapter.

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terials, all speak in agreement in favor of the alternating sign of the ratio  $\frac{I}{E}$ :

$$\frac{I}{E} > 0 \text{ for } P_1 \text{ and } P_5; \quad \frac{I}{E} < 0 \text{ for } P_3 \text{ and } P_7.$$

For the harmonics  $P_3 D_{sf}$ , as for  $P_1$ , we calculated the internal field  $I_3$  corresponding to the external field  $E_3$ . For  $n = 3$ , eq.(42) assumed the form:

$$\varphi_p(t) = \frac{42}{4} q^7 \left[ 2 \cdot 10^{-4} t e^{-\alpha^2 t} - x_p^2 \sum_{s=1}^{\infty} \frac{e^{-2 \cdot 10^4 x_s^2 t} - e^{-2 \cdot 10^4 x_p^2 t}}{x_s^2 (x_s^2 - x_p^2)} \right], \quad (82)$$

where  $\alpha^2 = \frac{\kappa_s^2 \Lambda}{\pi^2}$  and  $x_3$  is a root of the equation

$$J_{2.5}(x_s) = \frac{1}{\sqrt{\pi x}} \left[ \left( \frac{3}{x^2} - 1 \right) \sin x - \frac{3}{x} \cos x \right] = 0. \quad (83)$$

From the tabulated values of the roots of Bessel equations we selected values of  $\alpha_p$  satisfactorily representing the observed function  $E_3(t)$  (see the solid curve in Fig.45):  $\alpha_4 = 0.035$  and  $\alpha_6 = 0.074$ .

The approximate curve so obtained:

$$E_3 = 80 (e^{-0.035t} - e^{-0.074t}) \quad (84)$$

(t in hours) is given on this same figure by the dashed line.

The internal induced field corresponding to  $E_3$

$$I_3 = 80 (\varphi_6 - \varphi_4), \quad (85)$$

while computational expressions for  $\varphi_4$  and  $\varphi_6$  are given by the series eq.(82).

Figure 45 also gives the observed field of  $I_3(t)$  (solid line) and the calculated field (dashed line). It will be seen from the figure that the calculated values of  $I_3$  plotted with reversed sign very satisfactorily represent the observed  $I_3$ . Our attention is struck by the very high agreement in the course of the curves  $E_3(t)$  and  $I_3(t)$ , which forces us to the conclusion that the Lamb-Price induction theory well explains the ratio of the external and internal field of the third harmonic of  $D_{st}$  STAT in absolute value, but cannot explain the negative sign of this ratio.



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The negative sign of  $\frac{1}{E}$  is possible under the condition that the medium in which the currents are induced possesses a high magnetic permeability ( $\mu \gg 1$ ). In this case, eq.(42) is of the form

$$\varphi_p(t) = \frac{nq^{2n+1}}{n+1} \left[ -\frac{(n+1)(\mu-1)}{n\mu+n+1} (1 - e^{-\alpha t}) + \right. \\ \left. + 2(2n+1)\mu k^2 \sum_{s=1}^{\infty} \frac{e^{-\alpha n_s t} - e^{-\alpha t}}{\{n(\mu-1)(n\mu+n+1) - k_{ns}^2 q^2 a^2\} (k^2 - k_{ns}^2)} \right], \quad (86)$$

where the terms containing  $\mu - 1$  express the magnetic induction. If the inducing field does not depend on the time ( $\alpha = 0$ ), then eq.(6) is transformed into the ratio, mentioned in Section 1, between the inducing and induced permanent magnetic fields

$$-\frac{nq^{2n+1}(\mu-1)}{n\mu+n+1}.$$

Beginning at certain sufficiently large values of  $t$ , the terms expressing the electromagnetic induction and containing the exponential functions  $e^{-\alpha t}$  become small, and  $\varphi(t)$  passes into the region of negative values. But such an explanation of the negative sign of  $\frac{1_3}{E_3}$ , expressed at one time by McNish, appears to be implausible, since:

- 1) the ratio  $\frac{1_3}{E_3}$  gives no indications whatever that  $\mu \gg 1$ ;
- 2)  $\frac{1_3}{E_3}$  maintains an almost constant negative value beginning at  $t = 0$  hours to the end of the storm ( $t = 60$  hours). McNish's other assumption as to the role of the nonuniform conductivity of the surface conducting layers appears to be equally unfounded. The nonuniform conductivity of the medium leads to the result that to one harmonic of  $P_n^m$  in the inducing field there correspond a considerable number of harmonics in the induced field, with coefficients depending on the law of distribution of the conductivity of the medium. This argument at one time was used by me to explain the great variations of  $\frac{1}{E}$  in the longitudinal terms of the field of  $S_q$  (Bibl.9). However, as shown by the calculation described in Section 5, the in<sup>ST</sup>ATice of the surface layers, even under an exaggerated assumption as to the conductivity

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of the ocean, is still relatively small and cannot change the sign of the ratio  $\frac{I}{E}$

Thus the only reasonable explanation of the negative sign of  $\frac{I_3}{E_3}$  would appear to be the assumption of the nonuniformity of the structure and conductivity of the deep layers of the earth, located 500 - 1000 km below the earth surface. In the light of recent calculations of conductivity made by A.N.Tikhonov (Bibl.32), and also taking account of the results of certain works devoted to the main magnetic field, such an assumption does not appear impossible: but since I have had no opportunity to make any quantitative calculations supporting this view, the question as to the negative ratios for the harmonics  $P_3$  and  $P_7$  must remain open. It would be extremely desirable to conduct a study of other aperiodic variations with various rates of change of the magnetic field to obtain a broader experimental material, which would help to solve this question. Since at the present stage of the problem, the ratio  $\frac{I}{E}$  of the harmonic  $P_3$  cannot be quantitatively interpreted, it would appear reasonable, all the same, to consider the deep layers of the earth as uniform (more exactly, to consider that the physical properties of these layers depend only on the radius) and to base our judgements on the conductivity of the earth only on the data of  $S_q$ ,  $D_{st}$ , and the P-storms.

#### Section 7. Variation of Conductivity with Depth and the Internal Structure of the Earth

The results of the calculation described in Sections 2 - 5 are summarized in Fig.46, on which, for purposes of comparison, the conductivity data obtained by Chapman and Lahiri are also given. It will be seen from the figure that the boundary of the nonconducting shell passes, according to the data of  $P_2^1$  and  $P_3^2 S_q$  and  $P_1 D_{st}$ , at a depth of about 400 km (see curves 2, 3, and 4), while the conductivity of the core  $\kappa$  attains an order of  $4 - 5 \times 10^{-12}$  CGS. Information that is somewhat different (and with the data in less good mutual agreement) follows from the analysis of the polar storms (the results of calculations are shown by crosses whose abscissas are the thickness of the nonconducting layer  $d$ , and whose ordinates are the value of  $\kappa$ ). The

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value of  $\kappa$ , according to this material, ranges from  $10^{-14}$  to  $2 \times 10^{-12}$ , while  $d$  ranges from 45 to 250 km. Still, if we leave out of consideration the considerably aberrant point  $d = 45$  km,  $\kappa = 2 \times 10^{-12}$ , we shall note a tendency for the conductivity to increase with the depth. Still smaller values of  $\kappa$  follow from the Chapman data on  $P_3^2 S_q$  ( $\kappa = 0.37 \times 10^{-12}$ ,  $d = 250$  km, curve I).

Such differences in the values of  $\kappa$  and  $d$ , calculated from different experimental material, would tend to indicate the arbitrary nature of any sharp boundary between the conducting core and the nonconducting shell. The curves showing the increase of conductivity with depth appear more plausible. The two curves calculated by me for the model  $q, \kappa_0, s$ , that is, without allowing for the surface currents (cf. the curves 5 and 6 on Fig.46) are in good agreement. Both of them indicate the low values of  $\kappa$  ( $\sim 10^{-12}$  CGS) at depth of 400 - 700 km, and the sharp increase of  $\kappa$  for  $d = 900 - 1100$  km. In still deeper layers a further increase of  $\kappa$  is found, but the data from depths over  $>1500$  km, obtained from short-period variations, must be considered unreliable. According to Chapman's calculations, in a sphere of uniform conductivity, 70% of the induced currents corresponding to the harmonic  $P_2^1$  and 77% of the currents corresponding to  $P_3^2$  are concentrated in the peripheral shell of the sphere,  $0.9Rq < r < Rq$ , that is, with our values of  $q$ , at depths of 400 - 1100 km. The increase of  $\kappa$  with depth, of course, also increases the downward propagation of the currents, but still, a substantial part of the currents would hardly be induced at levels deeper than 1300 - 1500 km.

The allowance for the currents induced in the upper layers of the earth crust, as would be expected from simple physical reasoning, increased the values of  $\kappa$  (see curves 7 and 8). Thus, for example, at depth 1100 km, the value of  $\kappa$  increased from  $50 - 65 \times 10^{-13}$  to  $70 - 110 \times 10^{-13}$ . In the upper layers of the conducting core ( $d < 700 - 800$  km), however, the corrected values of  $\kappa$  are somewhat smaller than the uncorrected values.

From the series of curves of  $\kappa(d)$ , calculated by Lahiri, Fig.46 gives the two

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that he considers the most probable. Curve 9 is calculated on the assumption  $\kappa_0 = 4 \times 10^{-14}$  CGS,  $q = 1$ ,  $s = 37$ , and  $\kappa_1 \partial = 2 \times 10^{-6}$  CGS. Curve 10 assumes  $\kappa_0 = 2.3 \times 10^{-13}$ ,  $q = 0.903$ ,  $s = \infty$ , and  $\kappa_1 \partial = 5 \times 10^{-6}$ . Both curves indicate the shallow

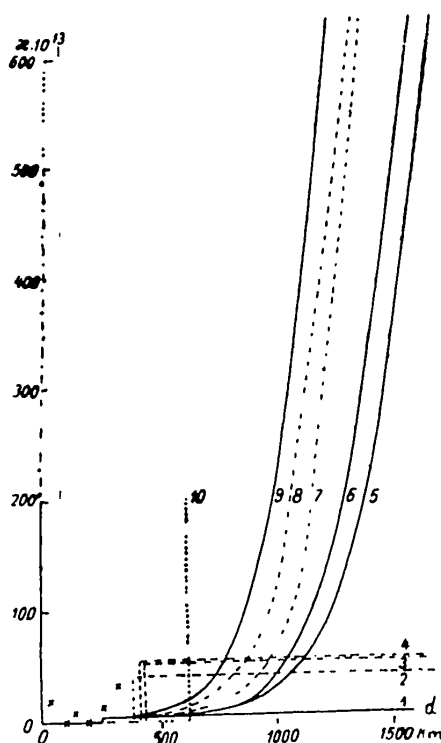


Fig.46 - Conductivity of the Earth ( $\kappa$ ) from the Data of Geomagnetic Variations

depth of the level at which  $\kappa$  increases ( $d \approx 600 - 700$  km). The discrepancy between the curves 7, 8, and 9, 10 can be explained not only by the different starting materials, but also by the different method of calculation. One of the Lahiri curves (not given on the figure) is calculated under the assumption  $\kappa_0 = 4 \times 10^{-11}$ ,  $q = 1$ , and  $s = 30$  (without allowing for the conductivity of the oceans) coincides almost completely with our own curve 7. Thus a consideration of Fig.46 shows that the following distribution with depth is the most probable. The surface layers (mainly on account of the oceans, which occupy 0.7 of the earth surface with a mean depth of 4.2 km) have a very high conductivity. The action of the ocean may be taken as

equivalent to a spherical layer of conductivity  $\kappa, \partial = 2 - 5 \times 10^{-6}$ . The conductivity of the first 200 km is roughly the same as that of the dry rocks on the earth surface, that is, it does not exceed  $10^{-14}$  CGS. The induction of currents at these depths may be practically disregarded. A substantial increase of conductivity begins at depths 200 - 300 km, while a sharp rise is located at  $d = 900 - 1000$  km, and a still steeper ascent of the curves is found at depths 1100 - 1200 km. The calculation of a model with a sharp surface of separation gives a moderate value of the conductivity. It is naturally greater than the actual value in the upper layers of

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the conducting core, and smaller than the actual value in the deep parts.

The distribution of conductivity so obtained does not contradict the modern idea on the structure of the earth. As will be seen from a comparison of Figs. 47 and 46, the region of the earth crust (to a depth of 60 - 80 km), is characterized by very low values of the conductivity which, in all probability, is connected with the anisotropic state of matter, and with the predominance of rocks with low iron content. A slight increase of conductivity begins from the upper layers of the outer shell downward, and a more substantial increase occurs in the lower layers of the shell, which are characterized by a change of chemical composition, an increase in the metallic content, and an increase in density and temperature.

A certain analogy is noted between the curves of  $\kappa(d)$  and the dependence of the velocity of longitudinal waves  $p$  on the depth. The second-order discontinuities of Repetti ( $d = 950$  km) and Gutenberg ( $d = 1200$  km) find their reflection in the curve of  $\kappa(d)$  as well: at these depths, as already remarked,  $\kappa(d)$  appreciably changes its direction. Thus the modification of the physical properties of matter at a depth of 900 - 1200 km, on the transition from the lithosphere to the barysphere, may be considered a confirmation of the change in the electric characteristics of the earth. It is true that the analogy between the curves of  $\kappa(d)$  and  $p(d)$  noted by us does not by any means indicate any parallelism of the curves. On the contrary, the increase of the gradient of the function  $\kappa(d)$  at depths of 900 - 1200 km is related to the decrease in the gradient of  $p(d)$ . The curve of temperature distribution given in Fig. 47 for comparison ( $T_G$  for Gutenberg and  $T_D$  for Jeffreys) and of density ( $P_1$  according to Gutenberg, and  $P_2$  according to Bullen) also confirm the changes in the physical properties of matter with depth.

This conclusion as to the variation of conductivity with depth may be considered a first approximation. The solution of the question of the negative sign of the third harmonic, and the more detailed study of the polar storms and other forms of local disturbances may possibly introduce substantial corrections in the conclusions

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so obtained. In order to judge of the nonuniformity of the deep layers it would seem advisable to apply the formulas of the plane problem to disturbances of local type

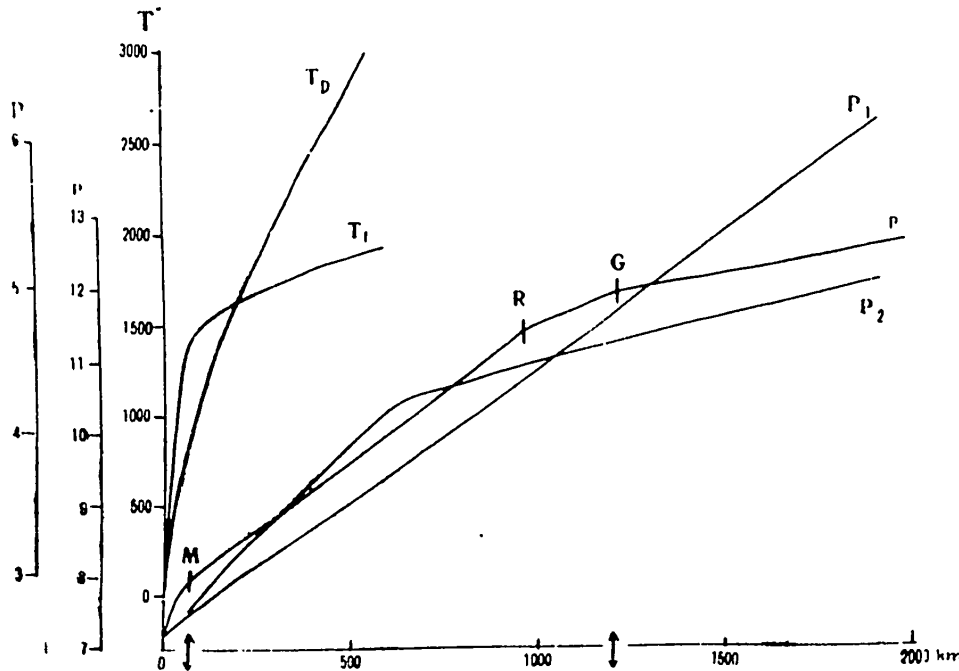


Fig.47 - Internal Structure of the Earth

(bays, pulsations). In this way it will be possible to obtain an extensive material on the conductivity of various depths and various areas of the earth.

### Conclusion

Section 1. It follows from all the above that the primary object of the present work, the construction of the electric currents causing the magnetic disturbances, has been accomplished. The calculations we have made are based on a sufficiently extensive empirical material (65 observatories) which allows us to expect that the field of calculated currents will be a good approximation to the field of observed variations. A consideration of the morphology of the disturbances, which preceded the calculation of the currents, shed light on certain questions of the structure and geographic distribution of the field. The most substantial of them are as follows:

1. The classification of magnetic storms and the separation of the storm field

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into its component parts. It appears to be most correct to divide storms into two main categories, worldwide and polar. Polar storms reach their maximum intensity in the auroral zone, and manifest themselves in the middle latitudes in the form of small bay-shaped disturbances. The field of polar storms depends on the local time and has no aperiodic part symmetric with respect to the earth axis, just as it has no prolonged after-effect.

The data on worldwide storms collected by us have confirmed the advisability of the separation of the regular parts, the  $D_{st}$  and the  $S_D$  variations, from the disturbance field, as proposed by Chapman. Worldwide storms, in our opinion, are always accompanied by polar storms superimposed on each other, and therefore the field of a worldwide storm should be divided by the means of the four-term formula

$$D_{st} + S_D + P + D_i.$$

2. The worked-up data on the  $D_{st}$  variations of a worldwide net of observatories have confirmed the fundamental features of the structure of the field described earlier by other investigators (position of the vector of the disturbance in the plane of the magnetic equator, low dependence of the field on the longitude, form of the  $D_{st}$  fluctuations of H and Z in the temperate latitudes). A more detailed examination of the question, however, by means of an evaluation of an estimate of the values of H and Z for the quiet intervals on days of worldwide storms, has shown that the  $D_{st}$  field does not have a sharp increase in the auroral zone, and varies smoothly from equator to the poles.

3. The  $S_D$  variations, on the other hand, do have a sharp increase in the auroral zone, and the form of  $S_D$  is determined primarily by the distance from that zone and by the local time. The  $S_D$  variations of the magnetic elements have been used to pinpoint the position of the zone. The data used by me have compelled me to place the position of the zone considerably further south than the Vestine zone. No dependence of the  $S_D$  variations on Universal Time was detected. The form and amplitude

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of the variations in the region near the pole have been elucidated.

4. It has been established that if the  $D_{st}$  field is considered as a function of  $\Phi$  and  $\tau$ , and  $S_D$  as a function of  $\Phi'$  and  $t_M$ , then there are no substantial anomalies in the geographic distribution of  $D_{st}$  and  $S_D$ . In particular, the complete normality of the disturbed variations at Huancayo has been specially noted.

5. A consideration of the geographical distribution of the  $S_D$  variations has shown that the linear current flowing in the auroral zone cannot explain the middle-latitude part of the field. For this reason it has been shown to be more correct to take the  $S_D$  system of currents as a system of surface spherical currents.

The currents of the polar disturbances have likewise been taken as surface currents, but extending only over the polar cap down to latitudes  $\Phi = 50^\circ$ .

Section 2. The extensive starting materials used made it advisable to calculate the currents of the disturbances by analytical methods.

The  $D_{st}$  currents were calculated on the basis of a spherical analysis of the  $D_{st}$  variations. For calculating the currents I used an expansion of the storm potential into a series of Bessel functions. The complexity of the geographic distribution of the  $S_D$  variations preventing me from using spherical analysis, and forced me to turn to the method of surface integrals. The method of calculating the external and internal parts of the potential from values of the potential and Z component, assigned on the surface of a sphere, proposed in 1941 by Vestine, has been further developed in the present work. A method has been given for calculating the density of the surface currents from the potential assigned on the surface of the sphere. The method is based on the extrapolation of the values of the external potential for points inside the sphere, the calculation from it of the current density from it (by solving a Fredholm equation of the second kind, to which the external Dirichlet problem leads), and extrapolation of the function of current density for external points at the distance of the hypothetical current layer. An analogous method of solution may be applied to the calculation of the internal current systems. All the laborious

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operations in the course of the calculations of the currents by the integral method are now reduced to a single type and allow use of the very same overlays to facilitate the calculations. A consideration of the accuracy of the method has shown that the errors of the mathematical operations themselves are considerably less than the accuracy of the initial geomagnetic data. The accuracy obtained as a result of the calculations performed is sufficient for the construction of a general picture of the currents.

The method can yield good results only in the case where the radius of the current-carrying layer differs little from the earth radius. But since in most geomagnetic problems, both for the main and irregular fields, this condition is satisfied, it follows that the method may be recommended for the investigation of a number of questions, as for example the construction of the currents responsible for the secular variations, the study of magnetic anomalies, and the like. The possibility is not excluded that the integral method may also find application in other branches of geophysics, replacing spherical analysis in the case of fields of rather complex structure.

Section 3. The current system of the  $D_{st}$  variations consists of current lines parallel to the circles of latitude. It differs substantially from the well known system of Chapman by the fact that there is no crowding of lines in the polar zones, and by the different signs of the current functions in the northern and southern hemispheres. On the basis of spherical analysis of the  $D_{st}$  variation, I also made a calculation of the equatorial ring current, which yielded the following results: radius of ring  $a = 3.8R \pm 0.8R$ ; current strength  $I = 7 \times 10^5$  amp. These values were calculated on the basis of the ratio between the harmonic coefficients of terms of different orders and is in good agreement with the ideas of other authors on the ring current.

The current systems of  $S_D$  variations, like the corresponding Chapman systems, consists of four current eddies. The intensity and location of the polar currents proved to be different from what would follow from the Chapman data. The signs of

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the current functions, which are different in each pair of eddies, also constitute a substantial difference.

The current system of the P-storms resembles the polar part of the  $S_D$  currents. A calculation of the linear current flowing along the zone from the data of the  $S_D$  variations is in good correspondence with the crowding of the current lines on the map of surface  $S_D$  currents. A calculation of the linear current, based on four pairs of Arctic stations, allowed me to establish the fluctuations of the height of the linear current, of its intensity, and of its position throughout the course of the day. The results proved somewhat different from the analogous results of other investigators.

In this work the seasonal and 11-year fluctuations of the  $S_D$  and  $D_{st}$  currents have been considered. It has been found that the intensity of the  $D_{st}$  currents varies rather regularly throughout the 11-year cycle, displaying the lag in the epochs of the maxima by 1 to 2 years with respect to the solar maxima, which is characteristic of all phenomena due to corpuscular radiation. The seasonal fluctuations of the  $D_{st}$  current can likewise be explained from the point of view of the corpuscular origin of the ring current: the maxima in the equinoctial epoch may be explained by the Corti effect, the additional maximum in summer by the Bartels effect.

The fluctuations of the  $S_D$  currents are much more complex, and are different at different latitudes: the 11-year fluctuations in the intensity of the middle-latitude eddies do not display a good correspondence with the march of the solar indexes. Small displacements of the lines of the centers of the middle-latitude eddies have been found. The 11-year fluctuations of the polar eddies are considerably greater with respect to their intensity and to the position of the auroral zone in years of high activity, there is a marked increase in the intensity of the currents, and there is also a shift in the position of the zone toward lower latitudes. The seasonal fluctuation of the middle latitude eddies are small, while those of the polar STAs are considerable. An intensification of the current in the equinoctial months and

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summer has been found, together with a shift of the zone from higher latitudes in the summer to lower latitudes in the equinox.

Section 4. The current systems calculated for individual instants of individual storms are in good correspondence with the average pictures depicting the regular part of the disturbance field. The 26 individual cases considered showed that in all cases the position and signs of the current eddies are the same as in the average systems. It is true that the form of the current lines and the intensity of the currents varies not only from storm to storm, but also from hour to hour within wide limits. But all the same the consideration of the individual storms confirmed the physical reality of the concept of a stable current system embracing the entire earth, and causing the magnetic disturbances.

Section 5. Data on the disturbed-day structure of the ionosphere have been adduced to judge the location of the currents of magnetic storms. A calculation of the  $D_{st}$  and  $S_D$  variations of the ionospheric parameters has shown that the greatest and most regular variations take place in the  $F_2$  layer. The  $D_{st}$  variations of ionization density of the  $F_2$  layers display a two-phase character at all latitudes; in the high and middle latitudes, the first phase is characterized by an increase in ionization density, the second by a decrease. In the equatorial latitudes, on the contrary, the first phase is negative, the second positive. This lack of correspondence between the  $D_{st}$  variations of the magnetic field and  $f^0F_2$ , and also the great regularity of the  $D_{st}$  of the magnetic elements - which is absent in the  $D_{st}$  of the ionospheric parameters, forces complete abandonment of any possibility of explaining the  $D_{st}$  variation by ionospheric processes, and, on the contrary, supports the hypotheses of an extra-ionospheric ring current.

The  $S_D$  variations of  $f^0F_2$  are similar in their geographical distribution to the  $S_D$  variation of the magnetic elements: at latitudes higher than  $\phi = 40^\circ$ ,  $S_D$  represents a simple wave with a maximum in the evening and a minimum in the morning, while in the low latitudes, this relative position of the extreme values is reversed.

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This resemblance makes it possible to explain the  $S_D$  variations by currents in the  $F_2$  layer. The possible mechanisms of formation of these currents have been investigated. The values of the conductivity of individually layers of the ionosphere in the direction of the magnetic field ( $\sigma_0$ ) and in two directions perpendicular to it ( $\sigma^I$  and  $\sigma^{II}$ ) have been reviewed, and it has been found that for the  $F_2$  layer, the conductivity along the velocity of the mechanical displacement of the gaseous masses ( $\sigma^{II}$ ) is greater than the conductivity of the dynamo effect ( $\sigma^I$ ). Thus it would appear that the dynamo effects can hardly have the decisive role in the formation of the  $S_D$  currents. The currents induced in the ionosphere by the alternating magnetic field of the equatorial ring current are likewise very small. In all probability the greatest part in formation of the current is played by the currents of latitudinal direction which either arise owing to the drift effect or owing to the motion of the earth in the field of the ring. The experimental data known from the literature as to the vertical motions or the vertical gradients of the ionization density in the  $F_2$  layer, which are particularly increased during the time of a disturbance, allow us to consider that the drift of charges under the action of the magnetic and gravitational fields is not eliminated, owing to the equilibrium between the force of gravity and the partial pressure in the gas, and consequently, may be adduced for the explanation of the magnetic variations. I have schematically shown here that, owing to the  $S_D$  variations of the ionization density of the  $F_2$  layer, currents of latitudinal direction may lead to the formation of current systems resembling the middle-latitude part of the  $S_D$  currents.

Section 6. The separation of the observed potential of the  $D_{st}$ ,  $S_D$  - variations, and P-storms into an external and an internal part led to the following results.

The ratio  $\frac{I}{E}$  for the first harmonic of the  $D_{st}$  field is about 0.40. Considering the internal field to arise by induction from the external field, I calculated by the Lamb-Price formula that the conductivity of the earth core (corresponding to  $\frac{I_1}{E_1} = 0.40$ )  $\kappa = 4.4 \times 10^{-12}$  CGS, and that its radius is  $0.94R$ . If the influence of

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the superficial conducting layers is taken into account, however, and the conductivity is assumed to increase with depth by the exponential law:

$$\kappa = \kappa_0 \left( \frac{r}{\rho} \right)^{-S},$$

then

$$S = 26, \rho = 0.91R \text{ and } \kappa_0 = 4 \cdot 10^{-13}.$$

The curve relating  $\kappa$  to the depth, calculated by this formula, discloses the sharp rise of  $\kappa$  at the depth 900 - 1200 km at which Gutenberg and Repetti found discontinuities in the variation of the velocity of longitudinal seismic waves. Thus the information on  $\kappa$  obtained from magnetic data does not contradict the modern idea of the structure of the Earth.

The mean ratio  $\frac{E}{E+1}$  for P-storms was found to be 0.86. To use these data to judge the structure of the Earth, I solved the Lamb problem in cylindrical coordinates. The numerical values of  $\kappa$  according to the data of the  $\frac{I}{E}$  of various terms of the P-storm potential ranges between  $10^{-12}$  and  $10^{-14}$ . In spite of the great scatter of the values of  $\kappa$ , these values do not contradict the conclusions drawn from the first harmonic of the  $D_{st}$  variations.

The ratio  $\frac{I}{E}$  for the third harmonic of the  $D_{st}$  variations was found to be negative. Comparison of this conclusion with the data of other authors compels belief in its authenticity, but it does not appear to be possible to verify it from the viewpoint of the induction theory (under the assumption of the spherical symmetry of  $\kappa$ ). The possibility is not excluded that this result may indicate the existence of great nonuniformities of conductivity in the depths of the Earth. At the present time, however, this question still remains open. The ratio  $\frac{E}{I+E}$  for the  $S_D$  variations ranges from 0.79 for the middle latitudes to 0.89 for the high latitudes.

This value differs appreciably from that adopted by Chapman for the entire Earth,

$\frac{E}{I+E} = 0.6$ . The latitudinal dependence of  $\frac{E}{I+E}$  for the  $S_D$  current may similarly be considered as an indication of the absence of spherical symmetry in the dis-

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tribution of  $\kappa$ .

Section 7. All the arguments of morphological and physical character advanced in the present work compel us to accept the following mechanism of formation of magnetic storms. A powerful corpuscular stream arriving from the Sun acts in several ways on the geomagnetic field. First, interacting with the geomagnetic field, it leads to the formation of an equatorial ring of currents, whose field produces the  $D_{st}$  variations of the magnetic elements. Second, as a result of the drift of charges, or of some other mechanism, electric currents are generated in the upper part of the ionosphere at both high and low latitudes. Since the conditions in the ionosphere are substantially related to the solar altitude, a characteristic property of these currents is their dependence on the local time (the  $S_D$  variations). And third, a certain part of the particles, becoming detached from the body of the ring (or from the corpuscular stream itself), are directed under the action of the magnetic field toward the polar regions, where they penetrate deep into the Earth atmosphere (to the levels of the E and D layers of the ionosphere), causing auroral displays and intense magneto-ionospheric disturbances there. The polar magnetic storms connected with the immediate processes in the ionosphere are of very local nature, and their course is governed by local time.

Thus the field of world-wide storms always contains three components: the  $D_{st}$  variations, the  $S_D$  variations, and the P-storms. The fluctuations of the  $D_{st}$  and  $S_D$  systems, and the superimposition of P-storms differing in form and intensity, gives the fluctuation of the magnetic elements a complex, random character during world-wide storms. The storm field also has smaller irregular fluctuations ( $D_i$ ), which may perhaps be connected with some ionospheric processes of more local type.

Less energetic solar streams do not lead to the formation of an equatorial ring nor of ionospheric currents. The particles of such streams, detaching themselves immediately from the body of the stream, proceed to the Polar regions and cause Polar storms there. Thus the P-storms can be observed even in the absence of world-

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wide storms.

It goes without saying, of course, that this work does not answer all questions connected with the construction of the current systems of magnetic storms. The possible investigations in the domain of the morphology of the disturbance field and of the disturbed ionosphere have not been exhausted, and no physical explanation of the origin of the currents has been worked out. In this work we have only considered the macro-structures of the disturbances in the geomagnetic field and in the ionosphere, and we have merely marked out paths along which the solution of the questions of the formation of the currents may be sought.

In studying the morphology of the field of magnetic storms in the future particular attention must be paid to the individual fluctuations, to the irregular part of the field  $D_1$  on which the present work has no bearing. It is necessary by elucidating the statistical regularities, or by analyzing the individual phenomena to confirm, on a large amount of material, the proposition here enunciated to the effect that the individual fluctuations during worldwide storms, noted on the magnetograms in the middle latitudes, are the result of the superimposition of polar storms piled one on top of the other.

In the present work we have collected a large amount of factual material on the regular variations, and have given a representation of it in the form of current systems. This material, it seems to us, may be of great use in the solution of the following practical questions: reduction of the magnetic observations to the middle of the year, and short-term magnetic forecasting. The methods of reduction existing at the present time, for days that are not magnetically quiet, are very imperfect, and are particularly unsuitable for high latitudes. The systematization of the regular disturbed variations, and the calculation of the current systems, will help to evaluate the possible deviations, during a disturbance, of the values of the magnetic elements from the normal, and to interpolate (or extrapolate) the observatory data for the points of observation. It also seems to us that the representation of the

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average current system of a magnetic storm will help to increase the accuracy of the geographical distribution of the degree of disturbance by time of day, and to evaluate the amplitude of the possible fluctuations, both of which accomplishments are necessary for short-time forecasting. Thus it is desirable to continue the consideration of the morphology of disturbance presented in this work, and to give it a form convenient for utilization in practical problems.

To a still greater extent it is necessary to continue the study of the morphology of the disturbed ionosphere. The regular variations of  $D_{st}$  and  $S_D$  of the disturbed ionosphere that have been considered in this work should be calculated for the largest possible number of years and points of observation. The study of the morphology of the disturbed ionosphere is not only of theoretical value but is also of great practical value for the maintenance of shortwave radio communication through the ionosphere. In view of this fact it is inadequate to have merely a schematic representation of the geographical distribution or time fluctuations of  $S_D$  and  $D_{st}$ , but it is necessary to have a distinctly elucidated picture of each observatory separately. Of particularly great interest is the study of the polar ionosphere, the processes in which are the cause of the polar magnetic storms and of the high-latitude part of the  $S_D$  variations.

As for the method of calculating the current systems from the data of geomagnetic variations, the integral method developed in this work has enabled us to obtain the numerical values of the external and internal potential, and of the current function, with sufficient accuracy. In future, however, in cases where it may be sufficient to obtain only a rough picture of the distribution of currents, or when the sparsity of data makes it impossible fully to utilize all the advantages of the method, it will still be possible to use an approximate method employing the analytical technique for solving the fundamental problems of geomagnetism on the basis of an extensive empirical material.

The question of the mechanisms of excitation of electric currents in the iono-

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sphere may be considered as having merely been posed in the present work.

I shall consider my object achieved if the present work attracts attention to the study of magnetic storms and thereby encourages the further development of the theory of geomagnetic variations.

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